

New approaches to multicriterial evaluation of alternatives in decision-making

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Abstract. This work is devoted to the development of mathematical models that make it possible to generalize the data obtained in the multi-criteria assessment of alternatives when making decisions under certainty conditions. A typical problem in multi-criteria assessment is the integration of particular criteria-based assessments of alternatives into some integral indicator reflecting the degree of attractiveness of alternatives. Traditionally, for these purposes, an additive method was used, which consists in the fact that the final assessment of the alternative is the sum of the assessments by the criteria, possibly multiplied by the weights of the criteria. However, this approach has a number of significant disadvantages, which include the following. When evaluating alternatives, such an indicator of their validity as feasibility is not taken into account, which leads to a distortion of the result due to too simple or difficult criteria. Evaluations of alternatives by qualitative and attributive criteria are subjective, since they are obtained by expert methods, which introduces subjectivity into the integral assessment of the alternative. The integral assessment of an alternative depends on the composition and number of assessed alternatives and on the set of assessment criteria. To eliminate the second drawback, a model for evaluating qualitative alternatives is proposed, which implies the introduction of a certain numerical indicator for a qualitative criterion, which for each alternative have a different meaning, while the attractiveness of the alternative depends on the indicator value. The set of alternatives in relation to the indicator is considered fuzzy, with the belonging function depending on the indicator.

1 Introduction

In any area of professional activity, and especially in the field of management and administration, the quality of the decisions made plays a key role. It is on the decision-making process that the result of the professional activity of the decision-maker (DM) largely depends, and as a consequence, the efficiency of the unit controlled by him.

Decision-making is a process that, on the one hand, is extremely important and necessary for every person, and on the other hand, it is a very complex process, which in most cases depends on many external factors and internal conditions. Currently actively developing decision support systems based on artificial intelligence technologies make it

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possible to automate decision making by providing significant assistance to decision makers.

However, these technologies require the development of new and improvement of existing decision support models, reflecting the realities of the functioning of the subject area. This emphasizes the importance and relevance of any scientific research related to the development of mathematical models in the field of modeling decision support systems.

2 Materials and methods

As mentioned earlier, despite the importance of improving decision support systems, this direction from the point of view of mathematical modeling is quite complex, due to the fact that the decision-making process is usually influenced by a huge number of both external and internal factors, most of which are stochastic in nature. This fact allows us to conclude that the decision-making procedure is subjective and probabilistic. This leads to the fact that the result of the decision-making activity of the decision maker largely depends on his personal characteristics and on the degree of influence of random factors affecting this result.

However, there is one direction in the theory of decision making, when the result of a decision made can be calculated with sufficient accuracy, when the influence of subjective and random factors is minimized or practically absent. This situation arises in cases when there is the most complete information about the alternatives and criteria, which makes it possible to obtain a fairly objective assessment of the attractiveness of alternatives and to select the optimal alternative with high accuracy. This situation is called a decision-making model in conditions of certainty [1].

A mathematical model of decision making under certainty conditions has a number of requirements for its applicability. Let's briefly describe these requirements.

1. It is necessary to accurately define the goals and objectives of decision-making, to identify a finite set of alternatives.
2. Select a finite number of evaluation criteria, describe methods for obtaining evaluations of alternatives according to these criteria from the point of view of their attractiveness for decision makers. Taking into account the fact that the criteria can be not only quantitative, the attractiveness of alternatives for which can be objectively measured, but also qualitative, attributive and others that do not allow obtaining objective assessments, then for such criteria unambiguous procedures for obtaining numerical assessments on a given scale should be prescribed.
3. There should be numerical ratings of the importance or weights for each criterion.
4. It is necessary, on the basis of private multicriteria estimates, to describe the procedure for obtaining a numerical integral assessment of the attractiveness of all alternatives, taking into account weights, which will allow the decision maker to choose the optimal alternative when making decisions.

The decision support model in conditions of certainty will allow the decision maker to objectively choose the optimal alternative, however, here too there are problems in the practical use of the model, which are associated with subjectivity and uncertainty.

The first such problem is associated with obtaining quantitative assessments of alternatives according to qualitative criteria. Usually this problem is solved by the methods of expert assessment (it is necessary to take a responsible approach to the selection of experts, assessing their professional competencies, in this case in the field of energy management), which are intensively developing today.

The second problem lies in the ambiguity of obtaining estimates of the integral indicator of the attractiveness of an alternative, which is commonly called the utility function [1, 2]. The utility function of alternatives is subjective, which is associated with the peculiarity of

mathematical methods for processing partial multi-criteria assessments of alternatives. Today, in practice, the method of obtaining integral estimates of the attractiveness of alternatives is used, which is usually called additive. Despite its simplicity, this method has several disadvantages. In this paper, these shortcomings will be considered and mathematical models will be proposed that make it possible to eliminate these shortcomings to one degree or another. In addition, computational experiments will be described, which will make it possible to substantiate the adequacy of the obtained integral estimates of alternatives and quantitative estimates by qualitative criteria.

Thus, the purpose of this work is to develop mathematical methods and models that allow one to obtain objective integral assessments of the attractiveness of alternatives for decision-makers in multi-criteria assessment, as well as to obtain quantitative assessments of alternatives by qualitative criteria.

For this, for the task of obtaining quantitative estimates of alternatives by qualitative criteria, the theory of fuzzy sets will be used, and for the task of obtaining an integral indicator of the attractiveness of alternatives - the Rush model for estimating latent variables.

3 The classical approach to making decisions in conditions of certainty

Let us first consider the classical method that is traditionally used when making decisions in conditions of certainty.

Take n alternatives, denote them: A_1, A_2, \dots, A_n . Alternatives are evaluated by m criteria, which we will designate as: K_1, K_2, \dots, K_m . We denote the weight of the criteria by w_j . Let us define through U_{ij} a particular estimate of the i -th alternative by the criterion K_j . We will assume that individual assessments of alternatives according to criteria can be measured on different scales: numerical, dichotomous, polytomical, attributive, or others, it is important that they are quantitative. If the criterion is qualitative, then it is necessary to obtain estimates for it in a quantitative form, using the methods of expert assessment [2, 3].

A necessary condition for determining the utility functions of alternatives is that all particular criterion assessments are measured on a single scale. In view of this, using linear transformations, the obtained estimates of U_{ij} must be normalized to a single scale. A single scale is usually chosen as such a scale. The following linear transformation of the U_{ij} measurements to the normalized estimates of the u_{ij} alternatives can be used:

$$u_{ij} = \begin{cases} \frac{U_{ij} - \min_i(U_{ij})}{\max_i(U_{ij}) - \min_i(U_{ij})}, & \text{quality growth;} \\ \frac{\max_i(U_{ij}) - U_{ij}}{\max_i(U_{ij}) - \min_i(U_{ij})}, & \text{decrease.} \end{cases} \quad (1)$$

In the classical approach, for each alternative, a certain utility function $F_i = F(u_{ij}, w_j)$, is calculated, which can be used as an integral assessment of the attractiveness of alternatives:

$$F_i = \sum_{j=1}^m w_j u_{ij} . \quad (2)$$

Having calculated the utility function (2), the decision maker chooses the alternative for which the value of the function is greatest.

The additive method is often used in practice due to its simplicity, however, it has several disadvantages.

1. When finding the utility function, it is not taken into account how strictly, or vice versa, loyally the alternatives are assessed for each criterion. In particular, when the main part of the alternatives have high scores on the criterion and only a small part of them are low, which is typical for a loyal criterion, then such a criterion will make a greater contribution to the integral indicator of the alternative than a strict criterion for which the main part of the alternatives have low scores. Failure to take into account the degree of influence of the criterion on the alternatives can lead to nonlinearity of the utility function.
2. As a result of the normalization of assessments of alternatives, part of the original information is lost.
3. Evaluations of alternatives by qualitative and attributive criteria are subjective, since they are obtained by expert methods, which introduces subjectivity into the integral assessment of the alternative.
4. The integral assessment of the alternative depends on the composition and number of the assessed alternatives and on the set of assessment criteria.

Consider alternative mathematical methods for obtaining an integral assessment of alternatives and partial estimates for non-quantitative criteria.

4 Indicator method based on fuzzy sets

It allows you to fairly objectively obtain a private assessment of the alternative by a qualitative criterion on a single scale, which allows you to avoid the normalization procedure. The idea of this approach is to find one or more numerical indicators for the criterion, which for each alternative have a different meaning, while the attractiveness of the alternative depends on the indicator value. For example, when evaluating possible projects according to the criterion "attractiveness of the project from the standpoint of energy management", such objective parameters as the cost of the project, implementation time, energy efficiency and the like can be used as such indicators.

Let us first take the case when for some criterion there is one indicator, and its value for the i -th alternative is equal to x_i . The degree of influence of the indicator on the attractiveness of the alternative can be calculated using the methods of the theory of fuzzy sets. Suppose that for the j -th criterion there is a fuzzy set of elements in the category "attractiveness of an alternative" with the function of belonging to this set $\mu_j(x)$. Then the particular indicator of the attractiveness of the alternative by the criterion u_{ij} can be written as:

$$u_{ij} = \mu_j(x_i). \tag{3}$$

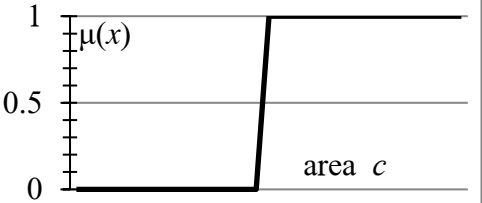
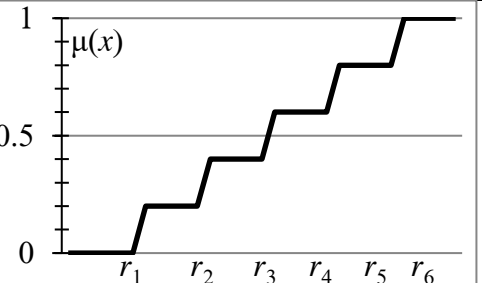
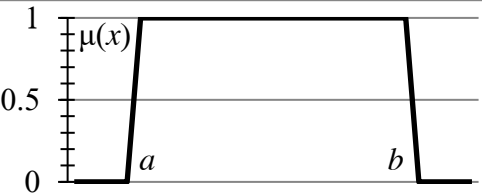
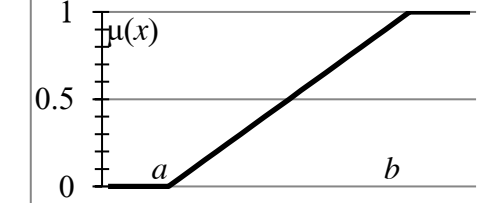
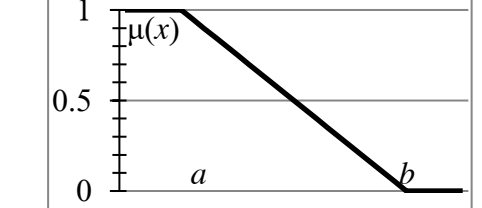
The integral estimate of the alternative in accordance with (2) will be equal to:

$$F_i = \sum_{j=1}^m w_j \mu_j(x_i). \tag{4}$$

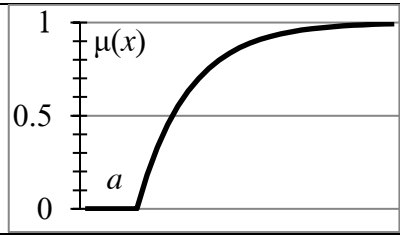
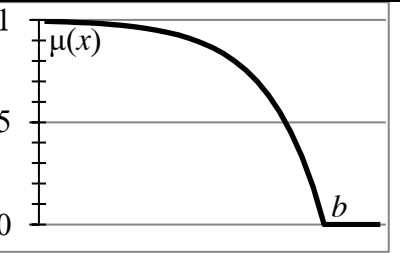
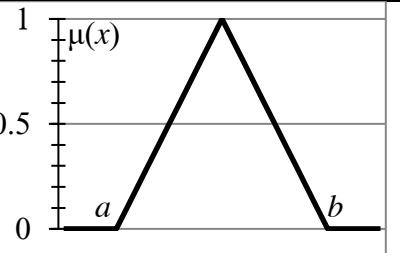
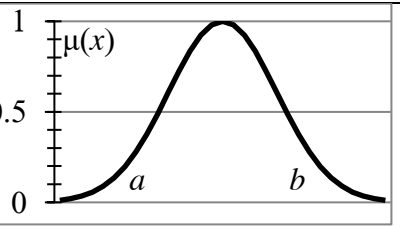
In the case of the presence of several indicators for the criterion, the private score is calculated using disjunction or averaging of the membership function for each indicator.

An important role in the proposed estimation model is played by the choice of membership functions to describe the influence of the indicator on attractiveness. Table 1 shows the main types of possible membership functions that can be used to evaluate alternatives.

Table 1. Types of membership functions $\mu(x)$

<p><i>Dichotomous function.</i> There is an attribute c and attractiveness - an element of a clear set belongs to it if the feature is satisfied.</p> $\mu(x) = \begin{cases} 1, & \text{by } x = c; \\ 0, & \text{by } x \neq c. \end{cases}$	
<p><i>Polytomic function.</i> There is an attributive indicator, but with k, attributes differing in the degree of correspondence $r(x)$.</p> $\mu(x) = \varphi(r(x)),$ $r(x) = 1, 2, \dots, k,$ $0 \leq \varphi(r) \leq 1,$ <p>With a uniform law:</p> $\mu(x) = \frac{r-1}{k-1}, \quad r = 1, 2, \dots, k.$	
<p><i>Uniform function.</i> Indicator x should fall in the interval from A to b.</p> $\mu(x) = \begin{cases} 1, & \text{at } x \in [a; b]; \\ 0, & \text{at } x \notin [a; b]. \end{cases}$	
<p><i>Function with optimum and minimum value.</i> Indicator x is desirable to be more than b, but not less than A.</p>	
$\mu(x) = \begin{cases} 0, & \text{at } x < a; \\ \frac{x-a}{b-a}, & \text{at } x \in [a; b]; \\ 1, & \text{at } x > b. \end{cases}$	
<p><i>Function with optimal and maximum value.</i> Indicator x must be less than A, but not more than b.</p>	
$\mu(x) = \begin{cases} 1, & \text{at } x < a; \\ \frac{b-x}{b-a}, & \text{at } x \in [a; b]; \\ 0, & \text{at } x > b. \end{cases}$	

Direct exponential function. It is necessary that the indicator x is not less than a and the more, the better.

$\mu(x) = \begin{cases} 1 - \exp\left(-\frac{x-a}{a}\right), & x > a; \\ 0, & x \leq a. \end{cases}$	
<p><i>Inverse exponential function.</i> It is necessary that the indicator x is not more than b and the less, the better. For practical calculations, the most acceptable is the value of the parameter $p=2$, although some estimates require smaller values of the parameter p.</p>	
$\mu(x) = \begin{cases} 1 - \exp\left(\frac{x-b}{b^{1/p}}\right), & x \leq b; \\ 0, & x > b. \end{cases}$	
<p><i>Triangular function.</i> Indicator x should fall within the interval from A to b, but is optimal in the middle of this interval.</p>	
$\mu(x) = \begin{cases} 1 - 2 \frac{ (a+b)/2 - x }{b-a}, & \text{at } x \in [a; b]; \\ 0, & \text{at } x \notin [a; b]. \end{cases}$	
<p><i>Gaussian function.</i> Those indicators that are formed as a result of the influence of a large number of factors are random values that are distributed according to the normal law. In this case, instead of a triangular function, it is better to use a function based on the normal law.</p>	
$\mu(x) = \exp\left[-8 \frac{(x - (a+b)/2)^2}{(b-a)^2}\right]$	

The technique described here makes it possible to objectively obtain assessments of the attractiveness of alternatives according to non-quantitative criteria, but it does not correct the shortcomings associated with the calculation of an integral assessment of the attractiveness of alternatives for decision makers. Consider the methodology for obtaining independent integral assessments of the attractiveness of alternatives, according to a linear interval scale.

5 Results

Obtaining integral assessments of the attractiveness of alternatives based on the theory of latent variables

The proposed method is based on the Rush model for estimating latent variables [4]. The main provisions of this model are described in the works [5-6].

We denote by u_{ij} the normalized partial estimate of the i -th alternative by the j -th criterion, which can also be found by the indicator method.

We introduce latent variables of the following form:

F_i – integral assessment of the degree of attractiveness for the decision maker of the i -th alternative or utility function;

Q_j – is a latent characteristic of the severity or loyalty of the j -th criterion, and the larger this characteristic, the more loyal the criterion is. In this approach, the probability p_{ij} that the j -th criterion will assess the attractiveness of the i -th alternative is higher than its assessment of its loyalty, is determined by an expression of the form [4]:

$$p_{ij} = \frac{e^{F_i - Q_j}}{1 + e^{F_i - Q_j}}. \tag{5}$$

To obtain numerical estimates of the latent variables θ_i and β_j , which are calculated based on the Rush model, we will use a modification of this model, with a mathematical kernel based on the least squares method [7-9]. According to this model, it is necessary to solve the optimization problem of the following form:

$$\sum_{i=1}^n \sum_{j=1}^m w_j \left(u_{ij} - \frac{e^{F_i - Q_j}}{1 + e^{F_i - Q_j}} \right)^2 \rightarrow \min, \tag{6}$$

$$F_i \geq 0; Q_j \geq 0; i = 1, 2, \dots, n; j = 1, 2, \dots, m.$$

The resulting estimates of the membership function and the feasibility of the criteria will be devoid of the disadvantages described above when obtaining estimates of alternatives using the classical additive method, and have the following advantages, which follow from the theoretical foundations of estimates obtained using the Rush model [10]:

1. The obtained integral estimates of the utility function for alternatives are their unique characteristics and will not depend on the number of alternatives and the set of criteria.
2. Integral assessments of the attractiveness of alternatives are measured on a linear scale that takes into account the influence on the utility function of the influence of criterion assessments.
3. In addition to assessing the degree of attractiveness for alternatives, the model allows you to obtain estimates of the degree of rigor or loyalty of the criteria.

As a disadvantage of the method, it is not possible to calculate the utility functions analytically, and to solve the optimization problem (6), it is necessary to apply numerical methods.

To check the adequacy of the proposed model, computational experiments were carried out, the essence of which was to generate matrices of partial estimates of alternatives for a different number of alternatives and criteria and to compare the integral estimates of alternatives obtained by different methods. One of the typical examples for data from table. 2 is shown in the figure (estimates are normalized to a unit amount).

Table 2. Evaluation data for 8 alternatives on 7 criteria

Alter-	Criterion
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native	K_1	K_2	K_3	K_4	K_5	K_6	K_7
A_1	0.872	0.961	0.746	0.655	0.075	0.242	0.199
A_2	0.182	0.408	0.613	0.310	0.175	0.609	0.127
A_3	0.404	0.199	0.482	0.288	0.127	0.531	0.318
A_4	0.688	0.291	0.414	0.304	0.295	0.732	0.543
A_5	0.877	0.149	0.266	0.495	0.292	0.431	0.436
A_6	0.155	0.634	0.806	0.813	0.522	0.499	0.541
A_7	0.473	0.054	0.268	0.094	0.058	0.418	0.329
A_8	0.927	0.247	0.123	0.209	0.964	0.331	0.178
The weight w_j	0.345	0.582	0.149	0.831	0.379	0.765	0.535

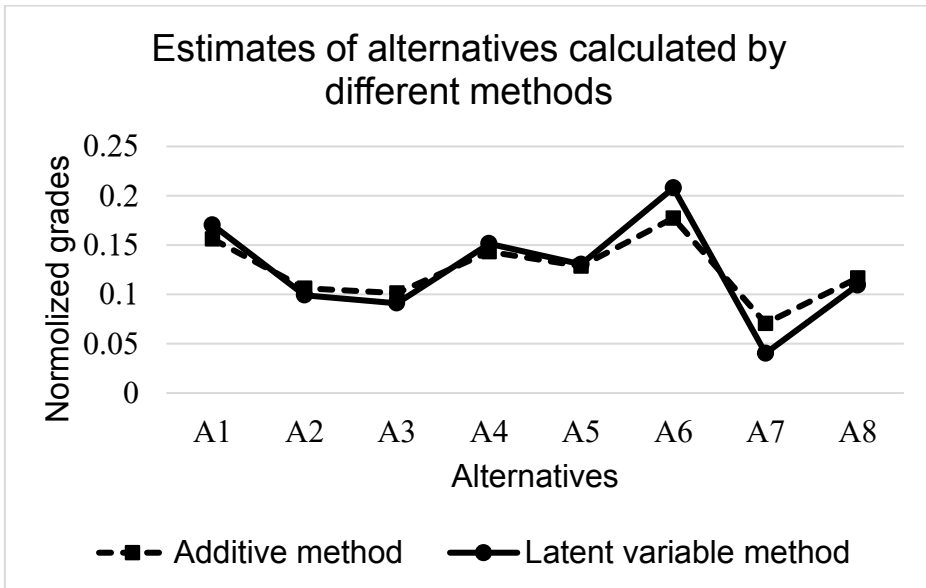


Fig. 1. Comparison of estimates of utility functions

The figure shows good consistency of the estimates of alternatives obtained by the classical additive method and the method based on the theory of latent variables. Similar results were obtained in all computational experiments, the correlation coefficient between the estimation results averaged 0,98.

The consistency of the estimation result according to the proposed model, with the estimates according to the approved additive method, confirms the adequacy of the estimates obtained.

6 Conclusion

In conclusion, it should be added that similar techniques, based on the Rasch model for estimating latent variables based on the maximum likelihood method, were used to assess the degree of personnel competence [13], to assess the quality of projects according to qualitative criteria [14], as well as to assess economic security of organizations [15].

Thus, the paper describes new methods for obtaining estimates of alternatives when making decisions in conditions of certainty, which can be successfully applied in energy management. The classical additive method is considered, its advantages and disadvantages are noted. To eliminate the disadvantages, two methods are proposed:

1. An indicator method based on the theory of fuzzy sets, which allows one to obtain objective private assessments of the attractiveness of alternatives by non-quantitative criteria based on the determination of one or a group of numerical indicators related to the degree of attractiveness of alternatives by the criterion. The main types of membership functions of alternatives are given in terms of their attractiveness.
2. The method for calculating the utility function in multi-criteria assessment, which can be interpreted as an integral assessment of alternatives. The method uses the Rush model for estimating latent variables, as a result of which it is possible to obtain independent estimates of alternatives on a linear scale and evaluate the properties of the criteria.

Computational experiments have shown that the models provide adequate estimates and can be used in decision-making in various fields of activity and used in the development of decision support systems.

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