Numerical Analysis of Cylindrical Shell under Axial Impact

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Abstract. The shell mainly resists the external load by the mid plane stress distributed uniformly along the thickness, rather than the bending stress varying along the thickness. Compared with the traditional flat plate, the shell can make full use of the material strength, so it has greater bearing capacity. In water conservancy projects, shells are widely used, such as double curvature flat shell gate, arch dam and so on. Thin shell theory is a classical theory in shell.

1 Introduction

Thin shell theory is based on the assumption of straight normal [1, 2]. These assumptions include: (1) the normal strain perpendicular to the middle plane is extremely small and negligible. (2) The normal of the middle plane is kept as a straight line, and the right angle between the normal of the middle plane and its vertical segment remains unchanged, that is, the shear strain in the two directions is zero. (3) The normal stress on the section parallel to the middle plane is far less than that on the vertical plane, so its effect on deformation can be neglected. According to the theory of elasticity, the basic equation of thin shell can be established by using the above hypothesis. They include: the equilibrium equation of the surface in the shell, the geometric equation, the physical equation and various boundary conditions on the boundary.

The allowable disturbance is determined by the statistics and probability distribution of the environment in which the structure is located, and the required safety

standards and economic factors must be considered [3-5]. Various disturbances are mainly initial geometric defects, load eccentricity and material inhomogeneity. The prescribed limits of motion require no interference with the normal function of the structure. The service life of the structure is mainly determined by the use value and economy [6-8]. According to Hoff's definition, it is necessary to have a detailed understanding of the structure's motion state to judge whether the structure has dynamic buckling or not.

2 Solution method

The in-plane deformation potential energy density, bending deformation potential energy density, external force work and kinetic energy density of the shell are as follows:

$$\Pi_{\varepsilon} = \frac{1}{2} K (\varepsilon_x^2 + \varepsilon_{\theta}^2 + 2\nu \varepsilon_x \varepsilon_{\theta} + \frac{1-\nu}{2} \varepsilon_{x\theta}^2) \quad (1)$$

$$\Pi_k = \frac{1}{2} D (\kappa_x^2 + \kappa_{\theta}^2 + 2\nu \kappa_x \kappa_{\theta} + 2(1-\nu) \kappa_{x\theta}^2) + \frac{N_x}{2} (\partial_x w)^2 \quad (2)$$

and

$$\Pi_t = \frac{1}{2} \rho h \left(\frac{\partial u}{\partial t} \right)^2 + \frac{1}{2} \rho h \left(\frac{\partial v}{\partial t} \right)^2 + \frac{1}{2} \rho h \left(\frac{\partial w}{\partial t} \right)^2 \quad (3)$$

So the Lagrange function, the Hamiltonian action and the dynamic boundary conditions are

$$L = \iint \left[\frac{\rho h}{2} (\partial_t u)^2 - \frac{Eh}{2} (\partial_x u)^2 - \frac{Eh}{2r^2} w^2 + \frac{2(1-\nu)}{r^2} (\partial_x \partial_{\theta} w)^2 - \frac{N_x}{2} (\partial_x w)^2 \right] r d\theta dx \quad (4)$$

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$$N_x = \begin{cases} -N & 0 \leq x \leq ct, t \leq l/c \\ 0 & ct < x \leq l, t \leq l/c \\ -N & 0 < x \leq 2l - ct, t > l/c \\ -2N & 2l - ct < x \leq l, t > l/c \end{cases} \quad (5)$$

and

$$\begin{cases} w \cdot [D \partial_x^3 w + D \frac{(1-\nu)}{r^2} \partial_x \partial_\theta^2 w + N_x \partial_x w] \Big|_{x=x_0} = -w \cdot Q_x \Big|_{x=x_0} = 0 \\ \partial_x w \cdot D(\partial_x^2 w + \frac{\nu}{r^2} \partial_\theta^2 w) \Big|_{x=x_0} = -\phi_x \cdot M_x \Big|_{x=x_0} = 0 \end{cases} \quad (6)$$

For the two cases of free support, fixed support and simple support, the above formula can be rewritten as follows:

$$\begin{cases} u = \frac{\nu}{r^2} \partial_r^2 w \\ w = 0 \\ M_x = D(\partial_x^2 w + \frac{\nu}{r^2} \partial_\theta^2 w) = 0 \end{cases} \quad (x = x_0) \quad (7)$$

$$\begin{cases} w = \frac{(1-\nu)}{r^2} \partial_x \partial_\theta^2 w \\ Q_x = D \partial_x^3 w + N_x \partial_x w = 0 \\ M_x = D(\partial_x^2 w + \frac{\nu}{r^2} \partial_\theta^2 w) = 0 \end{cases} \quad (x = x_0) \quad (8)$$

and

$$\begin{cases} Q_x = D \partial_x^3 w = 0 \\ w = 0 \\ \phi_x = \partial_x w = 0 \end{cases} \quad (x = x_0) \quad (9)$$

3 Numerical example

Fig. 1 and Fig. 2 describe the external pressure of the first and second order. It can be seen from the figure that the

critical load decreases with the increase of external pressure. For the case of only external pressure, buckling can also occur under axial tension impact.

Fig 1. External pressure of the first order

Fig 2. External pressure of the second order

4 Conclusion

The dynamic stability of a structure is defined as the dynamic buckling of a structural system in dynamic equilibrium, which is considered as the critical dynamic load if the motion reaches the specified limit within the required service life of the structural system after receiving a small allowable disturbance.

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