Application of wavelets and conformal reflections to finding optimal scheme of fiber placement at 3d printing constructions from composition materials

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Abstract. The article is devoted to finding the optimal schemes of fiber placement at the production of constructions, reinforced with continuous fibers by 3D printing method. As the optimization of the objective function one of the criteria for the destruction of the composite was chosen. For the process acceleration of multiple solution of the system of partial differential equations describing the stress-strain state of the structure, a computational algorithm based on wavelets built through subdivision schemes is proposed. To set the local coordinate system, it is proposed to use analytical functions, which will be constructed using the well-known Dini and Cisotti formulas, just by specifying the direction of laying the fiber at the product boundary. The article also presents a lifting scheme (lifting scheme) allowing to construct biorthogonal wavelet systems with specified properties using some initial biorthogonal wavelet systems with filters.

1 Introduction

At the present time in the high-tech areas of industries composite materials (CM) are widely spread, which consist of reinforcing material and a binder. In the quality of reinforcing material, the carbon fibers are used, which have high specific strength. In this case, the mechanical properties of CM products depend on the direction of the fibers. 3D printing is a perspective technology for manufacturing structures of complex shapes by sequential placement of composite materials. With the usage of 3D printing, in principle, it is possible to obtain the structure with spatial reinforcement along the given paths. The total control over the placement of fibers during the printing (100% of the fibers are at the right direction) allows them to be stacked according to the required operating conditions.

The geometry of the fiber placement is determined by the equations of CM mechanics themselves in the form of some (unknown) local orthogonal coordinate system [1]. To find the geometry of placement it is possible from these equations by solving them multiple

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times with different local coordinate systems. That is why it is necessary to work out rather fast algorithm for solving partial differential equations. To achieve this goal, it is proposed to use biorthogonal wavelets constructed by using subdivision schemes [2, 3, 4]. The choice of such scheme of decision is explained by the following:

- firstly, when using the lifting scheme, you can control the properties of the wavelets themselves [5],
- secondly, for a sufficiently accurate solution of the equations themselves, a small number of iterations is required.

To set the local coordinate system, it is proposed to use analytical functions that will be built at the usage of well-known Dini and Chizotti formulas just by specifying the direction of fiber laying at the product boundary [6]. Any of the CM fracture criteria can be selected as an optimization criterion for choosing fiber placement paths [7].

So, the goal is to develop methods and algorithms for finding the optimal fiber placement when 3D printing structures are made of composite materials reinforced with continuous fibers.

2 Experimental

Wavelets have a number of advantages over other basic functions. Firstly, the usage of the lifting scheme allows one to construct wavelets with the given properties: smoothness, compact support, symmetry, the required number of zero moments, vanishing on the boundary of the domain of functions corresponding to non-boundary mesh vertices [8]. Secondly, the high rate of decay of the wavelet coefficients, which allows, limiting themselves to a small number of terms in the expansion, to obtain sufficiently accurate approximations of the function. Thirdly, the presence of fast cascade algorithms for finding the coefficients of the wavelet expansion of the function. This section is devoted to the development of the technique for using biorthogonal wavelets in the approximate solution of partial differential equations.

Multi-scale analysis

In describing the basis of wavelet analysis, including notations, we will follow the works [2-3]. Let it be (X, B, μ) – a measurable measure space [7]. We will consider the real space $\mathrm{L}_2(X)$.

Definition 1.1. Let it be H – Hilbert space and $\aleph$ – countable set of indexes [9]. Family $\{f_n\}_{n\in\aleph}$ is called the Riesz sequence with constants $A, B > 0$, if for any $c = \{c_n\}_{n\in\aleph} \in l_2$ the row $\sum_{n\in\aleph} c_n f_n$ converges in H and

$$A \| c \|_{l_2}^2 \le \left\| \sum_{n\in\aleph} c_n f_n \right\|_H^2 \le B \| c \|_{l_2}^2$$

If the Riesz sequence is a basis, then it will be called the Riesz basis.

Definition 1.2. Multi-scale analysis at X is defined as a sequence of subspaces $V_j \subset \mathrm{L}_2(X),\ j \in J \subset \mathbf{Z}$, such, that

1. $V_j \subset V_{j+1}$;
2. $\bigcup_{j\in J} V_j$ tightly in $\mathrm{L}_2(X)$;
3. For any j there are scaling functions $\varphi_{j,k}$, $k \in K_j$ such that the set $\{\varphi_{j,k} : k \in K_j\}$ is a Riesz basis in V_j. Wherein $K_j \subset K_{j+1}$.

From the property 1 it follows that there is a sequence of coefficients $\{h_{j,k,l}\}$, such that [3]

$$\varphi_{j,k} = \sum_{l \in K_{j+1}} h_{j,k,l} \varphi_{j+1,l} \tag{1}$$

The values $h_{j,k,l}$ are defined for $j \in J$, $k \in K_j$ and $l \in K_{j+1}$.

Definition 1.3**.** Let it be $\{V_j\}_{j \in J}$ and $\{\tilde{V}_j\}_{j \in J}$ – two multi-scale analyzes at X with scaling functions $\varphi_{j,k}, k \in K_j, j \in J$ and $\tilde{\varphi}_{j,k}, k \in K_j, j \in J$ accordingly. Moreover, let it be $V_{j+1} = V_j + W_j$, $\tilde{V}_{j+1} = \tilde{V}_j + \tilde{W}_j$ and $\{\psi_{j,k},\ k \in M_j\}$, $\{\tilde{\psi}_{j,k},\ k \in M_j\}$ – Riesz basis in W_j and $\tilde{W}_j$ accordingly. If

$$(\varphi_{j,k}, \tilde{\varphi}_{j,k'}) = \delta_{k,k'},\ \forall k, k' \in K_j;$$

$$(\psi_{j,m}, \tilde{\psi}_{j,m'}) = \delta_{m,m'},\ \forall j \in J.\ \forall m \in M_j,\ \forall m' \in M_j;$$

$$(\tilde{\psi}_{j,m}, \varphi_{j,k}) = 0;\ (\tilde{\varphi}_{j,k}, \psi_{j,m}) = 0,\ \forall m \in M_j,\ \forall k \in K_j;$$

The family of functions $\{\psi_{j,k}\}_{j \in J, k \in M_j}$ and $\{\tilde{\psi}_{j,k}\}_{j \in J, k \in M_j}$, are called biorthogonal wavelet systems.

As $W_j \subset V_{j+1}$ и $\tilde{W}_j \subset \tilde{V}_{j+1}$, then

$$\psi_{j,m} = \sum_{l \in K_{j+1}} g_{j,m,l}\, \varphi_{j+1,l},\ \tilde{\psi}_{j,m} = \sum_{l \in K_{j+1}} \tilde{g}_{j,m,l}\, \tilde{\varphi}_{j+1,l},\ m \in M_j$$

In the case of biorthogonal wavelet systems for $f \in \mathrm{L}_2(X)$ the equality takes place [2]:

$$f = \sum_{n \in K_{j_0}} (f, \tilde{\varphi}_{j_0,n})\, \varphi_{j_0,n} + \sum_{j \geq j_0} \sum_{n \in M_j} (f, \tilde{\psi}_{jn})\, \psi_{jn} = \sum_{n \in K_{j_0}} \nu_{j_0,n} \varphi_{j_0,n} + \sum_{j \geq j_0} \sum_{n \in M_j} \gamma_{jn} \psi_{jn}, \tag{3}$$

where $\nu_{j,k} = (f, \tilde{\varphi}_{j,k})$, $\gamma_{j,m} = (\tilde{\psi}_{j,m}, f)$. Let it be

$$\tilde{\varphi}_{j,k} = \sum_{l \in K_{j+1}} \tilde{h}_{j,k,l}\, \tilde{\varphi}_{j+1,l}\ . \tag{4}$$

The sequences $h_{j,k,l}$, $\tilde{h}_{j,k,l}$, $g_{j,k,l}$ и $\tilde{g}_{j,k,l}$ are called the filters. From the biorthogonality conditions, we obtain [3]

$$\tilde{h}_{j,k,l} = (\tilde{\varphi}_{j,k}, \varphi_{j+1,l}),\ \tilde{g}_{j,k,l} = (\tilde{\psi}_{j,k}, \varphi_{j+1,l})\ .$$

As $V_{j+1} = V_j + W_j$, then $\varphi_{j+1,l} = \sum_{s \in K_j} c_{j,s,l} \varphi_{j,s} + \sum_{m \in M_j} d_{j,m,l} \psi_{j,m}$. From the biorthogonality conditions, we get

$$c_{j,s,l} = \tilde{h}_{j,s,l},\ d_{j,s,l} = \tilde{g}_{j,s,l}\ .$$

Consequently,

$$\varphi_{j+1,l} = \sum_{s\in K_j} \tilde{h}_{j,s,l}\, \varphi_{j,s} + \sum_{m\in M_j} \tilde{g}_{j,m,l} \psi_{j,m} \ . \tag{5}$$

From equalities (2) and (5) we obtain

$$\nu_{j,k} = \sum_{l\in K_{j+1}} \tilde{h}_{j,k,l}\, \nu_{j+1,l},\ \gamma_{j,m} = \sum_{l\in K_{j+1}} \tilde{g}_{j,m,l}\, \nu_{j+1,l} \ . \tag{6}$$

From the formula similar to (5), we obtain

$$\nu_{j+1,l} = \sum_{s\in K_j} h_{j,s,l} \nu_{j,s} + \sum_{m\in M_j} g_{j,m,l} \gamma_{j,m} \ . \tag{7}$$

The formulas (6) are wavelet decomposition formulas or analysis formulas, and formula (7) is wavelet reduction or synthesis formula.

Lifting scheme

Lifting scheme allows to construct biorthogonal wavelet systems with the specified properties using some initial biorthogonal wavelet systems with filters $h^0_{j,k,l},\ \tilde{h}^0_{j,k,l},\ g^0_{j,k,l},\ \tilde{g}^0_{j,k,l}$. By the lifting scheme the new family of filters $h_{j,k,l},\ \tilde{h}_{j,k,l},\ g_{j,k,l},\ \tilde{g}_{j,k,l}$, determining biorthogonal wavelet systems is found by the formulas [5, 3]

$$h_{j,k,l} = h^0_{j,k,l},\ g_{j,m,l} = g^0_{j,m,l} - \sum_{k\in K_j} s_{j,k,m} h^0_{j,k,l},$$

$$\tilde{g}_{j,m,l} = \tilde{g}^0_{j,m,l},\ \tilde{h}_{j,k,l} = \tilde{h}^0_{j,k,l} + \sum_{m\in M_j} s_{j,k,m}\, \tilde{g}^0_{j,m,l}$$

with any choice of sequence $\{s_{j,k,m}\}_{k\in K_j, m\in M_j}$. It should be noted that scaling functions $\varphi_{j,l}$ are the same in the original and raised multi-scale analyzes $\varphi_{j,k} = \varphi^0_{j,k}$.

Note that you don't need to change the function $\tilde{\varphi}_{j,k}$, but raise $\varphi_{j,k}$. This mechanism is exactly the same and is called the dual lifting scheme [5, 3]. It allows to improve the properties of the wavelet $\tilde{\psi}_{j,m}$. At the dual lifting scheme, the new filters are determined by the formulas:

$$\tilde{h}_{j,k,l} = \tilde{h}^0_{j,k,l},\ \tilde{g}_{j,m,l} = \tilde{g}^0_{j,m,l} - \sum_{k\in K_j} \tilde{s}_{j,k,m}\, \tilde{h}^0_{j,k,l},$$

$$g^1_{j,m,l} = g^0_{j,m,l},\ h_{j,k,l} = h^0_{j,k,l} + \sum_{m\in M_j} \tilde{s}_{j,k,m}\ g^0_{j,m,l}.$$

Obviously, that the lifting scheme is only useful if the original set of biorthogonal filters is available.

Let us dwell briefly at the method for constructing of biorthogonal wavelets on triangulated spaces with the finite set of simplices, presented in the work [10].

Let it be (T, g, X) – triangulated space with a finite set of simplices [11]. $T = \bigcup_{l=1}^{N} I^s_l \subset I^s_0 \subset \square^s$ – union of closed *s*-dimensional cubes of the form

$I_l^s = \prod_{i=1}^{s}[b_{i,l}; b_{i,l}+1]$, where $b_{i,l} \in \mathbf{Z}$, I_0^s – s-dimensional cube $g: T \to X \subset \mathbb{R}^s$ – homeomorphism, $g: \mathrm{Int}(T) \to \mathrm{Int}(X)$ – diffeomorphism of C^2 class.

Let it be $\{\varphi_{j,\alpha}\}_{\alpha \in K_j}$, $\{\psi_{j,\beta}\}_{\beta \in M_j}$ – scaling functions and wavelets on T. Let's define the scaling functions and wavelets on X by the following equalities:

$$\varphi_{j,\alpha}^X(X) = \varphi_{j,\alpha} \circ g^{-1}(X),\ \psi_{j,\alpha}^X(X) = \psi_{j,\alpha} \circ g^{-1}(X).$$

Then, if $f: X \to \mathbb{R}$ and $f \circ g \in \mathrm{L}_2(T)$, then [2]

$$f = \sum_{n \in K_{j_0}} \nu_{j_0,n} \varphi_{j_0,n}^X + \sum_{j \geq j_0} \sum_{n \in M_j} \gamma_{j,n} \psi_{j,n}^X,$$

in the sense that

$$\lim_{J \to +\infty} \int_T \left[\left(f - \sum_{n \in K_{j_0}} \nu_{j_0,n} \varphi_{j_0,n}^X - \sum_{j=j_0}^{J} \sum_{n \in M_j} \gamma_{j,n} \psi_{j,n}^X \right) \circ g(u) \right]^2 du = 0.$$

Application of wavelets to the approximate solution of partial differential equations

One of the methods for the approximate solution of partial differential equations that we will use is the method of least squares. It is widely used in solving boundary value problems in mathematical physics. [12]. Let's consider the differential equation and boundary conditions

$$L\omega = f \text{ for } X,\ L_i\omega = f_i \text{ for } \partial X,\ i = 1, 2, \dots, m \tag{8}$$

in Hilbert space $\mathrm{L}_2(X)$, where L – linear differential operator. Let it be $\{V_j\}_{j \in J}$ – multi-scale analysis X. We seek approximate solutions to equation (8) in the form

$$\omega_J = \sum_{n \in K_{j_0}} \nu_{j_0,n} \varphi_{j_0,n}^X + \sum_{j=j_0}^{J-1} \sum_{n \in M_j} \gamma_{j,n} \psi_{j,n}^X = \sum_{k=1}^{M(J)} c_k \omega_k \in V_J, \tag{9}$$

where for convenience the basic functions have the same index and are noted ω_k, and the coefficients $\nu_{j_0,n}$ and $\gamma_{j,n}$ are determined c_k and are found by the least squares method from solving the variational problem $\omega_J = \arg\min_{\omega \in V_J} I_J(\omega)$.

Functional $I_J(\omega)$ is defined by the equality

$$I_J(\omega) = \| L\omega - f \|^2 + \sum_{i=1}^{m} a_i \| L\omega - f_i \|^2,$$

where a_i – are positive weight coefficients. Taking into account the fact that when constructing wavelets it is possible to choose the sequence $s_{j,k,m}$ in the lifting scheme, one can zero out some of the basic functions on the boundary of the region (these are functions corresponding to the non-boundary vertices of the subdivision), one can find some of the expansion coefficients (9) from the boundary conditions. In this case,

$$\omega_J = \sum_{k \in IntX} c_k \omega_k + \sum_{s \in \partial X} c_s \omega_s ,$$

and the remaining coefficients can already be found from the minimization goal

$$\| \sum_{k \in IntX} c_k L\omega_k - g \|^2 \to \min_{c_k},$$

where $g = f - \sum_{s \in \partial X} c_s \omega_s$.

The second used approximate method for solving goal (8) is the collocation method, in which it is required that the equation and boundary conditions are fulfilled at grid nodes (so-called collocation nodes).

In this work, the example was calculated using spline wavelets. Let's briefly dwell on their construction.

Let it be $K_j = 2^{-j}\mathbf{Z}^n \cap T$. For each $i = 1, 2, \ldots, s$ let's choose the sequences $a^i_{s,\bullet} = \{a^i_{s,\alpha}\}_{\alpha \in \mathbf{Z}}$, $s \in \mathbf{Z}$, which are in the scheme

$$\nu_{j+1,\alpha} = \sum_{\beta \in \mathbf{Z}} a^i_{\alpha,\alpha-2\beta} \nu_{j,\beta}, \ \nu_{0,\alpha} = \delta_{\alpha,k} = \begin{cases} 1, & \alpha = k, \\ 0, & \alpha \neq k \end{cases}$$

lead to smooth functions $\varphi_{0,k}$ of the required smoothness class, and put $h_{j,s,k} = (a^1_{k_1,\bullet} \otimes \ldots \otimes a^n_{k_n,\bullet})_{k_1-2s_1,\ldots,k_n-2s_n}$, where $\mathrm{k} = 2^{-j-1}\left(k_1 \ \ldots \ k_n\right)$ and $\mathrm{s} = 2^{-j}(s_1 \ \ldots \ s_n)$. For example, the sequence $a = (\ldots \ 0 \ \frac{1}{8} \ \frac{4}{8} \ \frac{6}{8} \ \frac{4}{8} \ \frac{1}{8} \ 0 \ \ldots)$, as it is known leads to cubic B- splines [13]. Choosing the filter, we thereby constructed the scaling functions [14], which are determined by the scheme (7).

Let it be $\mathrm{e}_l \in \mathbf{Z}^n$ – nonzero vectors whose coordinates are equal 0 or 1, and $M_j = \left\{ \frac{2\mathrm{k} + \mathrm{e}_l}{2^{j+1}}, \ \mathrm{k} \in \mathbf{Z}^s \right\}$, $J = \{0\} \cup \mathrm{N}$. Let's consider the filter

$$g^0_{j,\mathrm{m},\mathrm{t}} = \delta_{\mathrm{m},\mathrm{t}}, \ \forall \mathrm{m} \in M_j, \ \forall \mathrm{t} \in K_{j+1}.$$

In this case, the wavelets $\psi^0_{j,\mathrm{m}}$ coincide with some of the functions of the space V_{j+1} :

$$\psi^0_{j,\mathrm{m}} = \varphi_{j+1,\mathrm{m}}, \ \mathrm{m} \in M_j .$$

Using the lifting scheme, we get a new filter

$$g_{j,\mathrm{m},\mathrm{k}} = g^0_{j,\mathrm{m},\mathrm{k}} - \sum_{\mathrm{r} \in K_j} s_{j,\mathrm{r},\mathrm{m}} h_{j,\mathrm{r},\mathrm{k}}$$

and spline wavelets

$$\psi_{j,\mathrm{m}} = \varphi_{j+1,\mathrm{m}} - \sum_{\mathrm{k} \in K_j} s_{j,\mathrm{k},\mathrm{m}} \varphi_{j,\mathrm{k}} .$$

The scaling functions won't be changed.

The tangent vectors to the curves along which the fibers are placed in 3D printing form a vector field **r** in *X*, which will be characterized by a complex number

$\mathrm{r} = r_1 + ir_2$, where $r_1 = r_1(x_1, x_2)$, $r_2 = r_2(x_1, x_2)$. We will consider this field to be harmonic, i.e. solenoidal and potential [15]. Such field has no sources or vortices. Moreover, let it be $X \subset \tilde{X}$, where the area $\tilde{X}$ will be considered simply connected, and the field is considered in this simply connected region. It means that expression $-r_2 dx_1 + r_1 dx_2$ is the total differential of some function v_2, determined for *X*. This function is called the current function [16]. Moreover, the expression $r_1 dx_1 + r_2 dx_2$ there is also the total differential of some function $v_1(x_1, x_2)$, which is called the field potential [16]. The current function $v_2(x_1, x_2)$ and field potential $v_1(x_1, x_2)$ are the conjugate harmonic functions [16]. The current lines and the lines of equal potential form the orthogonal family.
The analytical function

$$v_1(x_1, x_2) + iv_2(x_1, x_2),\ x_1 + ix_2 \in X$$

is called the complex field potential [17]. Thus, any analytic function in the domain $\tilde{X}$ also gives the scheme of fiber placement, and local curvilinear coordinate system in $X \subset \tilde{X}$. Let's agree the points $\mathrm{x} = (x_1 \quad x_2)^T$ depict at one complex plane, and the points $\mathrm{v} = (v_1 \quad v_2)^T$ at the other. Then the transformation $v_1 = v_1(x_1, x_2), v_2 = v_2(x_1, x_2)$ and its opposite $x_1 = x_1(v_1, v_2), x_2 = x_2(v_1, v_2)$ represent the transformation of some area $\tilde{X}$ plane x at the set Ω plane v. Level line network $v_1(x_1, x_2) = \text{const}, v_2(x_1, x_2) = \text{const}$ are called the isothermal network. The curves along which the fibers are placed are determined by parametric representations

$$\gamma_\alpha :\ \mathrm{r}_\alpha(v_1) = x_1(v_1, \alpha) + ix_2(v_1, \alpha),\ v_1 \in T_\alpha,$$

where T_α – some gap, $\alpha \in \square$ и $T_\alpha \times \{\alpha\} \subset \Omega$. Function $v_1(x_1, x_2)$ at $\tilde{X}$ can be searched from the Neumann problem $\begin{cases} \partial^{(2,0)} v_1 + \partial^{(0,2)} v_1 = 0; \\ \dfrac{\partial v_1}{\partial \mathrm{n}}(\mathrm{x}) = \partial_1 v_1(\mathrm{x}) n_1(\mathrm{x}) + \partial_2 v_1(\mathrm{x}) n_2(\mathrm{x}) = a(\mathrm{x}) \cos\theta(\mathrm{x}),\ \mathrm{x} \in \partial\tilde{X}, \end{cases}$

where $\theta(\mathrm{x})$ – he angle between the outer unit normal n(x) to the border of the area $\tilde{X}$ and fiber. Let's determine as $\eta(\mathrm{x}) = a(\mathrm{x}) \cos\theta(\mathrm{x})$.

For the Neumann problem to be solvable, it is necessary and sufficient that the following condition $\int_{\partial\tilde{X}} \eta(\mathrm{x})\, ds = 0$ should be valid. This condition will be satisfied if the piecewise continuous function $a(\mathrm{x})$ will be chosen by the next way:

$$a(\mathrm{x}) = \begin{cases} \dfrac{1}{\displaystyle\int_{\cos\theta(\mathrm{x})>0} \cos\theta(\mathrm{x}) ds}, \text{ if } \cos\theta(\mathrm{x}) > 0; \\ -\dfrac{1}{\displaystyle\int_{\cos\theta(\mathrm{x})<0} \cos\theta(\mathrm{x}) ds}, \text{ if } \cos\theta(\mathrm{x}) < 0. \end{cases}$$

Specifying the conformal display of the circle $\Gamma_\rho = \{\mathrm{w} = w_1 + iw_2 : w_1^2 + w_2^2 < \rho\}$ to the area $\tilde{X}$ You can reduce this Neumann problem to the Neumann problem for a circle, the

solution of which can be found by the Dini formula [18]. The conformal display itself can be specified by using the Chizotti formula [18].

3 Evaluation

This section describes the methodology for finding optimal fiber placement paths when 3D printing of CM structures with continuous fibers [19]. Let it be that $\sigma_1^{\pm}$, $\sigma_2^{\pm}$ – are the ultimate tensile and compressive strength along and across the fibers, and $\hat{\tau}_{12}$ –is the ultimate strength in the plane of the layer. As the objective function, we will use, for example, the criteria for maximum stresses

$$F(\sigma_1, \sigma_2, \tau_{12}) = \max\left(\frac{\sigma_1}{m_1(\sigma_1)}, \frac{\sigma_2}{m_2(\sigma_2)}, \frac{\tau_{12}}{\hat{\tau}_{12}}\right),$$

where $m_1(\sigma_1) = \begin{cases} \sigma_1^+, \text{ if } \sigma_1 > 0; \\ \sigma_1^-, \text{ if } \sigma_1 < 0, \end{cases}$ $m_2(\sigma_2) = \begin{cases} \sigma_2^+, \text{ if } \sigma_2 > 0; \\ \sigma_2^-, \text{ if } \sigma_2 < 0. \end{cases}$

The meanings σ_1, σ_2, τ_{12} can be found approximately from the equations of CM mechanics if we set the transformation v(x). We obtain the boundary conditions for these equations by specifying the angles $\theta(\mathrm{x}), \mathrm{x} \in \partial X$, which the fibers form with the outer normal to the boundary of the set *X*. Therefore, the objective function is the function of these angles θ

$$F : \theta|_{\partial X} \mapsto \mathrm{v(x)} = (v_1(\mathrm{x}), v_2(\mathrm{x})) \mapsto (\sigma_1, \sigma_2, \tau_{12}) \mapsto \max\left(\frac{\sigma_1}{m_1(\sigma_1)}, \frac{\sigma_2}{m_2(\sigma_2)}, \frac{\tau_{12}}{\hat{\tau}_{12}}\right).$$

Accordance $(v_1(\mathrm{x}), v_2(\mathrm{x})) \mapsto (\sigma_1, \sigma_2, \tau_{12})$ is carried out by using the technique described in Section 1.3, by approximate solving the equations of CM mechanics. We will maximize such a function using the genetic algorithm. Let's consider the example of a rectangular plate with a hole

$$X = [0; a] \times [0; b] \setminus \{(x_1, x_2) : (x_1 - x_{1,0})^2 + (x_2 - x_{2,0})^2 \le r^2\}.$$

At the drawing 1 the restriction of the conformal transformation of the unit circle to the polygon to a circle is $\Gamma_{0,75}$ of radius 0,75 and image $\tilde{X}$ of this circle. This conformal display was found by usage of the Chizotti formula. At the drawing 2 the optimal paths of fibers placement for 3D printing of the plate after two iterations of the genetic algorithm are presented [20].

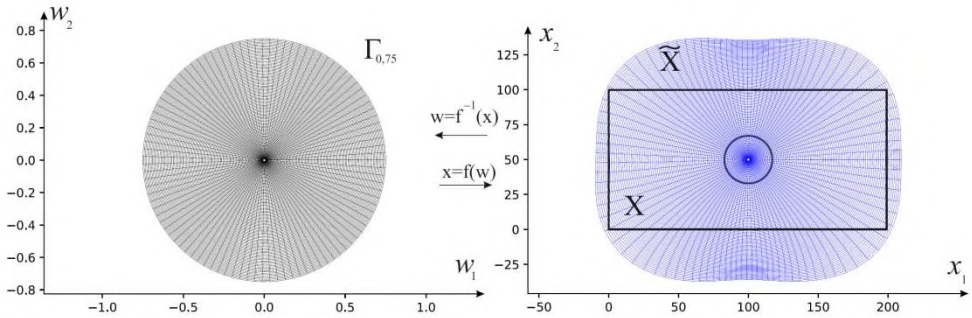

Fig. 1. The conformal transformation of the circle with radius 0,75 to the area $\tilde{X}$, containing the set of X

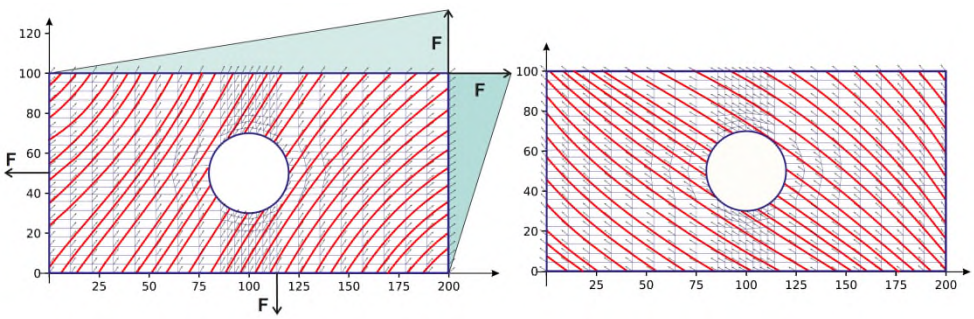

Fig. 2. Optimal fiber placement paths for 3D printing

4 Conclusions

The theoretical foundations of mathematical modeling of the process of manufacturing structures from composite materials reinforced with continuous fibers by the method of 3D printing have been developed. To set the local coordinate system, analytical functions were used in the work, built using the well-known Dini and Cisotti formulas by specifying the direction of laying the fiber at the boundary of the product. As an optimization criterion for the choice of fiber placement trajectories, the criteria for the destruction of a structural material were chosen. The proposed techniques are implemented in a CAD / CAE system for constructing such structures, written using the Python © programming language.

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