Numerical Study of One Prey-Two Predator Model Considering Food Addition and Anti-Predator Defense

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Abstract. This article examines the interaction between prey populations, juvenile predators, and adult predators. A mathematical model that considers adding food and anti-predators was developed. The equilibria of the existing system are that the system has four equilibria points with conditions suitable for the locale. Numerical simulations were carried out to describe the dynamics of the system solution. Based on numerical simulations, the varying of parameter causes changes in the extinction of prey or survival of prey populations, juvenile predators, and adult predators. Addfood parameters (A) encourae Hopf Bifurcation and Saddle-node bifurcation Numerical continuity results show that Hopf bifurcation occurs when the parameter value $A = 1.00162435$ and when the parameter value $A = 2.435303$ Saddle-node bifurcation occurs.

Keywords: *Mathematic Model, Simulation, Equilibria*

1 Introduction

The phenomena in ecology, especially in agriculture are interesting to study. Pests are plant-disturbing organisms ranging from roots, stems, and leaves. Planthonppers or insects are one of the pests for plants that can damage crops. Therefore, it is necessary to improve pest control effort that are environmentally friendly. Dragonflies can be used by farmers to control various pests, especially in rice fields. Dragonflies are natural enemies of planthoppers. An adult dragonfly can consume hundreds of leafhoppers every day [1]. Dragonfly juvenile that lives in water prey on leafhoppers or small insects. Adult dragonflies will eat on the other pests on the surface of the water ranging from leafhoppers to insects. The phenomenon of natural enemies can affect the development of pest populations, thereby protecting crops from damage and crop failure. The reduction in the number of pests makes dragonfly as predator look for alternative food other than leafthoppers. Additional food is needed to keep adult predatory species from becoming extinct so that the balance between pest and both of predator species is controlled [2].

Mathematical models provide more insight into prey-predator population dynamics. Many researches have developed in different models of one prey-two predator or two prey-one predators [3,4,5,6], and a stage-structure for predators [7,8,9,10]. Model [10] also discusses the Rosenzweig-MacArthur model with the effect refuge for immature prey. Tang and Xiao [11] has introduced anti-predator behaviour for predator. Panja [12] proposes effects anti-predator defense in a prey-predator model using Ratio-dependent functional response to make the model more realistic. Prasad [13], Ulfa [14], and Zhu et al [15] incorporate food additional for predators in their model. Based on the background of several studies of prey-predator models, we developed a one prey-two predators' model to provide an overview of the dynamics of change through numerical simulations.

1.1 Mathematical Model

1.1.1 Basic Model

A one prey-two predator model will be constructed based on several assumptions. First, the predators consist of a stage-structure, namely juvenile predators and adult predators. The prey model with two predators considers the effects of addfood and anti-predator defenses for juvenile predators.

The level of predation of predators on prey affects changes in population growth. Holling [16] has acquainted functional responses that depend only on the prey species, namely Holling types I,II, and III. Savitri [17], has introduced a mathematical model with two predators using ratio-dependent functional response. Salamah, et.al [18] considered an anti-predator in a modified Leslie-Gower model with Beddington-

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DeAngelis functional response. Apriyani, et al [19] discussed ratio-dependent functional responses and also considered the effect of anti-predator defense in which prey can attack juvenile predators.

1.1.2 Construction Model

Based on predator-prey model proposed [17-19], we developed three models with different functional responses. In this paper we evolve of the prey population, juvenile predators, and adult predators model to delve the dynamics and also incoporates addfood and anti-predator defense. The model is obtained as follows

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{\alpha x y_2}{m+x+nA},$$
$$\frac{dy_1}{dt} = \frac{\rho x y_2}{m+x+nA} - \beta y_1 - \gamma x y_1,$$
$$\frac{dy_2}{dt} = \beta y_1 - \mu y_2 \quad (1)$$

Suppose x is the population density of prey, y_1 represents of juvenile predator, and y_2 represents of adult predator the population density. All parameters relevant to the system (1) are positive and are described in Table 2. System (1) uses Holling type II functional response on predation rates of adult predators on prey.

1.2 Material and methods

We have studied local stability and system solutions and conceived of the existence of several equilibria points. Numerical simulations were performed using Runge-Kutta integrals of fourth-order which were displayed through phase portraits with Python 3.8 Software. To view the complete dynamics of the system, numerical continuity with MatCont is used, The parameters selected for continuity denote the effect of changing the stability of the equilibria point. Change in bifurcation dynamics showed by variations in add food parameter for predators, namely A.

2 Result and Discuss

2.1 Equilibria

The equilibria point is received from the equilibria solution of the system (1). The growth rates of each population is zero. The equilibria point describes the constant solution of the system. System (1) has four equilibrium points, as follows.

Table 1. The existence and type equilibria point.

The equilibria	Type of equilibria
E_0 = (0, 0, 0)	The extinction all population
E_1 = (k, 0, 0)	The extinction of both predator, namely juvenile predator and adult predator
$E_2 = (x^{2*}, y_1^{2*}, y_2^{2*})$	The interior equilibria point
$E_3 = (x^{3*}, y_1^{3*}, y_2^{3*},)$	The interior equilibria point

We provide sufficient conditions for the existence of a nonnegative root. For x^* we get equation.

$$\omega_1(x^*)^2 + \omega_2(x^*) + \omega_3 = 0 \quad (2)$$

With $\omega_1 = \eta\mu$,

$\omega_2 = A\eta n\mu + \eta m\mu - \beta\rho + \beta\mu$,

$\omega_3 = A\beta n\mu + \beta m\mu$

In contrast to all of the equilibria point, $E_2 = (x^{2*}, y_1^{2*}, y_2^{2*})$ and $E_3 = (x^{3*}, y_1^{3*}, y_2^{3*},)$ indicates that the prey, juvenile predators and adult predators can coexist.

2.2 The Stability of Equilibria

Point stability is executed by linearizing the system (1) using the Jacobi matrix [20].
The Jacobi matrix for equilibria point E_0

$$J(E_0) = J(0,0,0) = \begin{bmatrix} r & 0 & 0 \\ 0 & -\beta & 0 \\ 0 & \beta & -\mu \end{bmatrix}.$$

Eigenvalue of the Jacobi matrix $J(E_0)$ are $\lambda_1 = r > 0$, $\lambda_2 = -\beta < 0$, and $\lambda_3 = -\mu < 0$. Since there are positive eigenvalue, The equilibria point E_0 is also unstable, saddle point.

The Jacobi matrix for equilibria point E_1

$$J(E_1) = J(k,0,0) = \begin{bmatrix} -r & 0 & -\frac{\alpha k}{An+k+m} \\ 0 & -\eta k - \beta & \frac{\rho k}{An+k+m} \\ 0 & \beta & -\mu \end{bmatrix}.$$

With eigenvalue $\lambda_1 = -r < 0$, the other two eigenvalue λ_2 and λ_3 are determined by the quadratic equation $\lambda^2 - T_1\lambda + D_1 = 0$. Because $T_1 < 0$ and $D_1 > 0$, then the eigenvalue for $\lambda_2, \lambda_3 < 0$ are met, The equilibria E_1 is locally asymptotically stable. This indicates that the juvenile predators and adult predators are extinct so there is only a prey population.
The stability of the equilibria point E_2 and E_3 will be asymptotically stable if certain conditions are fulfilled. It shows that all population, namely pre population, juvenile predators, and adult predators will coexist.
Therefore, the extinction of juvenile predators and adult predators or extinction of all populations does not occur in the long term. We next investigated the stability of E_0, E_1, E_2, and E_3 by numerical simulation.

3 Numerical Simulations

The analytical solution of system (1) is not easy to decide. Therefore, numerical simulation can be performed to delve the system behavior. To see the behaviour of the solution as a whole, it is identified the local stability of equilibria point E_1 and $E_{2.3}$ by varying the value of the addfood parameter (A) and deliniate change occured of bifurcation diagram by MatCont. Next, we provide simulation applying the parameter in Table 2, all of the equilibria exist.

Table 2. Parameter model used for simulation.

Parameter	Description	value
r	The intrinsic growth rate of prey	6.6
k	Carrying capacity of prey	3.5
α	The attack rate of adult predators on prey	1.6
A	Addfood	2.416 65
n	Relative ability of the both predator to detect additional food to prey	1.2
m	The coefficient of environmental protection for the prey	1.3
β	The growth rate of adult predators comes from the transition of juvenile predators into adult predator	0.7
ρ	Proportionality constant, the conversion efficiency of predation	1.1
η	Anti-predator	0.3
μ	Natural death	0.2

3.1 Bifurcation Diagram

In numerical continuity, varying the value of the addfood parameter (A) is to indicate changes in the stability of several the equilibria point. Figure 1 presents the bifurcation diagram of the solution of the system (1) which is varying the parameter addfood for both of the predator.

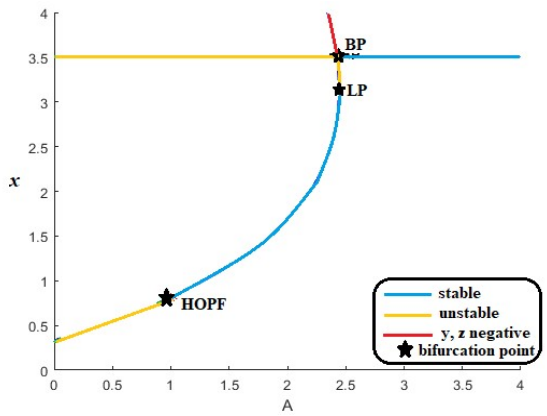

Fig. 1. Bifurcation Diagram of System (1).

a. Hopf Bifurcations

We perform numerical continuation by selecting the parameter of addfood for the predator which is A, the other parameter values remain constant from Table 2. Therefore, the emergence of Hopf bifurcation is driven by addfood parameter for the predator (A). The Hopf bifurcation point occurs when $A = 1.00162435$ as shown in Figure 2. The interior equilibria point for E_2 = (0.812441, 2.999504, 10.498265) when the Hopf bifurcation occurs.

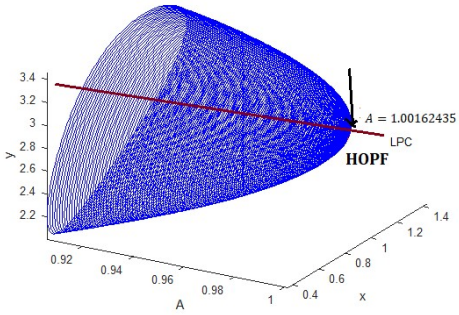

Fig. 2. Hopf Bifurcation of System (1) in equilibria point $\boldsymbol{E_2}$ at $\boldsymbol{A = 1.0016243}$

a. Saddle-node phenomenon

The results of the numerical continuation of the parameter $\boldsymbol{A}$ show the changes that occur in the stability behavior at the interior equilibria point ($\boldsymbol{E_2}$). The Bifurcation diagram in Figure 1 shows for $\boldsymbol{2.416667 < A < 2..435303}$. There are two interior equilibria points where the difference in stability is ($\boldsymbol{E_2}$) and ($\boldsymbol{E_3}$). The phenomenon of the emergence of LP namely *Limit Point* shows that there is a *Saddle-node* (*Fold*) bifurcation which is driven by the parameter of addfood for the predator.

3.2 Phase Portraits

The dynamics of the solution of the system (1) have been observed in Figure 4 – 6 of the above by phase portrait. The figure 3 show the dynamic solution of the system (1).

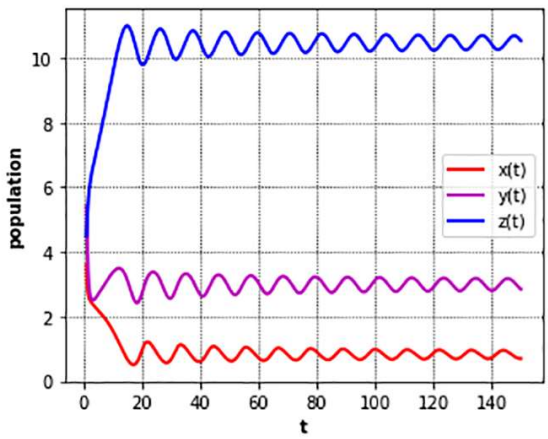

Fig. 3. Timeseries of System (1).

Based on the paremeter value in Table 2 and choose addfood parameter $A = 1.00162435$, the equilibria point E_0, E_1, and E_2 are exist. The value of each point of equilibria are E_0 = (0, 0, 0), E_1 = (3.5, 0, 0), and E_2 = (0.812441, 2.999504, 10.498265). Therefore, E_0, E_1 and E_3 are unstable and E_2 is asymptotically stable.

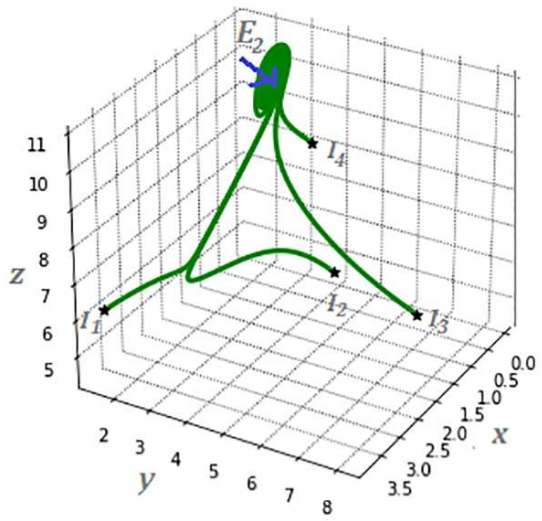

Fig. 4. The Phase Portrait of System (1) leads to the

3.3 Interior equilibria point for E_2 at $A = 1.00162435$.

The initial value in Figure 4 shows the initial density of all population. With initial value $I_1[3.8; 1.5; 6.2]$, $I_2[0.1; 3.9; 4.6]$, $I_3[2.1; 8.1; 6.5]$, $I_4[1.1; 4.3; 9.2]$. For different initial value, all numerical solution of system (1) convergen to the interior equilibria point (E_2), and E_2 is an asymptotically stable. The result are shown in Figure 4.

We use initial value $I_1[3.8; 1.5; 6.2]$, $I_2[2.1; 8.1; 6.5]$, $I_3[1.1; 4.3; 7.2]$, all numerical solution of system (1) lets to the extinction of the juvenile predator and adult predator. The equilibria point (E_1) is an asymptotically stable. The eigen value of $J(E_1)$ are $-3.9985 \times 10^{-9} + 0.5756I$, $-3.9985 \times 10^{-9} - 0.5756I$, and -1.4335.

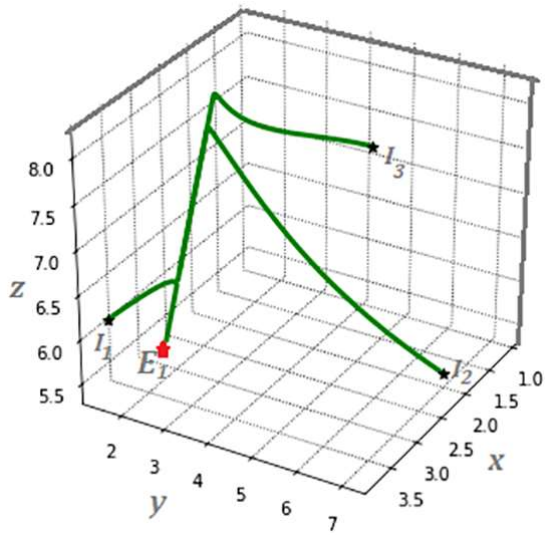

Fig. 5. The Phase Portraiet of System (1) in equilibria point E_1 at $A = 1.0016243$

3.4 Double stability phenomenon.

Another interesting dynamics behavior to observe as shown in Figure 5 is the appearance of two different stability of the equilibrium points, known as the bi-stability phenomenon. This phenomenon occurs at $A = 1.00162435$. There are two stabilities in the solution of system (3) namely the interior equilibria point (E_2) and the juvenile predator and adult predator extinction equilibria point (E_1) which are locally asymptotically stable. This phenomenon shows that the system (3) has a double stability

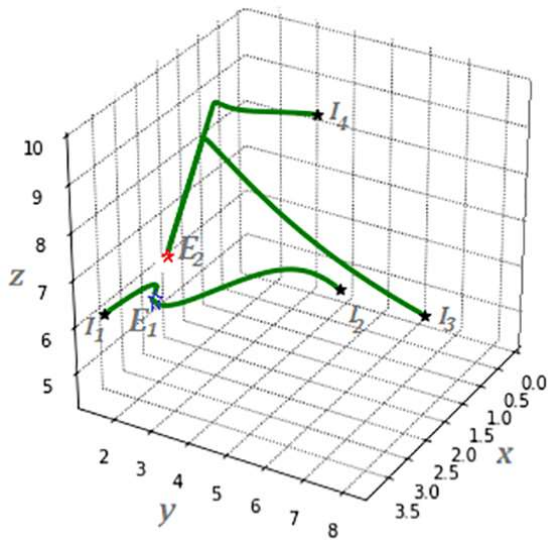

Fig. 6. The Phase Portriet of System (1) in equilibria point E_1 and E_2 at $A = 1.0016243$

The equilibria point $E_0 = (0, 0, 0)$, $E_1 = (3.5, 0, 0)$, and $E_2 = (2.8, 1.6, 5.7)$ are exist. We use the same initial value with $I_1[3.8; 1.5; 6.2]$, $I_2[0.1; 3.9; 4.6]$, $I_3[2.1; 8.1; 6.5]$, and $I_4[1.1; 4.3; 9.2]$. The numerical solution of system (1) lets to the extinction of the juvenile predator and adult predator (E_1) for initial value I_1 and I_2. The equilibria point (E_1) is an asymptotically stable. The eigen value of $J(E_1)$ are -6.6, -1.9, and -9.3×10^{-9}. Initial value I_3 and I_4 convergen to $E_2 = (2.8, 1.6, 5.7)$. The interior equilibria point (E_2) is asymptotically stable. The eigen value of $J(E_2)$ are -4.7, -1.7, and -0.002.

This study shows the addfood for predator has a meaningful on the dynamics of the prey population, juvenile predators and adult predators population. Therefore, it is prime for ecological models to combine predation affect both of the population interaction and additionalfood for predator. However, another possibilityis to maintain the ecology at coexistence working if possible uses the other parameter.

Finally, we notice that the all population can live together with some conditions described. The addfood of the predator helps two populations to survive for a long period of time without extinct.

4 Conclussion

1. We have numerically studied the population models of preys, juvenile predator, and adult predators model with Holling Type II functional response by considering addfood and anti-predator defense. The model has four equilibria points, namely the trivial point (E_0), the extinction point both of the predator (E_1), and the survival of prey, juvenile predators, and adult predators point (E_2 and E_3) are stable under certain conditions. Increasing the parameter addfood for both of predators (juvenile and adult) may stabilize equilibria point (E_1). It is can prevent extinction on the population of the juvenile predator.

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