

# Raven paradox: problem and solution given on the basis of Aristotle's logic

Anton Zamorev<sup>1</sup> and Alexander Fedyukovsky<sup>1,\*</sup>

<sup>1</sup>Peter the Great St. Petersburg Polytechnic University (SPbPU), Polytechnicheskaya, 29, 195251, St. Petersburg, Russia

**Abstract.** The paper is devoted to the solution of two well-known paradoxes of inductive logic: Hempel's and Goodman's, which the science has not solved unambiguously yet. The results of this research can be used in any natural science, but they are especially relevant for areas where there is an emphasis placed on environmental friendliness and sustainable development. The central problem of this research is the problem of limits of applying classical logic laws. The problem is solved by method of reduction of logical laws to those cases where they, according to Aristotle, act faultlessly, and refusal of their recognition in the cases where their action is questionable. The aim of the paper is to demonstrate the solution of both problems within Aristotle's logic. In that regard, the following results are received: common faults of previous solutions of Hempel's paradox, consisting in ignoring any of its parties, are revealed; the original nature of Goodman's paradox, consisting in wrong interpreting "inductive confirmation" criteria is opened; two methods of forming and assessing the subject volumes of statements are revealed: analytical and synthetical ones; it is proved that the theses treated in Hempel's paradox as equivalent ones are not always so, but only on condition of their subjects' reality and of their subject volumes' identity; it is established that the conditions of the statement equivalence correspond to the limits of applying logic laws in Aristotle's interpretation.

## 1 Introduction

This research is devoted to Hempel's paradox, which is of interest not only to experts in the field of logic, but also to a wide range of researchers [1-5]. Elliott, e.g., sees in a paradox a possibility of receiving evidence about the truth of some proposition by investigating matters that are seemingly irrelevant to that proposition [6].

Hempel's paradox (1) is called also as the raven paradox as it is connected with two theses: one states "All ravens are black", and the other states "All non-black ones are non-ravens" [7]. The problem is the following:

1) On the one hand, these statements are recognized as equivalent in classical logic, i.e. having equal value of truthfulness, so any fact admitted by inductive confirmation of the one thereby confirms also the second with the same probability.

---

\* Corresponding author: [fedyukovsky@mail.ru](mailto:fedyukovsky@mail.ru)

2) On the other hand, observing non-black objects, e.g., red apples, does not increase the confidence that all ravens are black.

Though C. Hempel approved the latter provision as purely intuitive, most scientists have not challenged it. We also do not intend to do it, and therefore the problem entirely rests against formal logic laws on the basis of which these statements are recognized as equivalent ones. Since C. Hempel's intuitive objection calls them into question.

The common fault of all solutions proposed for this paradox before is partial ignoring any of its parties. As a rule, this ignores the fact that statements "*All S are P*" and "*All P' are S*" are considered as equivalent ones in classical logic, or ignores the reasons of their equivalence approval. Peterson, e.g., claims that the raven paradox can be avoided without requiring the satisfaction of a relevant implication. For instance, the paradox can be avoided by rejecting the requirement of classical negation [8].

The most popular solution of Hempel's paradox is connected with using the Bayes theorem [9], which allows to calculate the event probability provided that there was the other event which is statistically interconnected with the first one. Different ways of using this theorem prove that contemplating red apples does not increase the probability of correctness of the conclusion "*All ravens are black*" at all, or increases it, but so little that it is intuitively intangible.

The lack of this solution is that it only confirms one of the paradox parties, namely: C. Hempel's intuition that red apples do not increase confidence in the conclusion "*All ravens are black*", and confirm only the conclusion "*All non-black ones are non-ravens*". But then these statements are not equivalent. I.e., it is necessary to reconsider the bases on which they were recognized so before. And it was just not made properly.

There are some, not less widespread at all, articles criticizing application for solving the paradox of the Bayes theorem, however claiming that "there is no analysis of this evidential relation that satisfactorily answers the Paradox of the Ravens, and the prospects for any answer along these lines are bleak" [10].

There is a similar lack in the solution proposed by the American philosopher N. Goodman [11], who suggested to introduce in logic the rule stating that the conclusion "*All S are P*" is impossible to be confirmed with the objects which confirm the conclusions "*All P' are not S*" or "*All P' are S*". Then C. Hempel's intuition that red apples do not increase our confidence in the conclusion "*All ravens are black*" acquires the status not of an intuition but that of a logical rule, i.e. the conflict between logic and intuition is eliminated externally.

But then there is a conflict in logic itself which insists on the equivalence of statements like "*All S are P*" and "*All P' are S*". Therefore, on the fact that any object confirming the second one confirms also the first one.

The American philosopher W. Quine who, on the contrary, based on the requirement to consider statements "*All ravens are black*" and "*All non-black ones are non-ravens*" as equivalent ones. At the same time, C. Hempel's intuitive objection was explained with N. Goodman's division of predicates into projective and non-projective:

1) statements which can be confirmed by the corresponding objects inductively were called projective ones. W. Quine classified as such, in particular, the conclusion "*All ravens are black*" which is confirmed only by contemplating black ravens.

2) statements which cannot be inductively confirmed with the objects corresponding to them were called non-projective ones, but they are confirmed only by equivalent projective conclusions. So the conclusion "*All non-black ones are non-ravens*" is classified by W. Quine as non-projective.

Consequently, contemplating red apples will not confirm not only the conclusion "*All ravens are black*", but even the conclusion "*All non-black ones are non-ravens*", owing to non-projectivity of the latter. But, according to W. Quine, on the contrary, by contemplating

black ravens it is possible to confirm inductively both conclusions, taking into account their equivalence.

The first lack of this solution is that W. Quine did not provide an accurate criterion to distinguish non-projective predicates. If we use the criterion of non-projectivity in Goodman's paradox, it will appear that the predicate "non-black" does not correspond to this criterion, i.e., from this point of view, it is projective. We shall explain it in more detail.

Goodman's paradox (2) is connected with fantastic assumption that all emeralds observed by us are green not always but only *until we can observe them*, i.e. till some time of *T* after which they become blue [11]. N. Goodman suggests to designate this property (to be green till *T*, but blue after it) with the predicate "grue" as something between "green" and "blue". The question is: Is it possible to draw, from the fact of observing only *grue* emeralds by us, an inductive conclusion that *all of them are grue* just as we, from the fact of observing only *green* emeralds, draw an inductive conclusion that *all of them are green* ?

The answer is: It is impossible. The predicate "grue" is a conjunction of two defining signs (green before *T* and blue after it). It means, to confirm that a certain emerald is grue is possible only having noted its two signs. For this purpose it is necessary to check it, at least, twice: before *T* to find it green, and after *T* to find it blue. But checking after *T* is impossible, according to the situation. It means, it is only possible to confirm that the emerald is *green before T*, but to conclude that it is *grue* is impossible.

Paradoxicality of this situation is only in the fact that usually we connect contemplating a thing with confirming its properties. Contrary to it, contemplating a grue emerald will not confirm that it is grue.

Therefore, the analysis of Goodman's paradox gives us the following criterion of "non-projectivity": a predicate is non-projective in only case when it is defined as *conjunction of signs from which at least one cannot be confirmed in principle*. As a result, if it is impossible to confirm this sign for a single thing, generalization is also impossible.

If this criterion is applied in the problem about "ravens" and "non-black", it will become clear that the predicate "non-black" is *projective*. It is defined not by conjunction, but disjunction of signs (it is either red, or blue, or yellow, etc.). Even one of them, by definition of disjunction, is enough to recognize a subject as "non-black", therefore, generalization is also possible.

The second lack of W. Quine's solution is that it ignores the very essence of C. Hempel's intuition that statement cannot be confirmed by objects which are not mentioned in it. W. Quine assumed that the statement "*All non-black are non-ravens*" is confirmed by contemplating black ravens. And it equally contradicts both C. Hempel's intuition, and the common sense to which W. Quine refers.

The following example allows to be convinced of it: we shall assume that we reduced the universe considered into the contents of a certain black box, did not find any raven in it, but investigated all non-black objects which are in the box (e.g., a pair of red apples). According to the concepts of traditional logic, it means that the conclusion "*All non-black ones in this box are non-ravens*" receives full inductive confirmation.

However, W. Quine's logic equates this full induction to total absence of confirmation as a confirmable conclusion is allegedly non-projective, and a projective conclusion, equivalent to it, "*All ravens in this box are black*" was not confirmed in any way. We ask: Is it not absurd to regard, on the one hand, *full induction as confirmation lack*? On the other hand, is it not absurd to insist on the equivalence of statements from which one is confirmed completely, but the second one is *not at all*?

Many philosophers have been induced by such counterexamples not to ignore the equivalence of C. Hempel's statements of "ravens" and "non-black", but to *deny* it directly. Koshy denies the mere possibility of applying paradoxical instances like white shoe and red pencil, since they do not specify what the contrapositive instance of the hypothesis all ravens

are black is [12]. However, at the same time they often ignore the reasons of approving their equivalence in classical formal logic.

## 2 Materials and methods

In this work, the problem is solved with the same method applied by the founder of formal logic Aristotle to solve the paradoxes he knew. Namely, with the method of reducing logical rules only into that universe considered within which they work faultlessly, and refusing their recognition in the cases where their action is questionable. If we study these cases in detail, there will appear two problems to be solved to fully remove Hempel's paradox.

Problem 1. It is known that the statements like "*All S are P*" and "*All P' are S*" are interconnected by means of two fundamental laws of Aristotle's logic. They form two conditional assumptions of a simple syllogism which proves the equivalence of these statements. Therefore, in order to deny the latter, it is necessary to consider the bases of each law and to clarify under what conditions at least one of them could not act.

The law of contradiction (1) is the provision stating that two contradicting each other statements  $P$  and  $P'$  about one subject of  $S$  cannot be simultaneously true in the same sense. And if all  $S$  are  $P$ , then any  $S$  is not  $P'$ , and any  $P'$  is not  $S$ . E.g., *if all non-black are non-ravens, then any non-black is not a raven, and any raven is not non-black*.

This conditional conclusion is the first assumption of the syllogism proving the equivalence of C. Hempel's theses about "ravens" and "non-black". Since this assumption results directly from the contradiction law, it is possible to cancel it only by *cancelling this law*. But such solution would destroy any proof theory. Because " $\langle \dots \rangle$  all who give the proof, bring  $\langle it \rangle$  to the provision as the last one: in essence, it is the beginning for all other axioms" [13], Aristotle claimed.

Actually, the contradiction law is a direct deduction from definitions of "truth" and "lie". We define "lie" as that contradicting the truth. It means that if at least one of the two statements is true, and the second one contradicts the first one, then the second one, thereby, *contradicts the truth*, i.e. it *lies*, by definition. The contradiction law also approves it.

Therefore, the contradiction law is not able to act only concerning senseless statements which are not true and are not false even hypothetically. But wherever there is even a possible truthfulness or falsehood of statements, there is a valid contradiction law, by definition. Since Hempel's paradox is just about statements which truthfulness is confirmed, both the law, and the conclusion from that, is to be valid here.

The law of excluded middle (2) is the provision stating that two contradicting each other statements  $P$  and  $P'$  about one subject of  $S$  cannot be simultaneously false in the same sense. And any  $S$ , if there is no  $P'$ , then it is either  $P$ , or not  $P$  at all. E.g., *if any raven is not non-black, then either all ravens are black, or there are no ravens in reality*.

The latter is important. Because it means that the law of excluded middle has, according to Aristotle, narrower limits of applicability than the previous law has. It is applicable to the statements not about all possible phenomena but only about real past ones, but is not able to act concerning the phenomena which are considered as a clear opportunity. Because " $\langle \dots \rangle$  in an opportunity the same can be opposite things together, but in real implementation it can not" [13], Aristotle taught.

Actually, the sets of  $P$  and  $P'$  are termed "contradicting", if one of them covers *all the reality* excluded from the other. It means, any single object  $S$  excluded from  $P$ , thereby is either included in  $P'$ , or *excluded from the real area*. In particular, to have black or non-black ravens, it is necessary that the ravens *existed* at least. But where ravens are not to *exist*, they *will be* neither black, nor non-black. Because " $\langle \dots \rangle$  about what does not exist, nobody knows what it is like" [14], Aristotle taught.

Therefore, in the syllogism confirming the equivalence of statements about "ravens" and "non-black" the weakest link is its second assumption which, on the basis of the law of excluded middle states: *if any raven is not black, then all ravens are black*. Because in such form the law of excluded middle acts only if ravens precisely exist.

Partly this provision explains why C. Hempel's intuition does not recognize the statements about "ravens" and "non-black" as equivalent ones. The fact is that these statements indeed appear as such *not always*, but only under the condition if the syllogism proving their equivalence is right. And it will be right for these statements if their subjects are real: "ravens" and "non-black". The intuitive contemplating red apples confirms only reality of the second one, but not reality of the first one at all.

Moreover, if there is no positive confirmation that ravens exist in general, then every non-raven we met, by the induction rule, is to increase just our confidence that there are no ravens in reality. And then the statement "*All non-black are non-ravens*" will inform nothing about ravens.

Hence the conclusion: if contemplating red apples is considered out of other experience, then the statement "*All non-black are non-ravens*" cannot be equivalent to the statement "*All ravens are black*", since in the syllogism proving their equivalence its second assumption does not act. But it does not mean at all as if philosophers who declared these statements nonequivalent are right since there appears the second problem.

Problem 2. It is known that C. Hempel stated the intuition of these statements not in the fictional world where there are no ravens, or where their existence was a question for someone. His total experience gave him the confirmation of reality of both "ravens", and "non-black". And under such condition, as we have ascertained, the statements investigated by us are equivalent. What is the reason of intuitive doubt in this fact? The answer is from the definitions:

1) equivalent statements are those which have identical truth value so the objects, confirming one of them, with the same probability confirm the second one.

2) objects confirming a statement with any probability are defined as those which correspond to the semantic value of this statement, i.e. constitute its subject volume.

Hence the conclusion: equivalent statements are those which have the same subject volume. And if the logic claims that two statements are equivalent under some condition, and the intuition under the same condition denies it, therefore, the logic and the intuition measure subject volumes of statements in different ways and in different units. That is how it is possible to explain that they coincide in one sense, and are various in the other.

The analytical way (1) is to consider simple categorical statement out of other experience, representing it as *conjunction of two signs: subject S and predicate P* inferred from the analysis of a single phenomenon intuitively contemplated. Since intuitive contemplation is always "<...> direct representation of a single thing" [15], I. Kant taught. Herewith, the statement volume is defined in two ways:

1) if the statement is affirmative, like "*S is P*", its semantic value corresponds only to *S*, which, at the same time, is *P*, that is, only such *SP*-objects form its analytical subject volume.

2) if the statement is negative, like "*S is not P*", its semantic value corresponds only to *S*, which, according to the law of excluded middle, either *is P*, or *is not at all*, that is, only such *SP'*-objects can form its real subject volume.

In both cases, however, the subject volume of the statement is constituted only by the objects included in the volume of subject *S*. Therefore, the statements with various subjects cannot be equivalent in the subject volume, formed in the analytical way.

For instance, if the subject of one statement is the term "ravens", and the subject of the other one - "non-black", then these statements will coincide in the subject volume only in case *all ravens are non-black*, and *all non-black are ravens*. But then the statements "*All ravens are black*" and "*All non-black are non-ravens*" will be simultaneously false. If, on the

contrary, both statements are true, then their analytical subject volumes not only do not coincide, but even never intersect in any common element.

Therefore, this analytical treatment does not allow to consider these statements as equivalent. And if their relative equivalence is inferred from the laws of classical logic, though not in general but only for some cases, in these cases the classical logic demands to measure their subject volume differently and in other units.

The synthetical way (2) is to consider simple categorical statement in synthesis with other experience. In this case, its subject volume is represented not as a set of separate things of class *S*, but as a set of all those possible conditions of the world in general (or those possible worlds as they are differently termed) when this statement would be true. Herewith, two options are allowed:

1) if the statement is true, then among the possible conditions of the world included in its subject volume, at least one occurs in reality during the time interval discussed.

2) if the statement is false, then among the possible conditions of the world included in its subject volume, none occurs in reality during the time interval discussed, i.e. it corresponds to an empty set.

Obviously, with such assessment of subject volumes *C*. Hempel's theses "*All non-black are non-ravens*" and "*All ravens are black*" are not always equivalent. We can imagine to ourselves the world in which there are non-black objects (e.g., red apples), but there are no ravens. Within such world the first statement will be true, and the second one – false, since its subject volume will be empty.

But if we, on the contrary, limit the universe considered by us only to the world where the existence of both ravens, and non-black objects is confirmed (e.g., red apples), in this framework the statements mentioned will be equivalent. Since in this case they have the same subject volume: each element, being a condition of the world in general, will confirm the statement of "ravens" with the existence of ravens in this world, and the statement of "non-black" will be confirmed with the existence of red apples or other non-black objects.

### 3 Results

Therefore, the following results were obtained:

1) Common faults of previous solutions of Hempel's paradox, consisting in ignoring any of its parties, are revealed.

2) The original nature of Goodman's paradox, consisting in wrong interpreting "inductive confirmation" criteria, especially concerning so-called non-projective predicates, is opened.

3) Two methods of forming and assessing the subject volumes of statements are revealed: analytical and synthetical ones.

4) It is proved that the theses treated in Hempel's paradox as equivalent ones are not always so, but only on condition of their subjects' reality and of their subject volumes' identity.

5) It is established that the conditions of the statement equivalence correspond to the limits of applying logic laws in Aristotle's interpretation.

### 4 Discussion

The relevance of the problem is connected with the fact that "classical logic" is often referred to as not a real logic of Aristotle, but its neopositivistic two-valued ersatz where the concepts of limits of applying the laws of logic are distorted. It is not strange that in it paradoxes multiply in a great number, creating new bases for revising the classical theory. Though some scientists (Ya. Lukasevitch, N. Brusentsov, etc.) repeatedly noted this fact, and even tried to



restore separate Aristotle’s ideas [16], but the logic of Aristotle in full of its provisions is seldom used. And the fact that it is free from paradoxes remains unobvious for many.

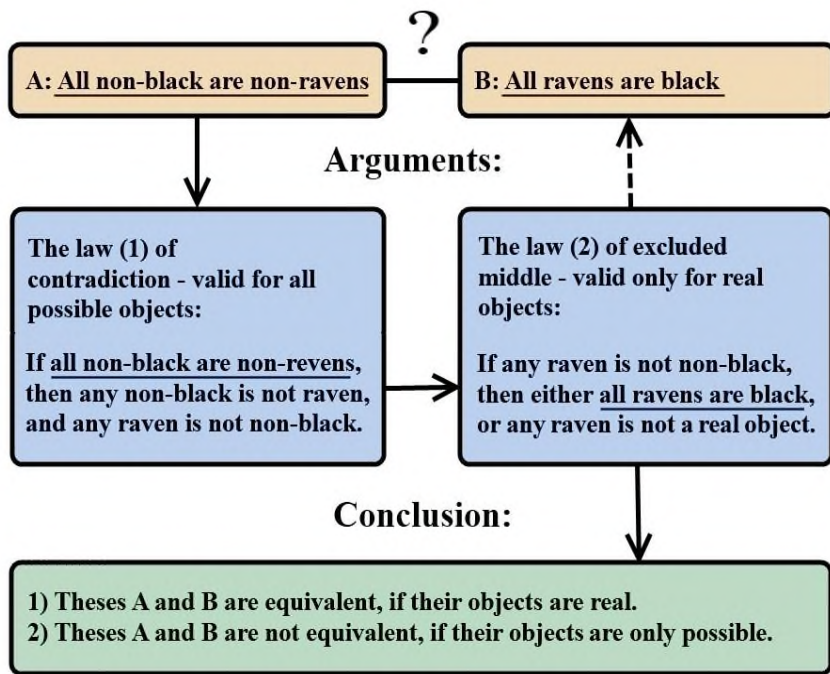


Fig.1. Raven Paradox: problem and solution.

## 5 Conclusion

The results of this research can be used in any natural science where the problem of inductive confirmation plays a key role. But they are especially relevant for those areas where there is an emphasis placed on environmental friendliness and sustainable development, namely: soil science, hydrology, oceanography, climatology, power engineering.

## References

1. M. Ramachandran, *A neglected response to the paradoxes of confirmation*, S. Afric. J. of Philos. **36(2)**, 179–185 (2017) <https://doi.org/10.1080/02580136.2016.1194152>
2. C. Hertogh, *Goodman’s Grue? Relativized Pluralism and Paradigmatic Thought Experiment (Whack)*, Humanities Bulletin **3(1)**, 64–103 (2020)
3. D. Bylieva, V. Lobatyuk, S. Tolpygin, A. Rubtsova, *Academic dishonesty prevention in e-learning university system. In A. Rocha, H. Adeli, L. P. Reis, S. Costanzo, I. Orovic, & F. Moreira (Eds.), 8th World Conference on Information Systems and Technologies, Advances in Intelligent Systems and Computing* **1161**, 225–234 (2020) [https://doi.org/https://doi.org/10.1007/978-3-030-45697-9\\_22](https://doi.org/https://doi.org/10.1007/978-3-030-45697-9_22)

4. D. Bylieva, Z. Bekirogullari, D. Kuznetsov, N. Almazova, V. Lobatyuk, A. Rubtsova, *Online Group Student Peer-Communication as an Element of Open Education, Future Internet* **12(9)**, 143 (2020) <https://doi.org/10.3390/fi12090143>
5. D. Bylieva, V. Lobatyuk, N. Ershova, Computer Technology in art (Venice Biennale 2019). Proceedings of Communicative Strategies of the Information Society (CSIS'19), 18 (2019) <https://doi.org/https://doi.org/10.1145/3373722.3373785>
6. K. Elliott, *How to Know That Time Travel Is Unlikely Without Knowing Why*, Pacific Philosophical Quarterly **100(1)**, 90–113 (2019) <https://doi.org/10.1111/papq.12256>
7. C. Hempel, Studies in Logic and Confirmation (Mind 54, 1945)
8. C. Peterson, *Studies in the logic of Confirmation*, Philosophical Studies **176(2)**, 437–471 (2019) <https://doi.org/10.1007/s11098-017-1023-1>
9. I. Lee, *Bayesian Confirmation Theory and Hempel's Intuitions*, Korean Journal of Logic **22(3)**, 351–395 (2019)
10. W. Peden, *The Selective Confirmation Answer to the Paradox of the Ravens*, International Studies in the Philosophy of Science **32(3–4)**, 177–193 (2019) <https://doi.org/10.1080/02698595.2020.1768014>
11. N. Goodman, *Fact, Fiction, and Forecast* (Cambridge, Massachusetts: Harvard UP, 1983)
12. P. Koshy, *A Solution to the Raven Paradox: A Redefinition of the Notion of Instance*, J. of Indian Council of Phil. Res. **34(1)**, 99–109 (2017) <https://doi.org/10.1007/s40961-016-0075-5>
13. Aristotle, *Metaphysics* (C.D.C. Reeve, Trans., Hackett Publishing Company, 2016)
14. Aristotle, *Analytica Priora et Posteriora* (Oxford University Press, 1964)
15. I. Kant, *The Metaphysical Foundations of Natural Science* (CreateSpace Independent Publishing Platform, 2015)
16. N. Brusentsov, *Three-valued interpretation of a syllogistics of Aristotle*, Historical and mathematical researches **8 (43)**, 317-327 (2003)