Study on drag reduction of flexible structure under flows

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Abstract. Flexible structures are common in nature and engineering. Under the action of fluid, the flexible structure bends to reduce drag. The study of this phenomenon has important scientific and engineering significance, so this article conducts research. Firstly, the differential equations are established, then solved by numerical methods, and finally numerically simulated.

1 Introduction

Drag reduction can be observed in plants. Vogel et al.[1, 2] have done pioneering work. The research found that compared with rigid body, the drag force of plant is reduced by elastic Reconstruction at high Reynolds number Re , which is no longer proportional to the velocity U^2 , and proposed the dimensionless Vogel exponent ν , $F \sim U^{2+\nu}$.

The structure of the paper is as follows: Section 2 introduces the simplified theoretical model of fluid force with Morrison formula. The third section, drag reduction under 2 different working conditions is discussed. Finally, it is summarized in Section 4.

2 Theoretical model

In order to simplify the complexity of the model, the following assumptions are used to simplify the model:

1. Uniform potential flow, no stickiness, no rotation, no compression
2. The model is the Euler beam, the slenderness ratio of the cantilever beam is large enough, only bending deformation is considered, and shear deformation is ignored
3. The beam cannot be extended
4. The potential flow acts on the normal direction of the beam surface, and the tangential direction of the beam surface is not subject to friction force

We use the curvilinear coordinate system shown in Fig. 1, in which S is the distance along the beam from the base and q is the local bending angle of the beam relative to the vertical.

Taking the force analysis of micro section dS , from the vertical force balance condition, we can get

$$V - f_D dS - \left(V + \frac{\partial V}{\partial S} dS \right) = 0 \quad (1)$$

where V denotes shear force, m denotes uniform mass per unit length.

The fluid force exerted by the uniform potential flow on the beam can be simplified as the drag force per unit length by using the Morison formula:

$$f_D = \frac{1}{2} C_D \rho (U \cos(\theta))^2 D \quad (2)$$

where C_D denotes the drag coefficient. ρ denotes the water density. U denotes the velocity of current. D denotes the diameter of beam.

Substituting Eq. (2) into Eq. (1), Eq. (1) can be rewritten as

$$\frac{\partial V}{\partial S} = -\frac{1}{2} C_D \rho (U \cos(\theta))^2 D \quad (3)$$

Taking the moment at the intersection of the right section of the segment and the s -axis based on the moment balance condition.

$$M + V dS - \frac{1}{2} \left(\frac{1}{2} C_D \rho (U \cos(\theta))^2 D \right) (dS)^2 - \left(M + \frac{\partial M}{\partial S} dS \right) = 0 \quad (4)$$

Ignoring the second-order trace in Eq. (4), we can get,

$$\frac{\partial M}{\partial S} = V$$

The relationship between bending moment and curvature of Euler beam is

$$\frac{M}{EI} = -\kappa$$

Substituting the obtained relationship into Eq. (3), we can get,

$$\frac{\partial^3 \theta}{\partial S^3} = \frac{1}{EI} \frac{1}{2} C_D \rho (U \cos(\theta))^2 D \quad (5)$$

Boundary conditions:

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$$\theta|_{s=0} = 0, \frac{\partial \theta}{\partial s}|_{s=L} = 0, \frac{\partial^2 \theta}{\partial s^2}|_{s=L} = 0$$

In order to make Eq. (5) dimensionless, S is

$$\text{normalized, } s = \frac{S}{L}$$

$$\frac{\partial^3 \theta}{\partial s^3} = C_Y \cos^2(\theta) \quad (6)$$

where. ρ_s is the density of the beam.

$$m = \frac{1}{4} \rho_s \pi D^2$$

$$C_Y = \frac{\rho U^2 C_D D L^3}{2EI}$$

Where C_Y is the Cauchy number, which the ratio of the hydrodynamic drag and the restoring force due to stiffness. The boundary conditions can be rewritten as

$$\theta|_{s=0} = 0, \frac{\partial \theta}{\partial s}|_{s=1} = 0, \frac{\partial^2 \theta}{\partial s^2}|_{s=1} = 0$$

Fig.1. Schematic the flow-induced reconfiguration of beam

3 Numerical analysis

3.1 Uniform beam under uniform flow

Following [3], two equations are introduced to obtain Vogel exponent.

$$R = \int_0^1 \cos^3 \theta ds \quad (7)$$

where R denotes Reconstruction number. When the deformed shape of the beam is obtained, i.e., θ is known, and the Reconstruction number R can be obtained through integration.

$$\nu = 2 \frac{\partial \log R}{\partial \log C_Y} \quad (8)$$

where ν denotes Vogel exponent.

The partial differential equations are calculated by direct integration method (DIM) and finite difference method (FDM), respectively. The beam is divided into 1000 evenly. Fig.2 shows the deformation shape of the

beam obtained by different numerical calculation methods under different Cauchy numbers C_Y . The deformation calculated by the direct integration method and the finite difference method is completely consistent with fig. 10 in [4], so the calculation results of the two methods are reliable. According to the deformation shape under different Cauchy number C_Y , the relationship between Reconstruction number R and Cauchy number C_Y can be obtained through Eq.7, as shown in Fig.3.

Fig.2. Deformation diagram of cantilever beam under uniform flow

Fig.3. Reconstruction number R and Cauchy number C_Y under uniform flow

Fig.4. Vogel exponent ν and Cauchy number C_Y under uniform flow

It can be seen from Fig.3 and Fig.4 that the results calculated by the two numerical methods almost overlap, which verifies the correctness of the two methods. The Vogel exponent first decreases and then increases with the increase of the Cauchy number C_Y , and finally remains unchanged. When the Cauchy number C_Y is large, the Vogel exponent is -0.667 ($-2/3$), which is the same as the Vogel exponent [3-5] is $-2/3$ are consistent. This further verifies the reliability of the results.

3.2 Uniform beam under shear flow

In deep sea, flow is usually shear flow. Therefore, the study of shear flow is of great practical significance. Let the velocity of shear flow be:

$$U = U_0 \left(\frac{y}{L} \right) \quad (9)$$

where U_0 is the velocity at the top of the vertical beam. The vertical height y is given by

$$y = \int_0^s \cos(\theta) ds \quad (10)$$

Incorporating Eq. (10) into Eq. (6) and integrating Eq. (6), we can get:

$$\left. \frac{\partial^2 \theta}{\partial s^2} \right|_s = -C_Y \int_s^1 \left(\int_0^{s^*} \cos(\theta) ds \right)^2 \cos^2(\theta) \cos(\theta - \theta^*) ds \quad (11)$$

Fig.5 shows the deformed shape of the cantilever beam under shear flow. With the increase of the Cauchy numbers C_Y , the bending of the cantilever beam gradually increases, but compared with the deformation shape of the cantilever beam under uniform flow, the bending curvature is smaller. The reason is that the flow rate of the shear flow decreases as y decreases compared with the uniform flow. When the Cauchy numbers C_Y reaches the order of 10^5 , the deformation shape of the cantilever beam is almost constant.

Fig.5. Deformation diagram of cantilever beam under shear flow

Fig.6 shows the relationship between the Reconstruction number R and the Cauchy numbers C_Y under shear flow. When the Cauchy numbers C_Y is greater than 1, it shows a linear decrease, and the linear slope is greater than that under the uniform flow.

Fig.6. Reconstruction number R and Cauchy number C_Y under shear flow

The relationship between the Vogel exponent and the Cauchy numbers C_Y under shear flow is shown in Fig.7, which also shows the same law as under uniform flow. That is, when the Cauchy numbers C_Y is less than 1, the Vogel exponent is 0. With the increase of Cauchy numbers C_Y , the Vogel exponent first decreased and then increased, and finally remained balanced, which was -1.18 , which was less than the Vogel exponent of $-2/3$ under uniform flow. It can be seen that under shear flow, the Vogel exponent is smaller than that under uniform flow, and the shear force drops more.

Fig.7. Vogel exponent v and Cauchy number C_Y under shear flow

4 Conclusion

We got the conclusion: Differential equations can use direct integration method and finite difference method, and the error of the results of the two methods is within the acceptable range. However, the numerical method is affected by the grid step size. It is recommended that the step size be as small as possible when the Cauchy numbers C_Y is large (i.e., large geometric nonlinearity). Under complex flow and complex external force conditions, the finite difference method is more stable than the direct integration method and easy to converge. The finite

difference method should be preferred.

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