Theoretical and experimental study on vibration modal for simply supported bridge

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Abstract. For the highway bridges with small or medium span, a vehicle will stay on the bridge after the excitation. So the measured values of the vibration characteristics of the bridge are related to the vehicle-bridge system. In order to analyse the natural vibration characteristics of simply supported bridge and the loaded frequency of vehicle-bridge system, theoretical analysis was carried out firstly, then the simply supported beam calculation model was built based on finite element method, and the results were compared with the measured values. The finite element model of a mass block-simply supported beam system was built by regarding the vehicles as a concentrated mass block attached. Both the vibration modals of the bridge system and the loaded frequencies of the vehicle-bridge system under different vehicle positions were obtained by software analysing. As a result, the variation characteristics of the loaded frequency for this simply supported bridge were extracted and discussed through the comparison of the results from both finite element method and the measured.

1 Introduction

With the sustainable development of national economy and engineering technology, achievements of bridge construction change with each passing day. The amount of existing bridges is increasing rapidly since a large number of newly built bridges were putting into operation. In recent years, the on-site testing technology is used widely in bridge test and evaluation. As an on-site test for highway bridges with short or medium span, their vibrations were excited usually by a single vehicle, and the vehicle will stay on the bridge after excitation, so the measured values of the bridge under the action of a single vehicle are the vibration characteristics of vehicle-bridge system. In other words, the measured frequencies are the loaded frequencies of the vibration system. The literatures show that the frequency differences between the vehicle-bridge system and bridge structure cannot be neglected [1], of even larger [2] than the changes of structural damage.

In this paper, the authors firstly analysed a simply supported steel beam theoretically, then a finite element model was built based on the software of Ansys, and the

analysis results were compared with the measured values. Next, the finite element model of a mass block-simply supported beam system was built by regarding the vehicles as a concentrated mass block attached. At last, the analysis results from software were compared with the tested values, and the authors gave some useful conclusions which are appreciated for reference.

2 Profile of the simply supported beam

The simply supported steel beam with a span of 690mm between two points, and with a rectangular cross section of 50mm (width) $\times$ 8.5mm (height) is shown in Fig.1. The elastic modulus and Poisson ratio of the steel beam are 2.06×10^5 MPa and 0.3 respectively. Furthermore, the mass density of steel is 7850kg/m³.

3 Modal frequency analyses

3.1. Theoretical analysis

As a simply supported beam with uniform section, its natural frequencies can be described as following [3]:

Fig.1. Sketch of a simply supported steel beam

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$$f_n = n^2 \pi^2 \sqrt{\frac{EI}{mL^4}} \quad n=1,2,3,\dots(1)$$

Where f_n is the frequency of each order, EI is the flexural stiffness of cross section, m is mass for per unit length, and L is the span between two support points.

According to the physical property parameters and size parameters of the steel beam described as above, the first 5 order natural vibration frequencies can be calculated as in Table 1.

Table 1 The first 5 order natural frequency from theoretical analysis (Hz)

order	frequency	order	frequency
1	41.449	2	165.797
3	373.044	4	663.189
5	1036.225		

3.2 Finite element analyses

The finite element model using commercial software Ansys [4-5] was built as shown in Fig.2. The conventional beam element was used to simulate the steel beam, and block Lanczos algorithm was adopted for modal extraction.

Fig.2 Finite element model for steel beam

The vibration modal parameters can be obtained by running Ansys modal analysis module. The first 5 order natural vibration frequencies are showing in Table 2. And the related vibration deformation diagrams are showing in Fig.3.

Table 2 The first 5 order natural frequency from Ansys (Hz)

order	frequency	order	frequency
1	41.469	2	165.840
3	373.030	4	663.880
5	1035.200

(a) The 1st order

(b) The 2nd order

(c) The 3rd order

(d) The 4th order

(e) The 5th order

Fig.3. The first 5 order vibration deformation diagram

Fig.4. The forced vibration test for steel beam

3.3 Experimental test

The ZHT-1 integrated experiment teaching system was adopted for vibration test. The size and the material parameters are the same with the Ansys model mentioned above. In this paper, the natural frequencies of the simply supported steel beam were obtained by free vibration and forced vibration methods. The test results were the mean values of the both methods, showing in Table 3. And Fig.4 is the picture of forced vibration test.

Table 3 The first 5 order natural frequency from test (Hz)

order	frequency	order	frequency
1	41.653	2	167.009
3	371.523	4	657.912
5	1028.969		

3.4 Results analysis

Three approaches are involved in carrying out the natural frequencies of the simply supported steel beam. The results are showing in Table 1 to 3. Supposing the experimental test results are the real values, the relative errors were calculated shown in Table4.

Table 4 Relative error for natural frequency (%)

order	Finite element method	Theoretical analysis method
1	0.441744892	0.489760641
2	0.699962277	0.725709393
3	0.405627646	0.409395919
4	0.755116186	0.802082953
5	0.605557602	0.705171876

From Table 4, it shows that the relative errors are all less than 1% for both methods. Furthermore, the errors from finite element method are less than theoretical analysis, it means, the natural frequencies from finite element method coincide well with test results. So the finite element method using beam element from Ansys can be adopted to simulate the steel beam vibration.

4 Loaded frequency analyses

4.1 Finite element method

The finite element model of a mass block-simply supported beam system was built by regarding the vehicles as a concentrated mass block attached. For convenience of comparing the results from experimental tests, the simply supported steel beam was divided in 16 equal parts, including 17 nodes, as shown in Fig.1. It's 40.295 gram for each mass block. For simulating the passing vehicles on the bridge, the position of the mass block changed along the length of the beam. The loaded frequencies of the mass block-beam system were carried out by running the Ansys software. The first 5 order of loaded frequencies are showing as Fig. 5.

Fig.6. Mass block-simply supported beam system

Fig.5. The loaded frequency of mass block-beam system

4.2 Experimental test

The ZHT-1 integrated experiment teaching system was adopted for vibration test again. The 80.59 gram mass block was fixed at note No.5 firstly, then at No. 8, No. 9 and No. 10 at last in order to simulate the passing vehicle on the bridge. Fig.6 is the picture of vibration testing when the mass block is at note No.5.

The natural frequencies of the mass block-simply supported steel beam system were obtained by free vibration and forced vibration methods respectively. The test results were the mean values of both methods, showing in Table 5.

Table 5. The first 5 order natural frequency for mass block-beam system from test (Hz)

Note order \	5	8	9	10
1	41.079	40.760	40.150	40.680
2	162.323	165.250	165.810	165.240
3	369.380	365.200	364.180	365.210
4	662.790	656.260	662.390	656.240
5	1023.540	1027.92	1007.460	1027.910

4.3 Loaded frequency results analysis

4.3.1 Finite element results analysis

Both the loaded frequencies of the mass block-beam system and the natural frequencies of the steel beam are obtained by Ansys software analysis. The relative errors

of the two kinds of frequencies can be calculated and shown in Fig. 7.

Fig.7. Relative error for loaded frequency by Ansys

4.3.2 Test results analysis

Regarding the limited positions can fix the mass block on the steel beam, four different positions carried out for loaded frequency testing. Comparison the loaded frequencies of the mass block-beam system with the natural frequencies of the steel beam, the relative errors can obtain, showing in Table 6.

Table 6. The relative error of loaded frequency for test (%)

Vibration order	Note NO.5		Note NO.8		Note NO.9		Note NO.10	
	Ansys	Test	Ansys	Test	Ansys	Test	Ansys	Test
1	1.71	0.94	3.21	1.71	3.33	3.18	3.21	1.90
2	3.23	2.12	0.49	0.36	0.01	0.02	0.49	0.36
3	1.54	0.98	2.20	2.10	3.13	2.37	2.20	2.10
4	0.05	0.01	1.54	1.00	0.05	0.07	1.54	1.00
5	1.61	1.13	0.99	0.70	2.94	2.68	0.99	0.70

5 Discuss and conclusions

In this paper, the authors selected the simply supported beam as research object. Firstly, the theoretically analysis was carried out for natural frequencies, and then the finite element model was built using Ansys. By regarding the vehicle on the bridge as mass block, the finite element model was built for mass block-beam system as well. And the comparison of the loaded frequencies of the mass block-beam system with the natural frequencies of the beam was carried out.

The following conclusions can be drawn from the analysis above:

(1)The loaded frequencies are slightly less than the natural frequencies of the simply supported beam. Fig 7 shows that the relative error drops of each mass block position except two ends. The same conclusion can be drawn from the theoretical analysis equation (1).

(2)Curve shape of the relative error for loaded frequency of each vibration order is similar with the vibration deformation diagram. For example, the double-wave curve shape of the 2nd order in Fig 7 is the similar with the 2nd order of Fig5.

(3)The position of the vehicle on the bridge has influence on the loaded frequencies. From Fig7, it shows that the relative error is larger than other position when the mass block is on the position of maximum amplitude. In other words, the position of maximum amplitude is the most sensitive position for loaded frequency.

(4)The beam element of Ansys can be adopted to simulate the vibration characteristics of simply supported bridge. Both Table 4 and 6 show that the frequency results from Ansys are very close to the results from experimental test. So the beam element from Ansys can be used to simulate the loaded frequency of vehicle-bridge system.

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