E3S Web of Conferences **236**, 02040 (2021) https://doi.org/10.1051/e3sconf/202123602040
ICERSD 2020

Thermal Effect of Thin Elastic Plates

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Abstract. Thermal effect refers to the heat released or absorbed by the object in the changing process at a certain temperature. In this process, the stress inside the material will change. Thermal stress refers to the stress produced when the temperature changes, because of the external constraints and the internal constraints between the various parts, so that it can not completely free expansion and contraction.

1 Introduction

To solve the thermal stress, it is necessary to determine not only the temperature field, but also the displacement, strain and stress fields. Time independent temperature field is called steady temperature field, which causes steady thermal stress. The temperature field changing with time is called unsteady temperature field [1-3]. In the traditional method, the temperature distribution is obtained from the heat conduction equation and boundary conditions, and then the displacement and stress are calculated according to the thermoelastic equation [4].

Elastomer generally refers to the material that can be restored to its original state after removing external force. However, the material with elasticity is not necessarily an elastomer. Elasticity is an important branch of solid mechanics. It studies the deformation and internal force of elastic body under the action of external force and other external factors, also known as elastic theory. Elasticity is the basis of material mechanics, structural mechanics, plastic mechanics and some interdisciplinary subjects, and elastic theory is widely used in construction, machinery, chemical engineering, aerospace and other engineering fields. When the external force does not exceed a certain limit, the object will return to its original state after removing the external force [5, 6].

In this paper, according to the properties of symplectic system and integral transformation, the original problem is reduced to Saint Venant solution problem and non-zero eigenvalue eigenvalue eigenvalue problem reflecting local effect.

2 Solution method

The relationship between internal force and strain, bending moment and curvature are

$$\begin{aligned} N_x &= K(\varepsilon_x + \upsilon\varepsilon_\theta) \\ N_\theta &= K(\varepsilon_\theta + \upsilon\varepsilon_x) \\ N_{x\theta} &= K(1-\upsilon)\varepsilon_{x\theta}/2 \end{aligned} \tag{1}$$

and

$$\begin{aligned} M_x &= D(\kappa_x + \upsilon\kappa_\theta) \\ M_\theta &= D(\kappa_\theta + \upsilon\kappa_x) \\ M_{x\theta} &= D(1-\upsilon)\kappa_{x\theta} \end{aligned} \tag{2}$$

According to the small deformation theory, strain, curvature and displacement can be described as follows:

$$\begin{aligned} &\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} - \frac{w}{r}, \varepsilon_{x\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}, \\ &\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \kappa_\theta = -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}, \kappa_{x\theta} = -\frac{1}{r}\frac{\partial^2 w}{\partial x \partial \theta}. \end{aligned} \tag{3}$$

The above equations should satisfy the following boundary conditions:

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$$w\cdot[D\partial_x^3 w+D\frac{(1-\upsilon)}{r^2}\partial_x\partial_\theta^2 w+N_x\,\partial_x w]\Big|_{x=x_0}=-w\cdot Q_x\big|_{x=x_0}=0$$
$$\partial_x w\cdot D(\partial_x^2 w+\frac{\upsilon}{r^2}\partial_\theta^2 w)\Big|_{x=x_0}=-\phi_x\cdot M_x\big|_{x=x_0}=0 \tag{4}$$

Applying the variational principle, we have displacements and stress components

$$\varsigma(x)=C_1\cos\alpha_{1n}x+C_3\cos\beta_{1n}x$$
$$\xi(x)=C_1e^{\alpha_{2n}x}\cos\beta_{2n}x+C_2e^{\alpha_{2n}x}\sin\beta_{2n}x \tag{5}$$
$$\eta(x)=C_3e^{-\alpha_{2n}x}\cos\beta_{2n}x+C_4e^{-\alpha_{2n}x}\sin\beta_{2n}x$$

$$\alpha_{1n}=\{\frac{N_x}{D}-\frac{n^2}{r^2}+[(\frac{N_x}{D}-\frac{n^2}{r^2})^2-4(\frac{Eh}{r^2D}+\frac{n^4}{r^4})]^{1/2}\}^{1/2}/\sqrt{2}$$
$$\beta_{1n}=\{\frac{N_x}{D}-\frac{n^2}{r^2}-[(\frac{N_x}{D}-\frac{n^2}{r^2})^2-4(\frac{Eh}{r^2D}+\frac{n^4}{r^4})]^{1/2}\}^{1/2}/\sqrt{2} \tag{6}$$

So the final dual governing equation are obtained as

$$\dot{\psi}=\begin{bmatrix}0 & -1 & 0 & 0\\ \partial_x^2 & 0 & 0 & 1/D\\ \frac{Eh}{r^2}+N_x\partial_x^2 & 0 & 0 & -\partial_x^2\\ 0 & 0 & 1 & 0\end{bmatrix}\psi \tag{7}$$

The symplectic orthogonal relation between the eigensolutions is

$$<\varphi_i,\varphi_j>=\int_0^{x_e}\varphi_i^T J\varphi_j dx=0$$
$$\psi(x,\theta)=\sum_{n=0}^{\infty}\left[a_n\varphi_n(x)\ e^{in\theta}+b_n\varphi_{-n}(x)\ e^{-in\theta}\right] \tag{8}$$

3 Numerical example

Fig. 1 and Fig. 2 exhibit stress response of the domain in a particular temperature field. The results show that the material has a strong dependence on the change of temperature. Even in the constant temperature field, the change of temperature conditions also has a significant impact on the stress and deformation of the material.

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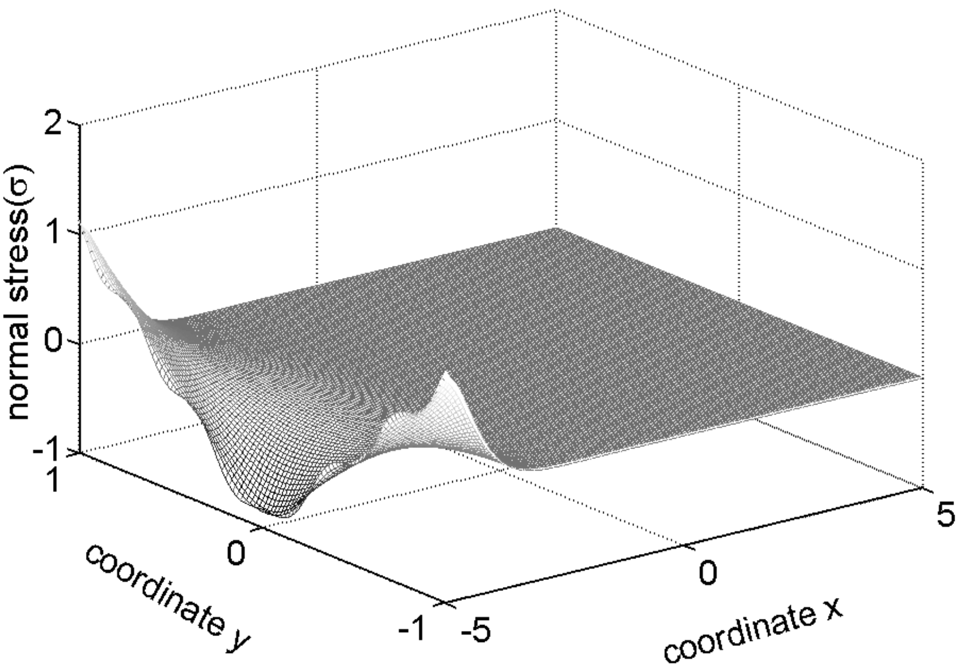

Fig 1. Normal stress

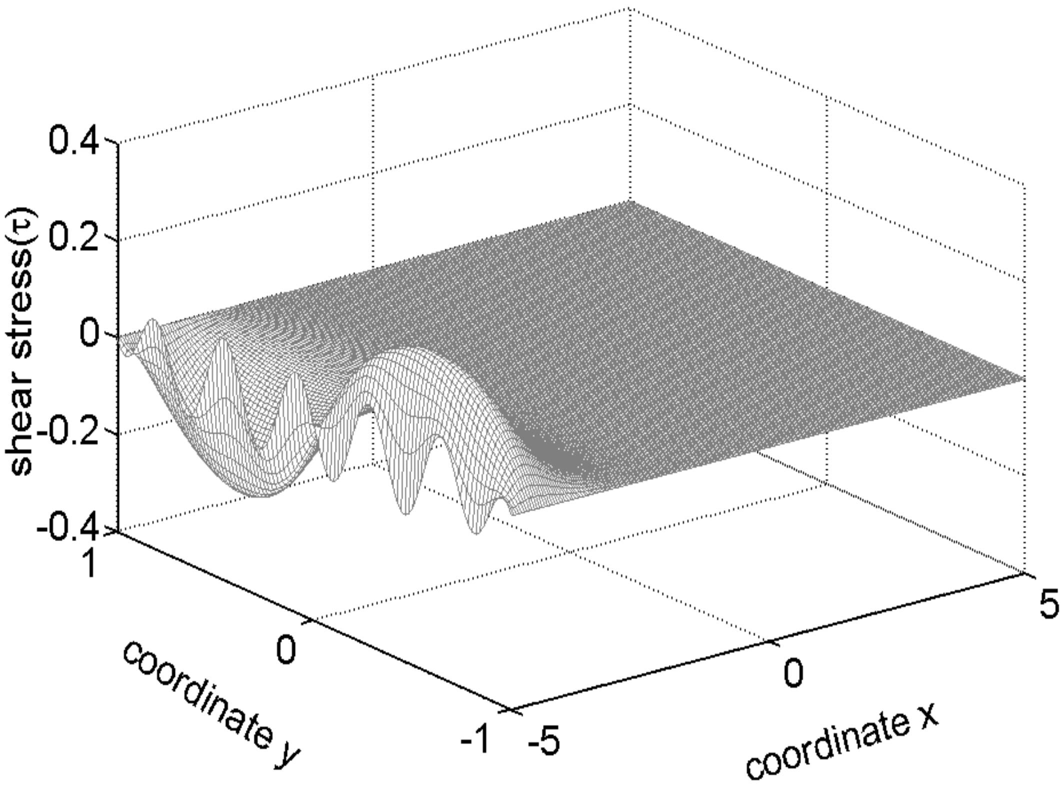

Fig 2. Shear stress

4 Conclusion

The elastic material is obviously dependent on the temperature condition. Based on the variational principle and Laplace integral transformation, the basic equations of thermoelasticity are transformed into solving the problem of nonhomogeneous boundary conditions. Using this technique, the stress and deformation of elastic materials in a specific temperature field are discussed.

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