Optimal paper web weight control system based on the Pontryagin's maximum principle

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Abstract. The paper describes the stages of paper production, considers the structure of a paper-making machine. Questions related to the proof and use of the Pontryagin's maximum principle in the theory of optimal control are considered. Optimal paper web weight control system based on the Pontryagin's maximum principle is presented. Adaptive learning methods for modeling nonlinear systems represent some of the latest advances in adaptive algorithms and machine learning techniques designed to model and identify nonlinear systems. Real-world problems always involve a certain degree of non-linearity, which makes linear models a suboptimal choice. This article may be of interest to research engineers and practitioners in the study and application of control systems using adaptive regulators. This book serves as an essential resource for researchers, graduate students and doctoral students working in the field of machine learning, signal processing, adaptive filtering, nonlinear control, system identification, cooperative systems, and computational intelligence. This book may also be of interest to the industry market and practitioners working with a wide range of nonlinear systems.

1 Introduction

Modern production is characterized by the complexity and high speed of technological processes, which is why complex automation takes an important place in industrial production [1,2].

Automatic control is widely used in many technical and biotechnical systems to perform operations that are not feasible by a person due to the need to process a large amount of information in a limited time, to increase labor productivity, quality and accuracy of regulation, to free a person from controlling systems operating in conditions of relative inaccessibility or hazardous to health [3-6].

The industry associated with the release and manufacture of pulp and paper products is considered the most complex branch of the forestry industry, since the production process is associated with chemical, mechanical and wood processing.

The production of paper and paper products is traditionally one of the leading sectors of the Russian economy. Until recently, the paper business was dominated by large manufacturing companies (various pulp and paper mills), which have a structure of widely branched holdings. However, with the advent of new technologies, this market is becoming

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more accessible for the profitable operation of small companies. The basis of the concept of investing in paper production at the level of small and medium-sized businesses are the following factors:

The papermaking market in Russia is growing steadily, adding on average 6% annually. This positive dynamics is caused by the fact that Russian business has become more competitive in the international paper market due to the factor of reducing overall costs. The growth of the market is also facilitated by an increase in demand for high-quality paper products from the corporate sector and private clients in Russia itself. In particular, a surge in demand for high-quality domestic packaging of industrial and household goods was noted.

The level of automation in the production of paper, including from waste paper, allows implementing a full production cycle at any acceptable scale. Moreover, an important fact here is that it is possible to purchase automatic lines designed to produce several types, varieties of products. On the market of such production lines for papermaking, there is a wide range of models - with a production capacity of 1 ton to 50 tonnes per shift, with a relatively low price range - from 1.5 to 70 million rubles.

It should be added that, despite the widespread computerization of the economy, the demand for paper products is growing steadily. This is primarily caused by the fact that the use of paper is gaining new uses, both in commerce and in everyday life.

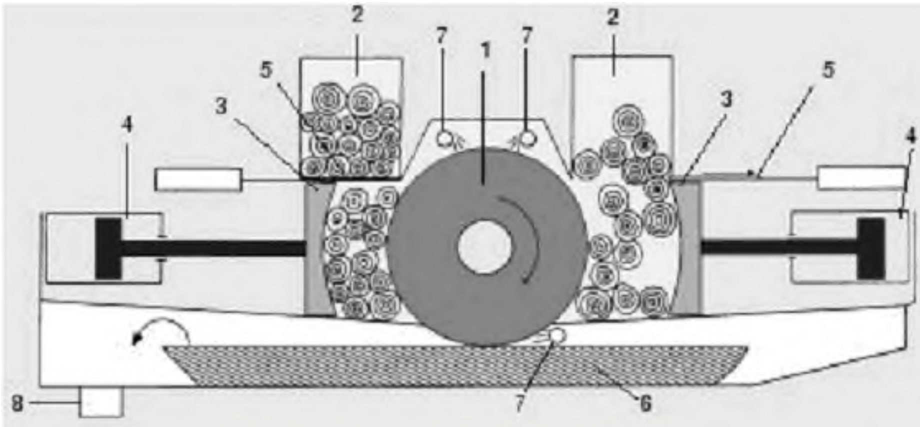
The intensive development of the printing industry, duplicating and copying equipment and other paper consumers sets the task of improving the quality of paper products. The quality of paper is determined by a number of mechanical and printing properties, depending on its type and grade. However, regardless of the paper purpose, there are quality indicators that affect most of its consumer properties. Unevenness of the profile of such quality indicators as weight of 1 m^2, ash content, moisture and paper thickness increases the number of scrap and paper web breaks, and an excessively high value of weight of 1 m^2 leads to overconsumption of fiber, fillers and energy resources.

The functioning of the pulp and paper industry, like other sectors of the economy, is influenced by many factors. In a market economy, the most acute issue is the rational use of raw materials by enterprises in this industry.

2 Paper making process

There are 4 stages in the paper making process.

At the first stage, the pulp undergoes preparation (the process of grinding plant fibers is started using special equipment, then substances for gluing, filling and painting are mixed). Wood pulp is obtained by mechanical abrasion in fiber separators at the wood-pulp shop. The principle of fiber separator configuration is shown in Figure 1.



1 – grindstone;
2 – wood container;
3 – presses;
4 – press cylinder;
5 – fasteners;
6 – tray;
7 – water sprinkler;
8 – discharge channel.

Fig. 1. The principle of fiber separator configuration.

At the second stage, the paper pulp, after preliminary sorting, goes to the paper-making machine. Forming of the paper web is carried out on the mesh part of the paper machine. The web is then dewatered under the pressure of the press rolls and dried as it passes through the drying cylinders.

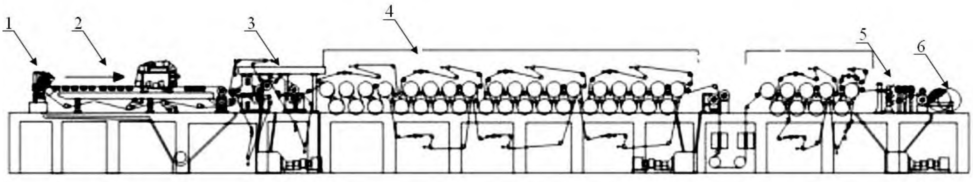
The third stage is the process of finishing the paper on the calender. The finished paper is cut into rolls of different sizes on a slitting machine.

The fourth stage is the stage of sorting and packing.

2.1 Structure of the paper-making machine

The functional diagram of the paper-making machine is shown in Figure 2. The paper-making machine consists of a mesh, pressing, drying, finishing parts and a drive. In addition, it includes a machine chest for accumulating paper pulp and feeding it to the machine.

The mesh part is intended for forming and dewatering the paper web. The headbox is designed for uniform and continuous filling of the mass onto the mesh.



1 – headbox
2 – mesh part
3 – pressing part
4 – drying part
5 – calender
6 – reel

Fig. 2. Functional diagram of a paper-making machine.

The mesh table is a horizontal plane formed by a mesh stretched between the chest roll and the suction couch roll.

The pressing part serves for further mechanical dewatering of the paper web after the mesh table and consists of two or three twin-shaft presses.

The drying part serves for the final dehydration of the paper web by evaporation of moisture. Then the paper goes to the reel, where it is reeled up into a roll.

3 Optimal control

Automated systems for monitoring and regulating the density of the paper web are designed to stabilize the specified modes of the technological process of papermaking by monitoring and regulating technological parameters and issuing control actions to the actuators.

The use of automated systems for controlling the paper web density allows:

- increasing the output of marketable products;
- reducing material and energy costs;
- making the choice of rational technological modes;
- improving the quality indicators of final products.

In addition, it is assumed that the achievement of the above goals should contribute to the improvement of the ecological situation by saving natural resources (wood).

The development of a control algorithm is the main and crucial stage in the creation of an ACS. The synthesis of the control algorithm requires the simultaneous accounting and control of dozens of frequency and time parameters of the system, which are difficult to implement with manual synthesis "on paper". As a result, a lot of time is spent, and regulators are rarely optimal according to the given criteria. Not to mention the fact that the controller synthesized by classical methods must be manually adjusted taking into account the nonlinearities of the control object [9-11].

To create control laws, it is necessary to apply advanced practices and approaches that allow automated synthesis of algorithms based on nonlinear models of objects, taking into account the necessary criteria and applying optimization methods for fine tuning taking nonlinearities into account. This makes it possible to obtain high-quality regulators in an extremely short time, including for multi-circuit systems.

Controlling complex systems with the help of classical linear controllers becomes more and more difficult, because they are subject to stringent requirements up to adjusting to changing environment and working conditions. This problem provoked the emergence of a large number of optimal and adaptive control methods, which are far from being as easy to use as classical linear control.

The development of the theory of optimal control is caused by the requirement for reliability, speed and high accuracy of control systems.

3.1 Pontryagin's maximum principle

Pontryagin's principle is used in optimal control theory to find the best possible control to transfer a dynamical system from one state to another, especially when there are constraints for the state or input controls. It says that for any optimal control, it is necessary, along with the optimal state trajectory, to solve the so-called Hamiltonian system, which is two-point, and a boundary value problem, plus the maximum state of the Hamiltonian. These necessary conditions become sufficient under certain convexity conditions for the objective function and the constraint function [12].

The maximum principle, widely regarded as a milestone in optimal control theory, is that maximizing the Hamiltonian is much easier than the original infinite-dimensional control problem; instead of maximizing in function space, the problem is transformed into point optimization. This logic leads to Bellman's principle of optimality, a related approach to optimal control problems, which states that the optimal trajectory remains optimal at intermediate times [13]. The resulting Hamilton - Jacobi - Bellman equation provides a necessary and sufficient condition for an optimum and allows a direct extension to stochastic optimal control problems, while the maximum principle does not. However, unlike the Hamilton – Jacobi – Bellman equation, which must hold for the entire state space, the Pontryagin's maximum principle is potentially more computationally efficient, since the conditions it sets must be satisfied only for a certain trajectory [14, 15].

3.1.1 Realization of the Pontryagin's maximum principle in time. Let's write the differential equation of the object:

$$0.1136\frac{d2y}{dt2}+0.942\frac{dy}{dt}+y=393x \quad (1)$$

Since only the first derivatives are used in the maximum principle formulated by Pontryagin, we introduce the notation:

$$y1=y, \qquad y2=\frac{dy}{dx}$$

Then equation (1) takes the form:

$$0.1136\frac{\mathrm{dy2}}{\mathrm{dt}}+0.942\mathrm{y2}+\mathrm{y1}=393\mathrm{x} \quad (2)$$

From (2) we can obtain $\frac{d2y}{dt2}=\frac{dy2}{dt}$

$$\frac{dy2}{dt2}=\frac{1}{0.1136}(-0.942\frac{dy}{dt}-y+393x)$$

Let's introduce auxiliary functions:

$$f1=y2; f2=\frac{1}{0.1136}*(-0.942\mathrm{y2}-\mathrm{y1}+393\mathrm{x}).$$

Then the optimal control will be implemented by the function:

$$H=\phi0*1+\phi1*f1+\phi2*f2.$$

Taking into account the previous transformations, we get

$$H = \varphi 0 * 1 + \varphi 1 * y2 + \varphi 2 * 1/0.1136*(-0.942y2-y1+393x) \quad (3)$$

Function $\phi 0$ is determined by the selected optimization criterion: minimum time, minimum consumed energy.

To find unknown functions $\phi 1$ and $\phi 2$, use the relations:

$$\frac{d\varphi 1}{dt} = -\frac{\partial H}{\partial y1} \text{ and } \frac{d\varphi 2}{dt} = -\frac{\partial H}{\partial y2}.$$

Then we get:

$$\frac{d\varphi 1}{dt} = \frac{\varphi 2}{0.1136} \quad (4)$$

$$\frac{d\varphi 2}{dt} = -\varphi 1 + \varphi 2 * \frac{0.942}{0.1136} \quad (5)$$

It is obvious from (4) that $\varphi 1 = \frac{1}{0.1136}\int \varphi 2 dt$. Substituting it into equation (5), we obtain the equation in operator form:

$$p\varphi 2(p) - p\phi 2(p) * \frac{0.942}{0.1136} + \frac{1}{0.1136 * p} * \varphi 2(p) = 0.$$

Let us write a characteristic equation, the roots of which show that these are two inertial links.

$$p^2 - p\frac{0.942}{0.1136} + \frac{1}{0.1136} = 0.$$

With a scheme implementation, the structure is important, and with software implementation using the Simulink system, it is enough to implement these links:

$$\varphi 2(p) = \frac{K2}{p^2 - p\frac{0.942}{0.1136} + \frac{1}{0.1136}}, \quad \varphi 1(p) = \frac{1}{0.1136}\frac{K2 * p}{p^2 - p\frac{0.942}{0.1136} + \frac{1}{0.1136}}.$$

It is possible to implement only $\varphi 2$, and obtain $\varphi 1$ by integrating the output $\varphi 2$. Also, in the software implementation of the function *f*1 and *f*2, one can use directly the result of differentiation of $\frac{dy}{dt}$ and $\frac{dy}{dt}$.

Time-optimal control:

$$H = -1 + \phi 1 * \frac{dy}{dt} + \phi 2 * \frac{d^2 y}{d^2 t}.$$

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3.1.2 Implementation of Pontryagin's maximum principle in Simulink

Let's consider the application of the Pontryagin's principle in the paper web weight control system. Figure 3 shows a Simulink model of optimal paper web weight control.

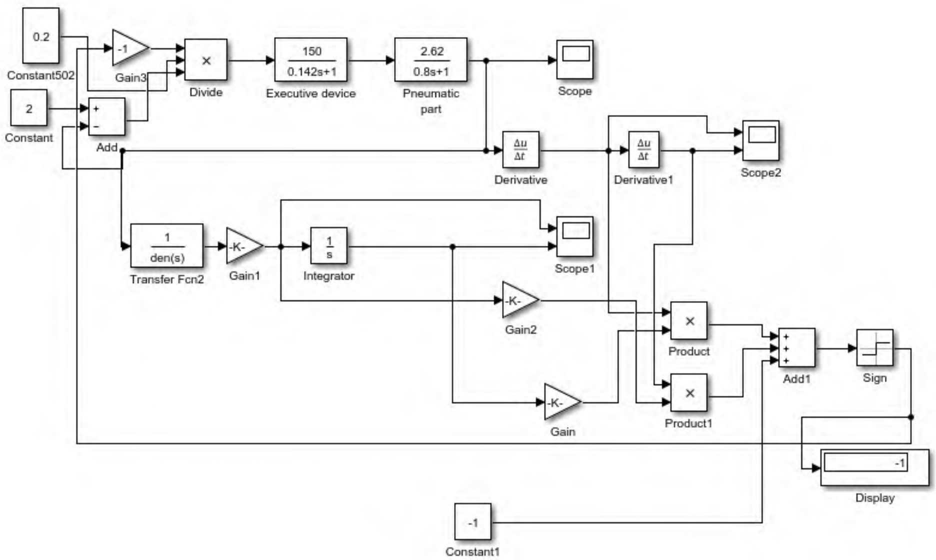

Fig. 3. Implementation of the Pontryagin's maximum principle in the paper web weight control system.

The results of modeling the optimal paper weight control system based on the Pontryagin's maximum principle are shown in Figure 4.

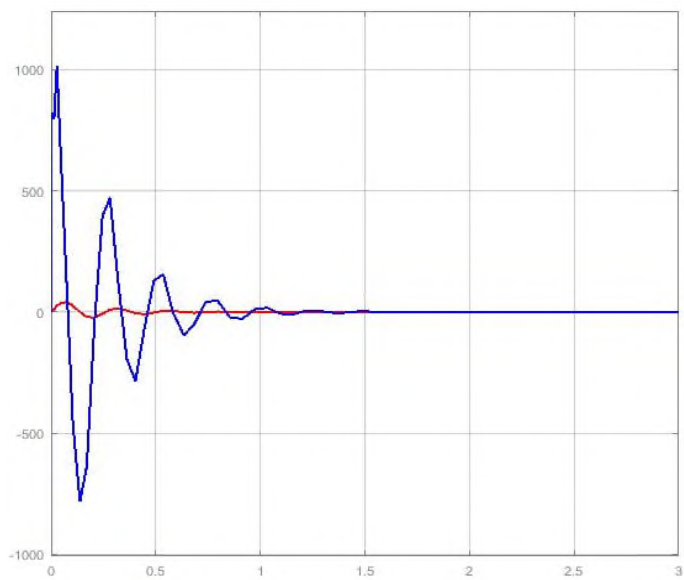

Fig. 4. Results of modeling the optimal paper weight control system based on the Pontryagin's maximum principle.

The blue graph is the result of the system operation without optimal regulation. It can be seen that it has a large overcontrol. The red graph is the result of the system operation using the optimal control based on the Pontryagin's maximum principle. It can be seen from the figure that the process indicated in red decays faster with less overcontrol. Application of the maximum principle allows achieving the optimal value of the transient time.

Acknowledgements

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