Management models for complex socio-economic systems

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Abstract. The article describes the management model of complex socio-economic systems, based on the synthesis of information that characterizes economic systems. To summarize the information, it is proposed to use the Rasch model for estimating latent variables. The management structure of complex economic systems is considered on the basis of generalization of system indicators according to the Rasch model. The mathematical basis for solving the problem is described, including taking into account the importance of the criteria. The case is considered when the values of indicators of the economic system are obtained from a variety of information sources. The results of computational experiments are presented, which allow analyzing the main properties of estimates of generalized indicators of economic systems. Computational experiments have shown that estimates of generalized indicators of objects are measured on a linear scale, are highly accurate and resistant to changes in the source data.

1 Introduction

In modern economic conditions, the forms of economic activity are changing at an ever-increasing pace. As a result, the processes in the economy are increasingly stochastic in nature, their dynamism is growing, and an uncertainty factor dominates in socio-economic systems. This, in turn, leads to a constant increase in management problems associated with the structural complexity of economic systems. The rapid progress of information and technological processes often outstrips the ability to organize and manage them with maximum efficiency.

Management problems are mainly related to the fact that modern socio-economic systems, the results of their functioning, are characterized by a large number of observed indicators, indicators and parameters. For effective management, it is necessary to take into account each of them, but this greatly complicates the process of managing the system. To build a viable management system, it is often necessary to develop a generalized model that integrates information in order to make decisions on the management of the socio-economic system. In particular, the problem of generalizing a large group of economic indicators to some integral indicator remains unsolved, which would make it possible to reduce the dimension of the data used and make them more convenient for analysis. This in turn would

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simplify the process of managing complex socio-economic systems and make it more transparent [1-4]. Currently, this task has not been fully resolved.

Thus, the aim of the work is to build a management model for complex socio-economic systems based on a synthesis of indicator indicators characterizing the system.

2 Choosing a model for generalizing indicator indicators

To solve the problem of generalization of indicator indicators of complex socio-economic systems, the Rasch model for estimating latent variables [5-7] is most effective, allowing a large sample of observed economic indicators to be collapsed within the framework of a generalized (integral) indicator reflecting the properties of the controlled object. This is due to the fact that the integral indicators of economic systems are typical latent variables.

The advantages of the Rasch model over other models that allow generalizing sets of parameters are as follows [8]:

1. The Rasch model allows you to convert measurements made in dichotomous, attributive or continuous scales into linear measurements, as a result of which qualitative data can be analyzed using quantitative methods.
2. Due to the fact that the scale of measurement of generalized indicators obtained on the basis of the Rasch model is linear, a wide range of statistical procedures can be applied to the results obtained.
3. Estimates of objects of socio-economic systems, obtained on the basis of the Rasch model, do not depend on a set of indicators and are an individual characteristic of each object.
4. Along with assessments of objects of socio-economic systems, the model allows one to obtain assessments of the impact of indicators on the socio-economic system. This allows monitoring the main indicators of the efficiency of socio-economic systems for the entire group of objects. Moreover, indicator assessments also do not depend on the set of evaluated objects and are individual characteristics of indicators.
5. Due to the rather simple structure of the estimation model, there are convenient computational procedures for obtaining estimates that can be implemented by information systems within the framework of available software products.

3 The structure of the management process of complex socio-economic systems

To solve the problem of generalizing indicator indicators of complex socio-economic systems, the Rasch model of evaluating latent variables [5-7] has the greatest efficiency, allowing a large sample of observed economic indicators to be minimized within a generalized (integral) indicator reflecting the properties of the control object.

In the traditional approach, the objects of economic systems were evaluated on the basis of indicator indicators obtained from each object, and on the basis of these estimates, management was carried out (Fig. 1 a).

The control scheme based on the Rasch model of evaluation of latent variables (Fig. 1 b) allows for more volumetric and objective control. This is due to the fact that the proposed management model is based on the calculation of some integral indicators characterizing the efficiency of economic activity for each object of the socio-economic system, which allow you to make decisions based on a variety of indicator indicators derived from both the economic system and the external environment, as well as take into account the properties of the indicators themselves.

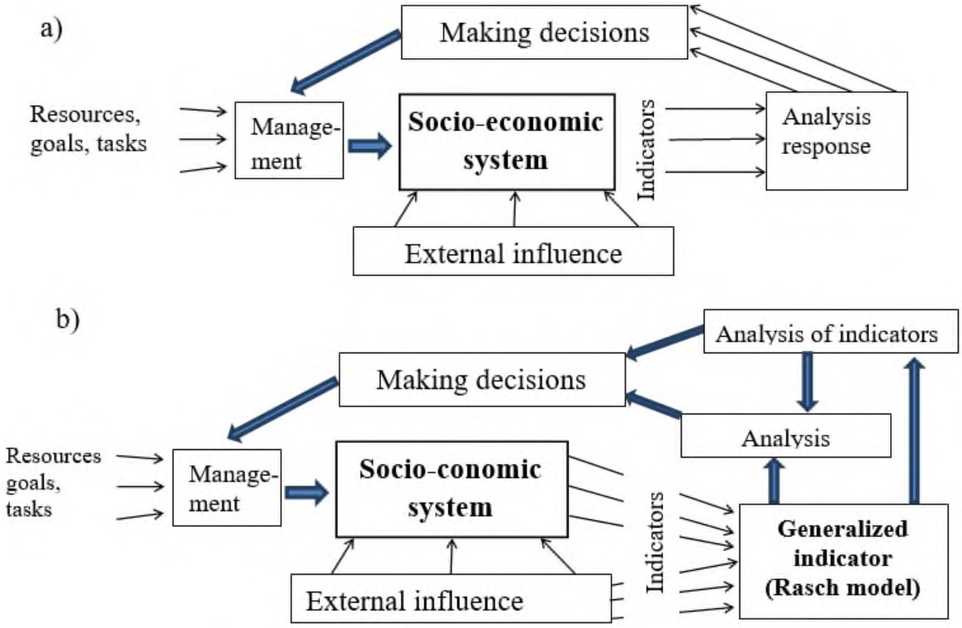

Fig. 1. The structure of the management process of socio-economic systems in: a) the traditional approach; b) in the approach based on the method of Rasch.

The implementation of this control model is possible thanks to the use of the Rasch model for evaluating latent variables as a mathematical basis for processing information. Rasch's model is a self-consistent model for evaluating latent indicators, which, on the one hand, allows one to evaluate each object of the socio-economic system based on the values of indicators, and on the other hand, to evaluate the feasibility of each indicator on the entire set of objects. In addition, the estimates obtained by the Rasch model are distributed on a linear scale and do not depend on the set of objects and indicator indicators. As a result, when making decisions on the management of the system, both integral assessments of the activities of each object and indicators of the degree of influence of each indicator on the socio-economic system are involved.

To measure the generalized indicator of objects are indicator variables [9], measured directly on the basis of evaluation criteria. The method of obtaining the numerical values of the indicator variables will be called the source of information. Sources of information can be either direct measurements in physical units or expert estimates.

The relationship between indicator variables and generalized estimates of objects is carried out on the basis of the Rasch model of measuring latent variables. The block diagram of the assessment is shown in Fig. 2.

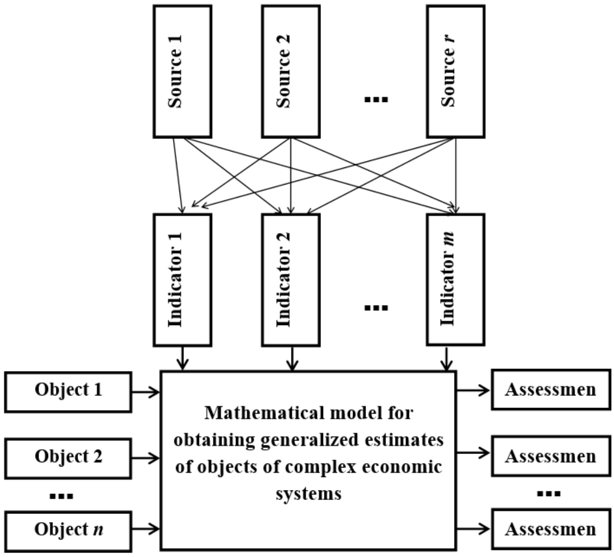

Fig. 2. Block diagram of measurement of generalized indicators of objects of complex socio-economic systems

Let us further consider a mathematical model for obtaining estimates of integral indicators of socio-economic systems.

4 Mathematical model for obtaining generalized estimates

First, a model based on the Rasch dichotomous model is presented. Let some generalized index be estimated for *m* objects of the socio-economic system. For evaluation *n* of indicator indicators, which will be referred to as criteria in the future. Let U_{ij} we denote the estimate of the *i*- th object by the *j*- th criterion. These estimates can be measured on different scales and have different dimensions. To bring the estimates to a single scale, the normalization procedure is carried out for a single segment [0; 1], we denote the normalized estimates by x_{ij}.

The latent variables will be: θ_i – estimation of the integral index of the *i*- th object of the socio-economic system; β_j - an indicator characterizing the influence of the *j*-th criterion on the socio-economic system; the smaller the value of β, the stronger the criterion affects the system. In this approach, the probability p_{ij} that the *i*-th indicator has a positive assessment according to the *j*-th criterion is determined by the logistic function:

$$P_{ij} = \frac{e^{\theta_i - \beta_j}}{1 + e^{\theta_i - \beta_j}}. \quad (1)$$

To find latent variables, we will use the Rasch model based on the least squares method [10-12]. According to it, the estimates θ_i and β_j are chosen so that the sum of the squares of the deviations of the empirical data x_{ij} from the theoretical probabilities (1) is the least. Mathematically, it boils down to solving an optimization problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ij}-P_{ij})^2=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}-\frac{e^{\theta_i-\beta_j}}{1+e^{\theta_i-\beta_j}}\right)^2\rightarrow\min;$$
$$\theta_i\geq 0;\ \beta_j\geq 0;\ i=1,2,...,m;\ \ j=1,2,...,n. \quad (2)$$

The described model for assessing generalized indicators of objects assumes that all criteria are of equal importance and make an equal contribution to the overall assessment. If the importance of the criteria is different, then this is taken into account by introducing weights. Let w_j be the weight of the j-th criterion. There are three ways to measure weights.

1) Private evaluations of objects according to the evaluation criteria u_{ij} must be taken into account taking into account the product by their weight factors w_j, and to solve the optimization problem (2) we will use indicator variables of the form: $\widetilde{x}_{ij}=x_{ij}w_j$.

2) When solving the optimization problem (2), each term is determined in proportion to the corresponding weight. As a result, an optimization problem of the form is solved:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}w_j\cdot\left(x_{ij}-\frac{e^{\theta_i-\beta_j}}{1+e^{\theta_i-\beta_j}}\right)^2\rightarrow\min;\ \theta_i\geq 0;\ \ \beta_j\geq 0;\ i=1,2,...,m;\ \ j=1,2,...,n. \quad (3)$$

3) We will take into account the fact that the latent variables β_j have the meaning of the importance of the evaluation criteria. If we define these variables in the form of parameters equal to the weights, we obtain the problem of finding estimates of the qualitative indicator of objects θ_i, taking into account the degree of importance of the evaluation criteria . The mathematical model of the optimization problem for this approach of taking into account weights will take the form:

$$S(\theta_i)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij}-\frac{e^{\theta_i-w_j}}{1+e^{\theta_i-w_j}}\right)^2\rightarrow\min;\ \theta_i\geq 0;\ i=1,2,...,m. \quad (4)$$

The given model allows one to obtain linear and independent estimates of generalized indicators of objects of socio-economic systems on an interval dimensionless scale and to carry out multilateral management of complex socio-economic systems.

5 Calculation of generalized estimates of objects of socio-economic systems for a variety of information sources

Let us now consider the approach to assessing the generalized indicators of objects for a variety of information sources. The sources of information can be multiple measurements of the objective characteristics of objects or the external environment, or expert assessment by a group of experts.

Let the generalized indicator be assessed for n objects of socio-economic systems: A_1, A_2, …,A_n, based on particular assessments by m criteria: K_1, K_2, …, K_m, and the data are taken from r sources of information: E_1, E_2, …, E_r.

The algorithm for solving the problem consists of two stages. At the first stage, the data are summarized by information sources. According to the data of each source E_k, k=1,2,…,r, conducts a multi-criteria assessment of the integral indicator, forming the initial data of continuous type on the interval [0; 1], which have the meaning of assessing the generalized indicator of the i-th object by the j-th criterion. As a result of solving the optimization problem (2), vectors of estimates of the generalized indicator of objects are formed for each source $\theta_i^{(k)}$. At the second stage, a matrix is drawn up, the initial data of which are estimates $\theta_i^{(k)}$.

As a result of solving optimization problems, we obtain the final estimates of the generalized indicator for each object from all sources according to all criteria. The solution algorithm diagram is shown in Fig. 3.

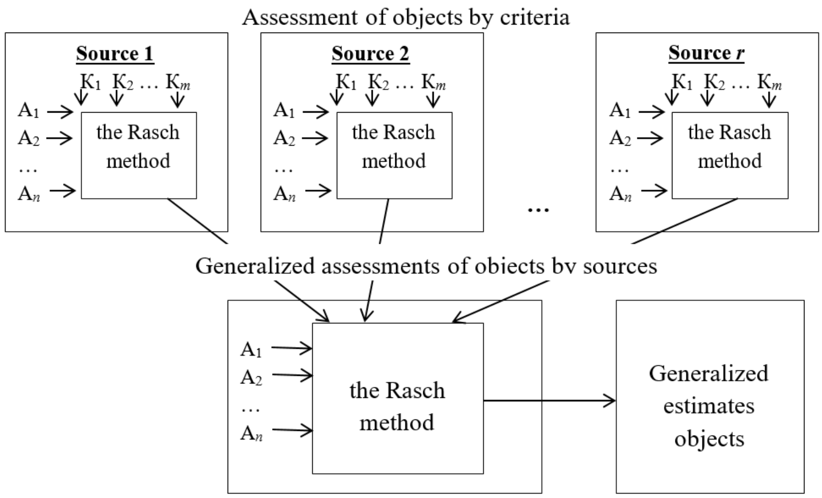

Fig. 3. Block diagram of the two-stage assessment procedure objects of complex economic systems for a variety of information sources

Thus, a two-fold generalization of the initial data is carried out: first, according to the criteria for each source of information, then according to the sources of information. The obtained estimates of the integral indicator for all objects and the estimates of the properties of criteria and information sources will be independent and measured on a linear scale.

6 The study of properties of estimates of generalized indicators of objects

Consider the properties of estimates of generalized indicators obtained by the Rasch model. For these purposes, algorithms for computational experiments were developed, which allowed us to check the adequacy of assessments, their linearity, accuracy, and resistance to changes in indicators.

To verify the adequacy, on a set of randomly generated data, the evaluations of objects of distributed economic systems and criteria obtained from traditional assessment models and the Rasch model were compared. The results for a large number of matrices of different sizes were analyzed. Using the methods of correlation analysis, it was shown that the estimates are in good agreement with each other.

To verify the linearity of the estimates, a matrix of values of indicator variables was repeatedly generated, which should give a priori linear estimates of generalized indicators. It is shown that the Rasch model gives linear estimates, when traditional estimation methods give a noticeable non-linearity of the estimates obtained, which can lead to a distortion of the result when evaluating objects of economic systems.

In addition, the assessment of the sustainability of assessments of integrated indicators was carried out when indicators changed, and the accuracy of the estimates obtained was assessed. For this, the methods of simulation modeling [12] were used and a computational experiment was carried out, the algorithm of which was based on the generation of indicators with given properties and analysis of the calculated generalized estimates. It was shown that the values of generalized indicators obtained by the Rasch model are highly accurate and resistant to fluctuations in indicator variables not exceeding 0.4 of their absolute values. This is illustrated by the graphs shown in Fig. 4.

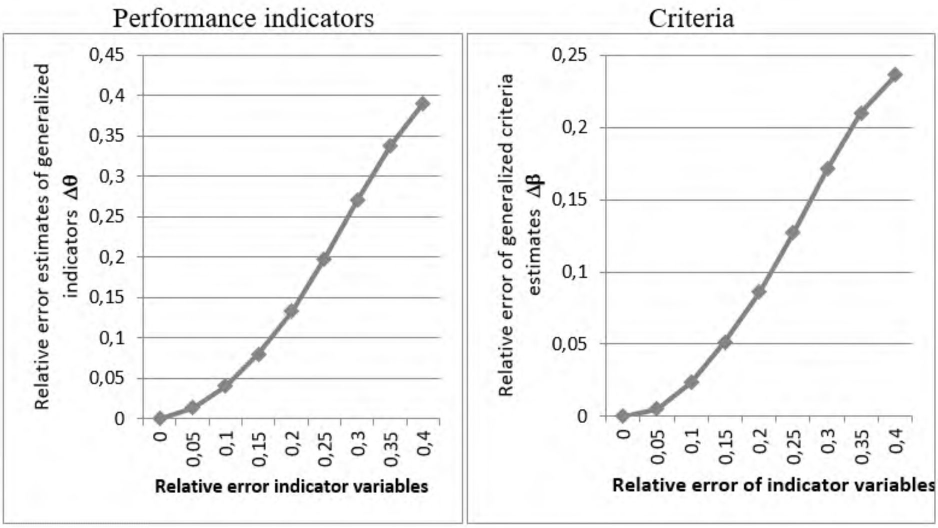

Fig. 4. The dependence of the relative accuracy of the estimates of generalized indicators and criteria of the error indicator variables

Thus, computational experiments to study the properties of integral estimates justify the adequacy of the measurement model of generalized indicators of objects of socio-economic systems according to the Rasch model and prove the linearity of the obtained estimates, accuracy and stability to changes in the source data.

7 Conclusions

Thus, the paper substantiates the application of the Rush model for estimating latent variables to the tasks of managing socio-economic systems. A mathematical model is presented for estimating generalized indicators of objects of complex socio-economic systems based on the Rasch method for measuring latent variables, which made it possible to obtain independent estimates of generalized indicators on a linear interval scale. Methods for calculating weights of indicators are proposed.

In the case of multiple sources of information, the model assumes a double convolution of the vectors of the initial data based on the Rasch method for evaluating latent variables.

To analyze the properties of estimates of generalized indicators, algorithms for identifying the properties of the model have been developed [13], which made it possible to organize computational experiments to analyze the adequacy, linearity, accuracy, and stability of the estimates obtained. The results of computational experiments showed that the proposed model provides adequate estimates that correlate well with estimates obtained by traditional methods. It is also shown that methods based on the Rasch model provide linear estimates and are highly accurate and resistant to changes in indicator variables.

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