

Research on Financial Market Risk Based on GARCH-M Model

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Abstract. Since 1970, with the gradual acceleration of economic globalization and the rapid development of information technology, the financial market has become increasingly unstable. Therefore, we must enhance our competitiveness in the financial market, enhance our ability to resist risks, and master effective measures such as measuring risks. In this paper, GARCH-M model and VAR method are used to study the value at risk of financial market and make an empirical analysis. Firstly, the VAR value calculation method based on GARCH-M model under generalized error distribution is given. Secondly, the closing price of Shanghai Stock Exchange Index is selected as sample data, and Eviews software is used to analyze its characteristics. The results show that the logarithmic yield series of the closing price of Shanghai Stock Exchange Index is not normally distributed, and the series has fluctuation aggregation effect, autocorrelation effect and heteroscedasticity effect. Finally, GARCH-M model is established, and VAR estimates at 95% and 99% confidence levels are calculated and tested. The results show that GARCH-M(1,1) model is more suitable for estimating the risk of logarithmic return rate of closing price of Shanghai Composite Index.

1 Introduction

In 1971, the Bretton Woods system collapsed, causing rapid changes in the global financial market, which made the financial market full of crisis. Therefore, it is necessary to master effective measures such as measuring risks. When evaluating the risks in the financial field, ARCH model can calculate the value of risks, which has become a very important and effective tool. Engel(1982)^[1] created ARCH model and used it to analyze the fluctuation of inflation index in Britain. In the following twenty years, the ARCH model has changed in various forms. Bollerslev(1986)^[2] first extended the ARCH model to GARCH model, which is an infinite order ARCH model and can be used as a simple GARCH model to replace a high-order ARCH model. In 1987, Engle et al. first proposed GARCH-M model, because GARCH-M model can explain risk premium and is often used to predict financial indicators closely related to risks, such as expected return on assets. Isabel Figuerola-Ferretti(2002)^[3] used GARCH-M model to study the high unemployment rate and sustained unemployment rate in Spain and, to a lesser extent, the high unemployment rate in Europe. Xiaoping Ren(2017)^[4] studied the fluctuation characteristics of A-share market index by means of stationary and arch effect tests, and established GARCH, Garch-M and EGARCH dynamic time series models. The conclusion is that Garch-M model has the best fitting effect on the index return rate of A-share market. Ling Lin et al.(2020)^[5] proposed a new hybrid forecasting model. Empirical results show that the proposed hybrid forecasting model WPD–EMD–ARMA–FIGARCH-M

achieves significant effect during periods of extreme incidents. The robustness test shows that this hybrid model is superior to traditional models. In this paper, GARCH-M is used to conduct in-depth research and analysis of financial market risks, find out the problems and find out the reasons for their existence, so as to accurately and effectively measure and prevent financial market risks.

2 Research method

GARCH-M model is established and VAR method is used to model, calculate and analyze the value at risk of China's financial market.

2.1 GARCH-M model

Engle et al. first proposed GARCH-M model in 1987, and its expression is as follows:

$$\begin{cases} r_t = \varphi_0 + \varphi_1 \sigma_t^2 + \varepsilon_t \\ \varepsilon_t = \sigma_t \eta_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^r \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2 \end{cases} \quad (1)$$

where φ_0 and φ_1 are constants, and φ_1 is a risk premium parameter, which means that the impact of risk fluctuation on r_t can be predicted. If $\varphi_1 > 0$, it means that the rate of return is positively correlated with its volatility. Other literatures have given specific expressions of risk premium, such as $r_t = \varphi_0 + \varphi_1 \sigma_t$, $r_t = \varphi_0 + \varphi_1 \ln \sigma_t^2 + \varepsilon_t$.

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2.2 VAR method

The VAR method, called the Value at Risk model, means the maximum possible loss of a financial asset or portfolio under normal market fluctuation. It is expressed by mathematical formula as follows:

$$\text{Pr ob}(\Delta P \leq -VAR) = 1 - C \quad (2)$$

where Prob represents the probability measure, $\Delta P = P(t + \Delta t) - P(t)$ represents the loss of the portfolio in the future holding period Δt , $P(t)$ represents the yield at time t, C represents the confidence level, and VAR is the value at risk of the portfolio under the confidence level C.

2.3 Calculation of GARCH-M-VAR under GED distribution

Generalized error distribution (GED distribution) proposed by Jp.Morgan in Risk Metrics can also describe the characteristics of peak and thick tail, and its distribution mainly reflects the distribution of yield series through the change of parameters. It is a flexible distribution form. The density function of the generalized error distribution is:

$$f(x, v) = \frac{v \cdot \exp\left[-\frac{1}{2} \left(\frac{x}{\lambda}\right)^v\right]}{\lambda \cdot 2^{\frac{v+1}{v}} \Gamma\left(\frac{1}{v}\right)} \quad (3)$$

In the generalized error distribution, if the parameter $v=2$, the distribution of the yield series is normal; If $v<2$, it reflects the characteristics of peak and thick tail of yield distribution; If $v>2$, it reflects the thin tail characteristics of yield distribution. Therefore, the generalized error distribution reflects the distribution of stock market yield series through parameter v .

Under the generalized error distribution, calculate the value of conditional standard deviation σ_t and calculate the quantile G_α under the distribution, which can be obtained :

$$VAR_{ged} = P_{t-1} G_\alpha \sigma_t \quad (4)$$

where P_{t-1} is the closing price of the stock at the previous moment, so as to obtain the VAR value under the generalized error distribution.

3 Empirical analysis based on GARCH-M-VAR

3.1 Sample data selection

Select the daily closing price of Shanghai Stock Exchange Index in Sina Finance Database, and record the closing price as P, the sample period is from January 4, 2001 to March 29, 2020, excluding holiday data, and

get a total of 4660 data. This paper adopts the form of logarithmic rate of return, as follows:

$$r_t = \ln P_t - \ln P_{t-1} \quad (5)$$

3.2 Characteristic analysis of sample data

Based on formula (5), the daily yield series of Shanghai Composite Index can be obtained. Firstly, descriptive statistical analysis is carried out on the yield series, and then the normality, autocorrelation, stationarity and heteroscedasticity of the yield series are tested. The analysis software is Eviews 8.

3.2.1 Basic descriptive statistical analysis

The basic descriptive statistical analysis mainly includes the mean value, maximum value, minimum value, standard deviation, skewness, kurtosis and normal distribution test statistics J-B. The specific results are shown in Table 1 below. Figure 1 shows the change trend of the yield series, and Figure 2 shows the QQ normal test chart.

Table 1. Descriptive Statistics of Sequence

Mean	Maximum	Minimum	Standard deviation
0.0002	0.094	-0.093	0.016
Skewness	Surtosis	J-B	P
-0.345	7.820	4602.53	0.000

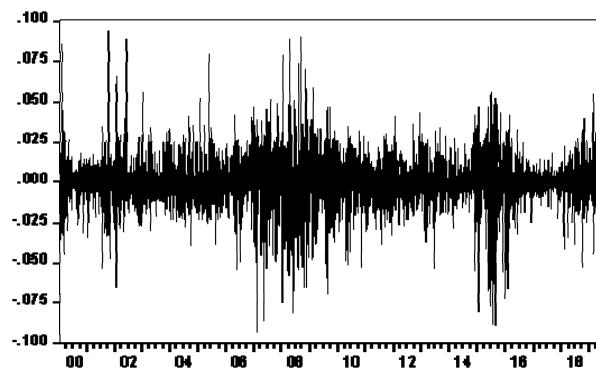


Fig. 1. Change trend of sequence

The results in Table 1 show that the average return rate of Shanghai Composite Index in the sample period is 0.0002, which is in the range of minimum value -0.093 and maximum value 0.094. The standard deviation is 0.016, and the ratio of standard deviation to average value is very large, indicating that the return rate series of Shanghai Composite Index fluctuates greatly. The skewness value is -0.345, less than 0, and the kurtosis is 7.820, which is obviously greater than 3, indicating that the yield series has the characteristics of peak and thick tail. Figure 1 shows that the yield series has obvious fluctuation aggregation effect. JB statistic is 4602.53, which is very large, and the corresponding P value is obviously less than 0.01, which shows that the return series of Shanghai Stock Exchange Index does not obey the normal distribution hypothesis. Fig. 2 is the normal

test chart of yield QQ, which shows that many points are scattered on the outside of the normal line, and both ends have obvious outward swing trend, which shows that there is a "thick tail" feature on both sides of the actual distribution of yield of Shanghai Stock Exchange Index.

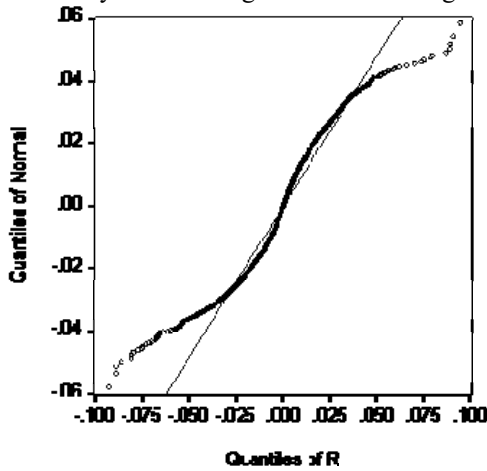


Fig. 2. Normal QQ diagram of sequence

3.2.2 Sequence autocorrelation test

In order to build the model better, the autocorrelation test is carried out on the return rate of Shanghai Composite Index, and the test results are shown in Fig. 3. Fig. 3 shows that the autocorrelation coefficient is 0.052, the q statistic is 20.185, and the corresponding p value is 0.000, which is less than 0.01, when the lag period is 4. The original assumption that there is no autocorrelation in the return of Shanghai Composite Index is rejected at the level of 1%, so the return series of Shanghai Composite Index has autocorrelation.

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.021	0.021	2.0267	0.155
		2	-0.023	-0.023	4.3946	0.111
		3	0.026	0.027	7.5403	0.057
		4	0.052	0.050	20.185	0.000
		5	-0.002	-0.003	20.206	0.001
		6	-0.052	-0.050	32.667	0.000
		7	0.030	0.030	36.967	0.000
		8	0.011	0.005	37.517	0.000
		9	0.002	0.006	37.543	0.000
		10	-0.001	0.002	37.552	0.000

Fig. 3. Autocorrelation test of yield series

3.2.3 Sequence stationarity test

In order to ensure the significance of statistical analysis, it is necessary to ensure that the model can pass the stationarity test. The ADF test is used to test the stationarity of Shanghai stock index yield series, See table 2 for inspection results. The results show that the T statistic under ADF unit root test is -66.797, and the absolute value is obviously higher than the absolute value of the critical value at 1%, which is 2.567. The original hypothesis that the sequence contains unit root is rejected at 1%, so the yield series is stationary.

Table 2. ADF Test

T	1% level	5% level	10% level	P
-66.797	-2.567	-1.941	-1.617	0.0001

3.2.4 Heteroscedastic test

Most financial yield series often show fluctuation cluster effect and heteroscedasticity effect (ARCH). Figure 1 above shows that the yield series of Shanghai Stock Exchange Index has fluctuation aggregation effect, and it is necessary to judge whether the yield series has heteroscedasticity effect. Engle's ARCH-LM test method is used to test the heteroscedasticity of the yield series. The test process of this method is divided into two steps. The first step is to establish a mean regression equation for the yield series, see formula (6); The second step is to extract the residual of the first step regression, calculate the square of the residual, and establish an autoregressive equation for the square of the residual again, as shown in Formula (7).

$$r_t = \eta + e_t \tag{6}$$

$$e_t^2 = \beta_0 + \sum_{l=1}^L \beta_l e_{t-l}^2 \tag{7}$$

where e_t in formula (6) is the residual sequence of regression. By judging whether the regression coefficient of formula (7) is 0 at the same time, that is, $H_0 : \beta_1 = \dots = \beta_L = 0$, the test statistic is $LM = nR^2 \sim \chi^2(L)$. If this hypothesis is rejected, it shows that there is heteroscedasticity effect. Table 3 shows the regression results of the residual square term. The results show that the regression coefficients of the three stages of residual square lag are all positive, and all of them have passed the significant test, indicating that there is a significant difference between the coefficients and zero, indicating that the yield series has heteroscedasticity.

Table 3. Regression results of residual square term

Variable	Coefficient	Standard deviation	t	p
Constant	0.000	0.000	14.566	0.000
RESID^2(-1)	0.123***	0.014	8.488	0.000
RESID^2(-2)	0.100***	0.015	6.883	0.000
RESID^2(-3)	0.152***	0.014	10.487	0.000
R-squared	0.063			
Adjusted R-squared	0.062			
F-statistic	103.410			
Prob(F-statistic)	0.000			

Note: ***, **, * are significant at 1%, 5% and 10% significant levels respectively.

3.3 Establish GARCH-M model

The above tests on the yield series of Shanghai Stock Exchange Index show that the series is not normally distributed, and there are fluctuation aggregation effect,

autocorrelation effect and heteroscedasticity effect in the series. OLS model is difficult to effectively capture the heteroscedasticity effect of financial time series, while GARCH model can effectively eliminate the heteroscedasticity effect of financial time series. GARCH is introduced as follows. GARCH(p,q) model includes mean equation and conditional variance equation, and the model is as follows:

$$R_t = \mu + e_t \tag{8}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i e_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{9}$$

Formula (8) is a mean value equation, and formula (9) is a conditional variance equation. e_t is the ordinary residual and σ_t^2 is the conditional variance. $\sum_{i=1}^p \alpha_i e_{t-i}^2$ is

ARCH term, $\sum_{j=1}^q \beta_j \sigma_{t-j}^2$ is GARCH term, and the sum of

ARCH term coefficient and GARCH term coefficient can reflect the fluctuation aggregation degree of change rate. In GARCH model, regression coefficient is required, which requires $\omega \geq 0$, $\alpha_i \geq 0, \beta_j \geq 0$, and

$$\sum_i \alpha_i + \sum_j \beta_j < 1$$

GARCH-M model considers the impact of risks on returns on the basis of GARCH. The mean equation adds the relevant terms of variance, and the variance equation remains unchanged. The mean equation is:

$$R_t = \mu + \theta \sigma_t + e_t \tag{10}$$

The VaR calculation formula is:

$$VaR_t = Z_\alpha \sigma_t \sqrt{\Delta t} \tag{11}$$

The GARCH-M model results are estimated by Eviews software, and the conditional variance σ_t^2 of the return rate of Shanghai Stock Exchange Index is obtained, then σ_t is obtained by prescribing, and then the quantile Z_α under a certain distribution of 99% or 95% confidence level is determined, which is brought into the formula (11) to calculate the VaR value of the return rate of Shanghai Stock Exchange Index.

4 Results and Discussion

4.1 Empirical result analysis

Based on GARCH-M model, it is assumed that the residual error obeys generalized error GED distribution, and the results are summarized in Table 4. There are two parts, one is the regression result of mean equation, and the other is the regression result of variance equation.

Table 4. GARCH-M(1,1) Model Results

Mean value equation				
Variable	Coefficient	Standard deviation	Z-Statistic	P
θ	0.069**	0.034	2.006	0.045

μ	-0.0002	0.0004	-0.572	0.567
Variance equation				
ω	1.42E-06***	3.97E07	3.588	0.000
α_1	0.068***	0.007	9.439	0.000
β_1	0.928***	0.007	134.08	0.000
GED PARAME-TER	1.178***	0.031	38.385	0.000

Note: ***, **, * are significant at 1%, 5% and 10% significant levels respectively.

The regression equation is written as:

$$R_t = 0.0685\sigma_t - 0.00023 \tag{12}$$

$$\sigma_t^2 = 1.42e - 0.6 + 0.0682e_{t-1}^2 + 0.9281\sigma \tag{13}$$

The results in Table.4 show that the regression coefficient of θ in the mean equation is 0.0685, and the corresponding P value is 0.045, which is less than 0.05. The regression coefficient is positive after passing the significant test at the significant level of 5%. The results show that there is a positive correlation between the return rate and the risk in the sample period.

In the variance equation, the regression coefficient of ω is positive, which passes the significant test at 1% significance level, the regression coefficient of α_1 is 0.068, and the regression coefficient of β_1 is 0.928, both of which pass the lower significant test at 1% level, and the coefficient sum is 0.996, which is less than 1, which meets the requirements of the model. Because the coefficient sum is close to 1, it shows that the Shanghai Stock Exchange yield index has a strong fluctuation aggregation effect. The regression coefficient of β_1 is 0.928, which indicates that the variance of Shanghai Stock Exchange's yield series mainly comes from GARCH term in earlier period, and the variance change has strong persistence.

4.2 VaR calculation and posterior inspection

The posterior test of VaR model refers to the coverage of the actual loss by the measurement results of VaR model. Kupiec test method is selected, and two confidence levels of 95% and 99% are selected at the same time. The dynamic VaR value one day ahead of the model with significant coefficient test is obtained, and the posterior test is carried out.

Table 5. VaR Results and Posterior Inspection Results

Confidence level	Average	Maximum	Minimum
95%	0.0242	0.0710	0.0089
99%	0.0391	0.1147	0.0143
Standard deviation	Days of failure	Failure rate	LR
0.0105	230	0.049	0.04
0.0170	59	0.013	3.079

The results in the above table show that under 95% confidence level, the average value of VaR is 0.0242, the

maximum value is 0.071, the minimum value is 0.0089, the actual failure days are 230, the actual failure rate is 0.049, which is slightly lower than the expected 5% yield, and the LR value is 0.04, which is less than 3.841 of the 95% confidence level. Therefore, the original assumed actual failure rate and expectations cannot be rejected.

At 99% confidence level, the average value of VaR is 0.0391, the maximum value is 0.1147, the minimum value is 0.0143, the actual failure days are 59, the actual failure rate is 0.013, which is slightly higher than the expected 1% return rate, and the LR value is 3.079, which is less than 6.635 at 99% confidence level. Therefore, it is impossible to reject the original hypothesis that the actual failure rate and the expected failure rate (. It shows that the GARCH-M(1,1) model is suitable for calculating the VaR with 99% confidence level, but it is slightly higher than the expected failure rate and tends to underestimate the risk. Generally speaking, GARCH-M(1,1) model is suitable for calculating the risk of the return rate of Shanghai Stock Exchange Index.

5 Conclusion

Through the related research on VAR at risk, such as defining characteristics and calculating methods, etc. Combined with GARCH-M model, the empirical analysis is carried out. Eviews software is used to analyze the characteristics of Shanghai Stock Exchange Index, which shows the relevant characteristics of sample data. The GARCH-M model with high coincidence degree was fitted, and the VAR values with confidence levels of 95% and 99% were calculated and tested to ensure the accuracy and precision of the model.

The conclusions are as follows:

1. The daily return series of Shanghai Stock Exchange Index has obvious characteristics of "fluctuation aggregation effect" and "peak and thick tail", which does not conform to normal distribution and is a stationary series. When the fourth order is delayed, the autocorrelation coefficient is large and there is autocorrelation. The regression coefficients of the three stages of residual square lag are all positive, and they all pass significant tests, which shows that there are significant differences between the coefficients and zero, indicating that there are heteroscedasticity in the yield series.

2. It is feasible to study financial market risk based on GARCH-M-VAR method. The GARCH-M(1,1) model of logarithmic return series of Shanghai Stock Exchange Index is established under the generalized error distribution, which has passed the failure tests with confidence levels of 95% and 99%. It is suitable for calculating the risk of return of Shanghai Stock Exchange Index, and has high precision and accuracy.

3. Under different confidence levels, VAR values are different, that is to say, the higher the confidence level, the more accurate the estimated value of VAR is, and the more likely it is to avoid risks. Therefore, in order to get the maximum benefit, investors should choose their

investment according to their risk preference and adjust their strategies in time. If they do not have strong risk tolerance, investors can choose higher confidence level to reduce risks.

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