https://doi.org/10.1051/e3sconf/202125201004

Power transformer life analysis based on Lambert W function

Jingyang Wang[1], Liyou Fu[2]

[1] Business College, Shanghai Dian Ji University, Shanghai, 201306, China
[2] Business College, Shanghai Dian Ji University, Shanghai, 201306, China

Abstract. As the core equipment of many construction electric fields, reasonable life management of power transformer plays an important role in ensuring the safety production and economic benefits of public and enterprises. Based on Lambert W function, the exact solution is obtained by calculating the annual apportioned total cost. The variable values that may have errors in the function model are deeply analyzed. Combined with the inherent characteristics of power transformer, the factors influencing the economic life in the results are analyzed. The reliability of this method is proved by an example. Compared with the traditional tabulation method, this method is convenient for further analysis of the influencing factors and changing rules of economic life. At the same time, this method can also be used in other equipment to study its economic life since the specific problems are analyzed. It has the value of popularization.

1 Introduction

Transformers are widely used, and are used in almost all electronic products, especially power transformers in modern industrial enterprises. As one of the core equipment of an enterprise, the transformer needs to be fully guaranteed in terms of equipment installation, smooth operation, safety and reliability, and studying the economic life of power transformers is a strong guarantee for these factors. Establishing economic and reasonable maintenance and update plans through studying the economic life, will more effectively promote the utilization of the core assets of the enterprise, thereby bringing greater economic profits to the enterprise.

There are no mandatory specific standards for the life span of transformers. And for now, most of the research at home and abroad is mainly focused on economic evaluation, and the economic life of transformers has not formed a unified definition. At the same time, there is very little research on the operation and management of power equipment in China, and there is not enough information to form a scientific and reliable decommissioning decision-making criterion. In recent years, the research on the economic life of transformers and the frequency of citations analyzed by CiteSpace are very few. Literature[3] comprehensively considers the impact of the life cycle cost of the active transformer, the economic performance of the selected reference transformer, and the random fuzzy uncertainty of the parameters on the economic life of the active transformer, but its quantitative data is less and there are more qualitative components; Literature[4] defined economic life span is the year in which the net benefit value obtained from comprehensive depreciation, energy saving, loans and other economic indicators is the largest; Literature[5] is based on the principle of maximum annual average return based on the comparison of maintenance and replacement; Literature[6] considers the investment, maintenance, failure loss of the transformer during the full life cycle; Literature[7] puts forward the economic life model of power transformer based on the lowest equivalent cost of transformer life year; none of the above documents has an accurate solution to the economic life and combined with sufficient influencing factors. Literature[9] proposes a general equipment economic life solution, but it is not completely applicable to any equipment. A specific analysis is required for power transformers.

This paper combines the advantages of the above documents and uses the Lambert W function to derive an accurate solution for the economic life of power transformers. By analyzing the factors that affect the economic life and the change law of the economic life of equipment, it is of great significance to improve the economic life of power transformers and increase their economic benefits.

2 Mathematical Model of Economic Life of Power Transformer

The calculation method of the economic life of the power transformer should consider the purchase cost of the equipment, the inspection and maintenance cost, and the residual value. Taking these into account, before the service life of the power transformer is not over, there are other reasons that the machine cannot be used again. This is because in the later life of the power transformer, the loss caused by excessive failures caused by frequent use of the equipment. As the service life increases, the annual

---

Liyou Fu: e-mail: fuly@sdju.edu.cn

investment will be less, but on the contrary, the maintenance and repair costs of the equipment have increased in the opposite direction, making the annual allocation of the total cost increase sharply at the lowest point.

Therefore, in order to reduce the cost as much as possible, according to the annual cost method, the economic life of the equipment is the service life with the lowest average cost per unit time, and find the lowest value.

This article quotes the idea of establishing the model used in the literature[9]. By combining with the inherent characteristics of transformer, the original model is improved and the model is rebuilt to improve the accuracy of the model.

It is assumed that the annual apportionment of the purchase cost and the residual value is C_1. C1 should be the purchase cost multiplied by the annuity present value coefficient minus the residual value multiplied by the annuity terminal value coefficient. Based on experience, The residual value of transformer decommissioning is usually 30%~40% of its original value. Take 30% here to get

$$C_1 = K\left(A/P,i,n\right) - 0.3K\left(A/F,i,n\right) \tag{1}$$
$$= Ki\frac{\left(1+i\right)^n - 0.3}{\left(1+i\right)^n - 1}$$

In formula (1), K represents the purchase fee, A represents the equivalent annual value of the funds, P represents the capital limit, F represents the final value of the funds, i represents the annual interest rate, and n represents the year.

Assuming that the annual apportionment of inspection and maintenance costs is C2, you can get

$$C_2 = \begin{pmatrix} C\left(1\right)\left(P/F,i,1\right) + C\left(2\right)\left(P/F,i,2\right) \\ + \cdots + C\left(n\right)\left(P/F,i,n\right) \end{pmatrix}$$
$$\left(A/P,i,n\right) = \frac{i}{\left(1+i\right)^n - 1}\sum_{j=1}^{n} C\left(j\right)\left(1+i\right)^{n-j} \tag{2}$$

In formula (2), j represents the year. It can be assumed that the increase in inspection and maintenance costs per year is, and C_2 can be changed to

$$C_2 = \frac{i}{\left(1+i\right)^n - 1}\sum_{j=1}^{n}\left[C\left(1\right) + \left(j-1\right)\Delta C\right]\left(1+i\right)^{n-j} \tag{3}$$

Through simplification, the final expression of C2 can be obtained as

$$C_2 = \Delta C\frac{i}{\left(1+i\right)^n - 1}\cdot\frac{\left(1+i\right)^n - ni - 1}{i^2} + C\left(1\right) \tag{4}$$

Assuming that the annual apportionment of the total cost is CT, we can get

$$C_T\left(n\right) = C_1 + C_2 = Ki\frac{\left(1+i\right)^n - 0.3}{\left(1+i\right)^n - 1} \tag{5}$$
$$+\Delta C\frac{i}{\left(1+i\right)^n - 1}\cdot\frac{\left(1+i\right)^n - ni - 1}{i^2} + C\left(1\right)$$

Make

$$\frac{\mathrm{d}C_T\left(n\right)}{\mathrm{d}n} = 0 \tag{6}$$

$$\frac{\Delta C - \Delta C\left(1+i\right)^n - \left(1+i\right)^n\left(0.7Ki - n\Delta C\right)\ln\left(1+i\right)}{\left(-1+\left(1+i\right)^n\right)^2} \tag{7}$$
$$= 0$$

From formula (7), we can see that if the denominator on the left side of the equation is equal to 0, we can get

$$n\ln\left(1+i\right) - 1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C} = -\frac{1}{\left(1+i\right)^n} \tag{8}$$

Multiply both sides

$$e^{n\ln\left(1+i\right) - 1 - \frac{0.7K_0 i\ln\left(1+i\right)}{\Delta C}} \tag{9}$$

Get

$$\left(n\ln\left(1+i\right) - 1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}\right) \tag{10}$$
$$\cdot e^{n\ln\left(1+i\right) - 1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}}$$
$$= -\frac{1}{\left(1+i\right)^n}\cdot e^{n\ln\left(1+i\right) - 1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}}$$
$$= -\mathrm{e}^{-1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}}$$

It can be seen that the formula (10) is in the form of Lambert W function, which can be rewritten as

$$W\left(z\right) = n\ln\left(1+i\right) - 1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C} \tag{11}$$

$$z = -e^{-1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}} \tag{12}$$

The exact solution of the economic life is as follows

$$n^* = \frac{0.7Ki}{\Delta C} + \frac{W\left(-e^{-1 - \frac{0.7Ki\ln\left(1+i\right)}{\Delta C}}\right) + 1}{\ln\left(1+i\right)} \tag{13}$$

In formula (11), the purchase cost K and the annual interest rate i are all values greater than 0, the value range of z in the Lambert W function should be ($-e^{-1}$, 0), according to the Lambert W function It can be seen from the nature that W(z) must have a real number solution, and the exact solution of the Lambert W function can be obtained through Mathematica. The curve drawn by Mathematica is as follows.

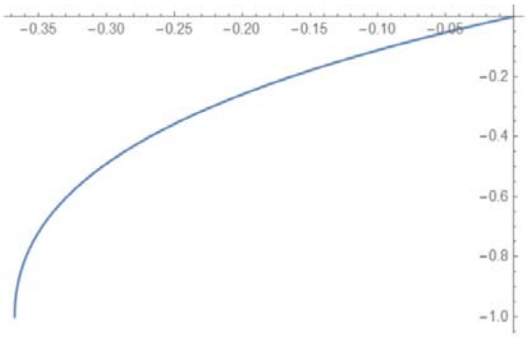

Fig 1. Lambert W function graph

As shown in figure 1, the horizontal axis represents z, and the vertical axis represents W(z). W(z) increases with the increase of z, and increases monotonically on (-e^{-1}, 0).

3 Analysis of Factors Affecting Economic Life of Power Transformer

Included $C(1)$ in the formula for apportioning the total cost of the initial model, But the exact solution for economic life does not include $C(1)$. Therefore, its value has no effect on the solution of the economic life of the power transformer. Within the value range where the Lambert W function has a real number solution, the function image is monotonically increasing, When ΔC increases, z will decrease, W(z) decreases accordingly, At the same time, $0.7Ki/\Delta C$ will also decrease, which will eventually lead to a decrease in economic life. Therefore, the solution of the economic life of power transformers will decrease with the increase in the annual increase in inspection and maintenance costs. For the purchase fee K, an increase in the purchase fee K will increase z, and W(z) will increase accordingly. At the same time $0.7Ki/\Delta C$ will increase, Therefore, the solution of the economic life of the power transformer increases with the increase of the purchase cost.

For the annual interest rate i, it is mentioned in literature[9] that the larger the annual interest rate i, the greater the annual apportionment value of the purchase cost, but as the annual interest rate i increases, the annual apportionment value of the inspection and maintenance fees is also increasing, so It is impossible to prove the influence of annual interest rate i on the economic life of power transformers only through economic life equation and capital recovery formula. The effect of the value of i on the economic life of the transformer is affected by the ratio of K to ΔC. It is necessary to use professional mathematics software to judge by the controlled variable method. The following figure shows the change trend graph of the economic life of the power transformer with the annual interest rate i in the range of 0-0.3 when the other variables are unchanged.

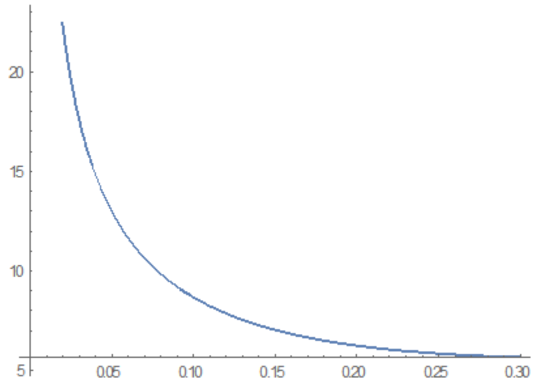

Fig 2. Economic life curve of power transformer with annual interest rate I in the range (0-0.3)

As shown in figure 2, the economic life of a power transformer continues to decrease with the increase of the annual interest rate i, which shows that the annual apportionment of the inspection and maintenance costs occupies a large proportion in the process of solving the accurate solution of the economic life. This also verifies as the number of years of use increases, the annual apportionment of inspection and maintenance costs continues to increase, while the proportion of annual apportionment for purchases is becoming less and less.

In the process of solving, the values of purchase cost K and annual interest rate i are easy to obtain, but for the value of ΔC, it is often difficult to obtain a value with almost the same increase in inspection and maintenance costs every year. Therefore, C_2 needs to be further analysis.

The annual apportionment of inspection and maintenance costs should be

$$C_2(t) = \frac{r(t)C_{MAD}(t) + (1 - r(t))C_{MAL}(t)}{(1+\delta)^t} \tag{14}$$

In formula (14), $C_2(t)$ represents the inspection and maintenance cost in year t, $r(t)$ represents the failure rate function of the power transformer, power transformers installed in different years have different failure rate functions, and $C_{MAD}(t)$ represents the loss and maintenance costs caused by the failure of the power transformer that year. $C_{MAL}(t)$ represents the periodic inspection fee of the current year, and δ represents the discount rate.

It can be seen from the formula that if the annual amortization increase of the inspection and maintenance cost is required, $C_2(t+1)$ minus $C_2(t)$ is required to obtain the annual increase, and the optimal solution is found by analyzing the relationship.

Based on the analysis of the above influencing factors, it is necessary to start with these influencing factors to improve the economic life of power transformers. The annual interest rate i cannot be controlled, and for the power transformers that have been purchased and put into use, the purchase fee K cannot be changed, that is, consider reducing the annual apportionment of the inspection and maintenance fee. To reduce maintenance costs, economic and reliable maintenance methods can be adopted. For example, the main internal components of power transformers are iron cores, windings and transformer oil. In the event of problems, simply replacing these components will greatly increase the cost of maintenance. To repair the iron core, the first step is to remove the oil stains attached to the iron core, and to ensure that dust and impurities do not enter the transformer tank during the disassembly and assembly process, and then check the shape and position of the iron core (mainly including color, neatness, connection Tightness, grounding, paint film, etc.), and adjust in time if there is a problem. The initial steps of winding repair are the same as iron core repair, degreasing, checking the appearance position, and testing whether there is aging problem by pressing. If pressing is brittle, you need to replace the insulation package. For transformer oil pollution, because the amount of transformer oil in power transformers is relatively large, considering the high cost of oil, the most economical way is to suck out the oil and impurities at the bottom of the transformer through a suction device.

While minimizing the inspection and maintenance costs of power transformers, considering that the design life of power transformers specified by the national standard is 20 years, but few power transformers can operate beyond this limit, the increase in inspection and maintenance costs is only part of the reason, but it is still It must be eliminated in advance by industry regulations based on operating conditions. Take a batch of thin-insulated transformers with aluminum coils produced around 1976 as an example. The state has issued a document to eliminate them in advance, so the innovation of basic equipment and technology still has an important impact.

4 Case analysis

Take an in-service power transformer in a substation as an example for analysis and calculation. The voltage level of the power transformer is 10000±5%/400V, the purchase cost is 80,000 yuan, and the design life is 25 years. It was put into operation in 1980. The annual interest rate is 5%, the discount rate is 5%, the average loss cost caused by the initial power transformer failure is 10,000 yuan/time, and the regular inspection cost is 1,500 yuan. Try to analyze the economic life of the power transformer (The failure rate function is $r(t)=0.10t^{0.93}$ (%/unit·year)).

Establish an image of the loss and maintenance cost caused by power transformer failure.

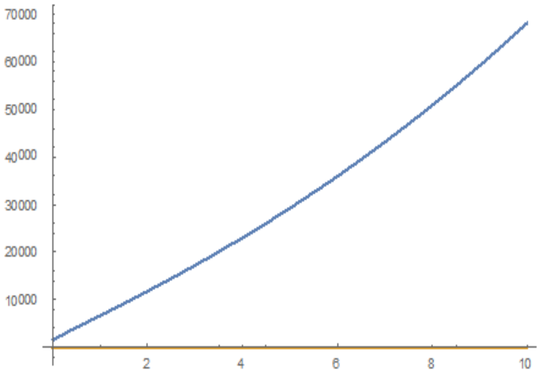

Fig 3. Loss caused by power transformer failure and maintenance cost chart

As shown in figure 3, the horizontal axis represents the year, and the vertical axis represents the loss and maintenance costs. Although the annual loss and maintenance costs increase with the increase in the number of years of use, the curve in the figure still shows an approximately linear increase. Trend, so ΔC can be approximated to 5000 yuan.

Bring the value of ΔC into the Lambert W function and solve it by the product logarithm formula.

$$\mathrm{ProductLog}\left[-e^{-1-\frac{0.7*80000*\ln(1.05)}{5000}}\right]=-0.2825$$

Which is

$$W\left[-e^{-1-\frac{0.7*80000*\ln(1.05)}{5000}}\right]=-0.2825$$

Then the economic life of the power transformer is accurately solved as

$$n^{*}=\frac{0.7*80000*0.05}{5000}+\frac{-0.2825+1}{\ln(1+0.05)}=15.26\text{ years}$$

The economic life is much shorter than the design life, and the best economic life does not mean that the longer the economic life, the better. In fact, the power transformer can still be put into use when it reaches the economic life, until the power transformer's maximum annual average net income is lower than its inspection and maintenance costs, until it no longer generates positive income. It is difficult to make the service life of a power transformer reach its design life. Various reasons may lead to a decrease in the economic life of the power transformer during service, thereby reducing the service life.

In this example, based on the comprehensive consideration of the operating status of the power transformer, it is recommended that the optimal operating years of the transformer be within 15-18 years.

5 Conclusion

This paper uses the Lambert W function to establish an analytical model for solving the economic life of power transformers. Although the establishment of the initial model was derived under the same conditions as the annual increase in inspection and maintenance costs, the average annual increase in inspection and maintenance costs obtained through in-depth analysis of the annual allocation of inspection and maintenance costs in the establishment of the complete model later is convincing and accurate. The results obtained based on the Lambert W function can be applied in practice. This method provides a reliable and novel idea for calculating the economic life of power transformers, and is not only for calculating the economic life of transformers. It has great reference value for the future planning of other types of equipment and even its substations.

References

1. Wu Z.H. (2018) Economic life model of power transformer and its application example. Technology Wind, 98-99.
2. Wang W.B., Bai W.G., Shi L.L. (2019) Comprehensive life cycle economic physical life assessment method for power transformer. Power System Protection and Control, 91-98.
3. Li R., Han B., Lu Y. (2014) Transformer's economic life assessment based on random and fuzzy theory. Power System Protection and Control, 42: 9-16.
4. Yu J.L., Wang C.F., Zhang B. (2010) Economic Life Evaluation of Power Transformer in Service. Proceedings of the CSU-EPSA, 86-90.
5. Liu Y.W., Ma L., Wu L.Y., Zhou Y., Lian C. (2012) Economic Life Model of Power Transformer and Its Application. Power System Technology, 235-240.
6. Luo X.C., Li L., Wei Z.L., Ge J.B. (2011) Applications of Life Cycle Cost Theory in Decision-Making of Investment for Distribution Transformer Renovation. Power System Technology, 207-211.
7. Wang H.F., Zhao W.F, Du Z.D. (2015) Economic Life Prediction of Power Transformers Based on the Lifetime Data. Power System Technology, 39: 810-816.
8. Hu B., Hu W.P., Jia Z.H. (2010) Life cycle cost and economic life analysis of transformer. The 9th Symposium of equipment life cycle cost Committee of China Equipment Management Association, Shanghai. 227-230.
9. Wu S.H., Liu X.D., He B. (2016) Economic life analysis for weapon equipment based on Lambert W function. Systems Engineering and Electronic, 38: 844-851.