

# Research on Priority-based Flexible Job Shop Scheduling Algorithm

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**Abstract.** Research on the Flexible Job Shop Scheduling (FJSP) problem in the manufacturing process of aircraft engines has been carried out. It is found that there are real requirements such as order mandatory nodes and equipment selection in the current job scheduling process. Order mandatory node means that some important orders must be delivered on time at a certain point in time, which often affects the quality of workshop scheduling results. In addition, equipment selection is also an important factor affecting the quality of job shop scheduling results. On this basis, an order priority calculation method with manual intervention is designed. Experiments show that the proposed shop job scheduling algorithm effectively improves the quality of job shop scheduling results.

## 1 Introduction

China is the world's largest industrial manufacturing country and manufacturing plays a crucial role in the development of the country's economy. Thanks to advanced technologies such as information physical systems[1], the Internet of Things [2], big data [3], deep learning[4], information management systems[5] and cloud manufacturing [6,7], manufacturing has shifted from traditional mechanised manufacturing to digital and intelligent manufacturing [8]. The main problem that restricts the productivity of enterprises is the job scheduling problem. Job scheduling requires the completion of scheduling objectives under multiple constraints, which is a long-standing NP hard problem. The existing branch delimitation method [9], integer programming method [10], genetic algorithm [11], forbidden search [12], and existing algorithms do not take into account the existence of mandatory delivery time for the machining process and do not further optimize the machining process according to the equipment load in the equipment selection process. This paper defines and solves the flexible shop scheduling problem according to the actual situation, introduces the forced delivery node to propose a priority calculation function, improves the hierarchical analysis method applied to the machining process, and establishes a real and effective mathematical model. Experiments are made on real cases, and the experimental results show that our proposed solution algorithm is effective, reducing costs and increasing completion rates compared to existing algorithms.

Chapter 2 of this paper defines the flexible shop floor job scheduling problem, describes in detail the constraints in the real processing environment and designs an evaluation function based on the actual

experience, and evaluates the good and bad job scheduling results through the results of the evaluation function. Chapter 4 compares the job scheduling results of the existing algorithm with the present algorithm under real cases, and shows that our algorithm can satisfy the flexible shop scheduling problem and outperform the existing algorithm by several indicators.

## 2 Problem definition

The Flexible Job Shop Scheduling Problem (FJSP) is defined as follows: In a machining process the set of workpieces  $W(W_1, W_2, \dots, W_n)$  is machined on a set of machines  $M(M_1, M_2, \dots, M_m)$ , each process can be machined by one or more machines, and the following constraints are satisfied:

- (1) The machine can only machine one workpiece at a time.
- (2) The workpiece can only be machined by one machine at a time.
- (3) The workpiece must be machined in the sequence specified for the process.
- (4) The process must not be interrupted.
- (5) Different workpieces and processes do not interfere with each other.
- (6) Priority is given to workpieces with mandatory delivery nodes.

Storage and maintenance costs are incurred when artefacts are completed before the delivery period, and when artefacts are completed beyond the delivery period then they cannot be delivered on schedule, we set up an evaluation function as follows:

$$F = f_1 + f_2 \quad (1)$$

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$f_1 = \sum e(l_i - d_i)$  ,  $f_2 = \sum a(d_i - l_i)$  ,  $e$  is Evaluation weights for overdue artefacts ,  $a$  is Evaluation weights for early completion artefacts ,  $e, a > 1$  ,  $l$  is Workpiece completion time ,  $d$  is Workpiece delivery time. The scheduling objective is to obtain the minimum value of the evaluation function.

### 3 Algorithms

We solve the flexible workshop job scheduling problem in two stages, depending on the priority of the workpiece and the load on the equipment during machining. The first stage identifies the current workpiece to be machined and the second stage identifies the equipment for machining the workpiece. Below, we present the detailed steps of the two phases.

#### 3. 1 Phase 1: Calculation of artifact priority

Each machine can only process part of the process, and there is a sequential relationship between the processes of the workpieces. When there is a free machine, the machinable process is determined, and the machinable workpieces are determined according to the process route of the workpieces. In order to ensure that workpieces with mandatory delivery nodes are delivered on time, priority is given to processing workpieces with mandatory delivery nodes. There are two situations in which the remaining workpieces can be machined: one is when the remaining processes of a workpiece with priority are still not delivered on schedule, and the other is when the remaining processes of a workpiece with priority are delivered earlier. When there is a difference between the expected delivery period of the workpiece and the specified delivery period, the larger the difference the greater the impact on the evaluation function, the early delivery and overdue delivery have different weights in the evaluation function, in order to get a better value of the evaluation function, we design the workpiece priority calculation formula as follows:

$$P = \frac{r_i}{\lambda|r_i - (d_i - t)| + \eta} + \delta \quad (2)$$

$r_i$  is remaining machining time for the workpiece,  $t$  is current time, Overdue or early delivery according to workpiece,  $\lambda$ ,  $\eta$  have different values. On overdue delivery,  $\lambda$  values are overdue evaluation weights  $e$ ,  $\eta$  values is  $(d_i - t)$ , when delivered ahead of schedule, values are early completion weights  $a$ ,  $\eta$  values is  $r_i$ . As shown in Figure 1, when delivery is overdue, the remaining machining time for the workpiece is  $d_i - t$ , the time required to machine the workpiece is  $r_i$ , work is overdue for  $r_i - (d_i - t)$ , the effect on the evaluation function is the overdue delivery time multiplied by the overdue evaluation

weight, the longer the overdue time, the larger the denominator and the smaller the ratio, because  $e > 1$ , so, the fractional ratio is less than 1. As shown in Figure 2, When delivered early, the workpiece is delivered early at  $(d_i - t - r_i)$ , the effect on the evaluation function is the early delivery time multiplied by the early completion weight, the longer the lead time, the larger the denominator and the smaller the ratio, because  $a > 1$ , the fractional ratio is less than 1.

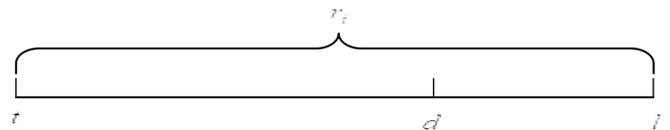


Figure 1. Workpiece overdue chart

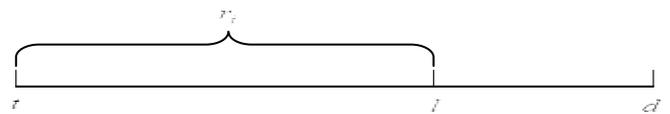


Figure 2. Workpiece completion ahead of schedule

When there is a mandatory delivery node for the artifact,  $\delta=0$ , otherwise  $\delta=1$ . The smaller the value of  $P$  as a function of the time the part is completed ahead of schedule or overdue, the longer the delivery time. Since the fractional ratio is less than 1, with the introduction of  $P$ , workpieces with forced delivery nodes must be machined before other workpieces.

#### 3. 2 Phase 2: Selection of processing equipment

Once a workpiece has been identified for machining, the selection of different machining equipment can produce different results when multiple machinable machines exist, so it is important to investigate equipment selection strategies. The choice of equipment is influenced by various factors, such as the machine load, the time the workpiece starts machining on the machine. The key to the study is how to select the optimal processing equipment, taking into account all the influencing factors.

The hierarchical analysis method can organically combine qualitative and quantitative factors in the decision-making process, so we use the hierarchical analysis method for the selection of equipment. The general steps of the hierarchical analysis method are: determining evaluation indicators, constructing a system hierarchy diagram, constructing a judgement and comparison matrix, consistency testing, and calculating the overall hierarchical ranking.

Hierarchical analysis constructs comparison matrices from expert ratings, and the constructed matrices often do not satisfy the consistency test. In addition, the job scheduling process requires multiple equipment selections and it is not reasonable to perform an expert scoring for each equipment selection. In this paper, we improve the hierarchical analysis method based on the information of the matrix construction method [13], job

scheduling and equipment load that satisfies the consistency test. The improved hierarchical analysis method can automatically construct the comparison matrix that satisfies the consistency test. The steps of the improved hierarchical analysis are as follows:

(1) Influencing factors

There are number of factors that influence the choice of equipment, and this paper uses the earliest point at which the equipment can start machining as  $C_1$ , the total load on the equipment as  $C_2$ , and the total time that all workpieces can be machined on the equipment as  $C_3$ . The earlier the  $C_1$ , the lower the probability that the workpiece will be overdue, giving preference to equipment with a small  $C_1$ . The greater the  $C_2$ , the greater the probability that the workpiece will wait and

the greater the probability that it will be overrun, giving preference to equipment with a small  $C_2$ . The greater the  $C_3$ , the greater the probability of resource competition for this equipment (two workpieces requiring the same machine for processing in the same time period) and the greater the probability that the workpiece will overrun, giving preference to the equipment with the smaller  $C_3$ .

(2) Building a hierarchy chart

As shown in Figure 3, the hierarchy diagram is divided into three layers. The target layer is the final choice of equipment, the guideline layer is the individual influencing factors and the program layer is the equipment available.

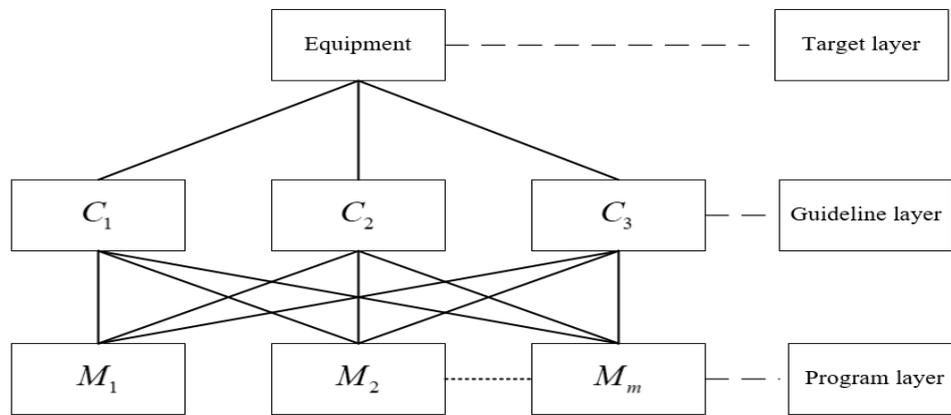


Figure 3. Hierarchy diagram

(3) Automatic construction of comparison matrices

There are two types of comparison matrices in hierarchical analysis, the first of which is the hierarchical single ranking comparison matrix  $A$ . The hierarchical single ranking refers to the ranking of the importance of the factors at this level in relation to a factor at the previous level. Before starting the process, construct a comparison matrix  $A$  and satisfy the consistency test.

The other is the hierarchical total ranking comparison matrix  $B$ . The hierarchical total ranking is the process of determining the ranking weights of the relative importance of all factors in a given level to the overall objective, for the same influencing factor in the criterion level, Program level with equipment  $M_1, M_2, \dots, M_m$ , Calculate  $C_1, C_2$  and  $C_3$  for each device, Comparing devices with each other  $C_1$  is worthwhile to the comparison matrix  $B_1$ . Similarly  $B_2$  and  $B_3$  can be obtained.

$$b_{ij} = \begin{cases} 2 & M_i > M_j \\ 1 & M_i = M_j \\ 0 & M_i < M_j \end{cases} \quad (3)$$

Calculate the importance index  $r_i$  of each device ,

$$r_i = \sum_{j=1}^n b_{ij} \quad (i = 1, 2, \dots, m) \quad (4)$$

The magnitude relationship of the importance index reflects the important relationship between elements of the same influencing factor. If  $r_i > r_j$ ,  $M_i$  is more important than  $M_j$ . Use  $M_{max}$  and  $M_{min}$  to denote the elements corresponding to the largest sorting index  $r_{max}$  and the smallest sorting index  $r_{min}$  respectively. Suppose the relative importance of  $M_{max}$  and  $M_{min}$  relative to the upper element is given according to a certain standard, denoted as  $g_k$ . The conversion formula is as follows:

$$g_{ij} = f(r_i, r_j) = g_k^{(r_i - r_j)/R} \quad (5)$$

$R = r_{max} - r_{min}$ , take  $c_k = 9$ . The matrix constructed by the transformation formula meets the consistency requirement [16].

(4) Total ranking of calculation levels

Calculate the normalized eigenvectors of  $A$ , and calculate the eigenvectors of  $B_1, B_2, B_3$ . Calculate the inner product of the standardized feature vector and the feature vector to obtain the total ranking weight, and select the equipment with the largest weight for processing.

### 3.3 Algorithm flow

With workpiece set  $W(W_1, W_2, \dots, W_n)$  and equipment set  $M(M_1, M_2, \dots, M_m)$  to be processed

Step (1) Based on the process route of the workpiece, the set of processes to be machined is obtained.

Step (2) Calculate the priority of each process using Equation (2) to determine the process to be machined first.

Step (3) Select the machining equipment for the process using an improved hierarchical analysis.

Step (4) Determine if the process has a successor, and if so, add the successor to the set of processes to be processed.

Step (5) If the set of processes to be processed is not empty go to step (2).

## 4 Experimental results and analysis

In this paper, we use the real calculations of a domestic aircraft engine group as shown in Table 2.

**Table 1.** Production schedule

	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	Y	T
$W_1$	1	1	2	3	4	-	8
$W_2$	3	3	2	1	4	-	9
$W_3$	2,4	2,4	1	-	3,5	-	9
$W_4$	1	1	3	5	2,4	0	10
$W_5$	1	1	-	3	2	-	7
$W_6$	1	1	-	3	1	-	10

$W$  is the workpiece to be processed,  $M$  is the processing equipment, Y indicates whether the workpiece requires mandatory delivery, a value of 0 indicates that the workpiece has a mandatory delivery period, and T is the workpiece delivery date. The same row represents the processing process of the same workpiece, the smaller the value the more it is processed first. The same column represents the processes that can be processed on the machine.

We use the currently available algorithms EDD, LWR, MWR, LPT, and SPT algorithms to compare with our proposed algorithm DA, and the experimental results are as follows.



**Figure 4.** Evaluation value and completion results

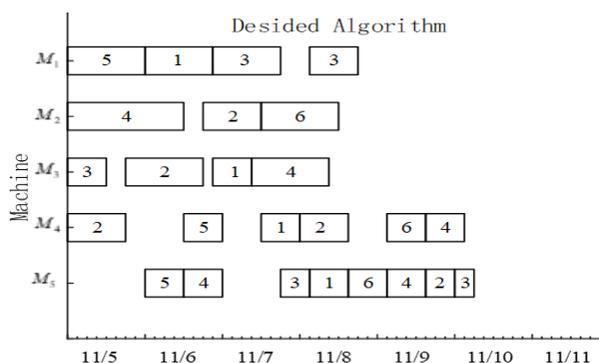
In Figure 4, F is the evaluation function value, and the smaller the F, the better the job scheduling result. As shown in the figure, our algorithm has the smallest F value and the best job scheduling result. Completion Rate is the workpiece processing completion rate. As shown in the figure, our completion rate is also the best.

**Table 2.** Whether to complete the mandatory delivery target

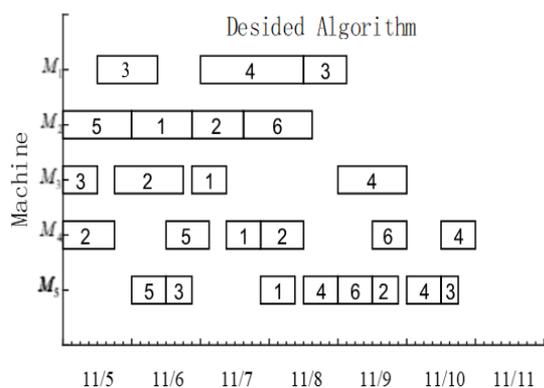
EDD	LWR	MWR	LPT	SPT	DA
NO	YES	NO	YES	NO	YES

According to Table 4, it can be seen that our algorithm accomplishes the forced delivery goal. Combining Figure 4 and Table 4, our algorithm can get the best results.

We compare the results of the DA algorithm with and without the device selection strategy with the following processing order.



**Figure 5.** No device selection strategy is used



**Figure 6.** Adopting the device selection strategy

Observing Figure 5 and Figure 6, it is obvious that better results can be obtained by using the device selection strategy.

In summary, the algorithm in this paper ensures that the mandatory node workpieces are delivered on schedule while the evaluation value is minimized, which not only ensures the completion rate of the workpieces, but also significantly reduces the inventory and improves the productivity and capital liquidity of the enterprise.

## 5 Conclusion

The order priority formula is given to accurately measure the risk of order overruns, where the greater the calculated order priority, the greater the risk of that order overrun. At the same time, decision makers are able to set a priority for mandatory node orders in advance, which in turn ensures that mandatory node orders are processed on a priority basis. An equipment selection method based on hierarchical analysis is designed based on the consideration of various influencing factors. This method allows a comprehensive evaluation of each machine, so that the most suitable machine can be selected and the purpose of improving the scheduling efficiency can be achieved. A shop operation scheduling

algorithm based on priority and considering equipment selection strategy is designed.

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