Delay Margin Computation and Controller Design of Time-delayed AGC System Based on Root Locus Analysis

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Abstract. To investigate the small-signal stability of automatic generation control (AGC) system with constant communication delay, this paper establishes its state-space based on Padé approximation and conducts eigenvalue calculation. Then, by tracking the change of eigenvalues with time delay, the time-delay margin can be confirmed. Different controller parameters have complex influence on time delay margin and damping performance, so this paper utilizes particle swarm optimization (PSO) to obtain the optimal PI controller parameters to derive delay margin largest and an ideal damping performance. Finally, the simulation studies verify the correctness of time-delay margin calculation and indicate the AGC system has an ideal damping performance with optimal PI controller parameters

1 Introduction

Automatic generation control (AGC) plays a crucial role in maintaining frequency stability and tie-power exchange at scheduled values between different control areas. With the development of modern power system, AGC requires a wide area open communication network to transmit information. But it brings some unavoidable unreliable factors, such as time delay, packet losses, etc. It should be noted that time delay is one of the most unreliable factors [1]-[3]. Large delays over delays margin can threaten the stability of the AGC system.

The research on time-delay AGC system has attracted much attention in these years. There are several methods for investigating the properties of time-delayed AGC system. These methods can be divided into two categories which are named frequency-domain methods and time-domain methods. The frequency-domain methods can recognize the stability of the AGC system by calculating all the eigenvalues. A well-known sufficient condition for the stability of a closed-loop system is that all eigenvalues lie in the left half s-plane. In [4], a method for calculating time delay margin is proposed, and the core of this approach is to eliminate exponential terms in characteristic equation without making any approximation and form a new regular polynomial. Besides calculating delay margin, this paper also investigates the influence of some parameters of proportional-integral (PI) controller on delay margin. Base on [4], a graphical method applied to compute all stabilizing PI controller gains for a constant delay is proposed. The main aim of the approach is to extract the stable region and the stable boundary trajectory from the parameter space of the PI controller with user-defined gain and phase margin [5]. Authors in [6] proposes a method to analyze the stability of AGC system and estimate delay margins. The main result is nicely based on a sufficient and necessary condition. This paper takes the real-world AGC system of East China Power Grid as an example. The simulation results verify the effectiveness of the method. Although the frequency-domain methods can get an accurate delay margin, it is important to point out that they can only deal with the constant delay.

The time-domain methods are implemented to analyze the stability of AGC system with time-varying and constant delay. They utilize Lyapunov stability theory and linear matrix inequalities (LMIs) to estimate the delay margin. A new stability criteria of time-delay AGC system based on Lyapunov-Krasovski (L-K) is provided in [7] to generate less conservative stability test conditions in the form of LMI. Based on L-K and truncated second-order Bessel–Legendre (B-L) inequality, [8] proposes a new method to analyze the stability of PI-type AGC system with time-varying delay. The simulation of two-area interconnected power system verifies the effectiveness and superiority of the method. A sufficient delay-distribution-dependent stability and stabilization criterion is provided in [9]. This paper also provides the gain of PI-type AGC and the allowable upper bound of the communication delay simultaneously while preserving the desired performance. Considering the practical AGC is a sampled-data system, author in [10] undertakes stability analysis of AGC with both sampling and time-varying delay. Based on Wirtinger based integral inequality and its affine version, this paper proposes a new stability criterion for linear system with both sampling and transmission delay.

The purpose of this paper is to calculate the time-delay margin of AGC system. Moreover, it utilizes particle swarm optimization (PSO) to obtain the optimal PI controller parameters to derive delay margin largest

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and an ideal damping performance. The main contributions of this paper can be generalized as follows:

1) In this paper, the AGC system state-space model based on Padé approximation is established to realize the linearization of AGC system. According to the model, the state space equation of the system is derived. The eigenvalues of AGC system were obtained by solving the characteristic equation.

2) A well-known sufficient condition for the stability of a closed-loop system is that all eigenvalues lie in the left half s-plane. Based on the proposed model, tracking eigenvalues with the change of time delay is an effective method to obtain delay margin. Moreover, PSO is introduced to obtain the optimal PI controller parameters for making the delay margin largest and having an ideal damping performance.

3) Based on the proposed model, the validity of eigenvalue analysis is verified by time-domain simulation and Prony analysis. Then, the model is applied to root locus analysis and delay margin calculation. Finally, the parameters of PI controller are designed by PSO.

The remaining parts of this paper are organized as follow. Section 2 proposes the dynamic model of AGC system with constant delays. Section 3 introduces time-delay margin computation and controller design based on root locus analysis. A case study is given in Section 4 to illustrate the correctness of the proposed method. Finally, conclusions are drawn in Section 5.

2 Linearized model of time-delay AGC system

2.1 AGC system containing time delay

The real power system including AGC system has complex and nonlinear characteristics. So, it is usually described as a set of nonlinear differential equations. When a small disturbance occurs in AGC system, the linearized model of AGC can be established. The linear model is enough to assess the small-disturbance stability of AGC system around an equilibrium point. The dynamics of single-area AGC system containing time delay are presented as follows [11]-[13]

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + F\Delta P_d(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where

$x(t) = [\Delta f \quad \Delta P_m \quad \Delta P_v \quad \int ACE]^T$,

$y(t) = \begin{bmatrix} ACE & \int ACE \end{bmatrix}^T$

$$A = \begin{bmatrix} -\frac{D}{M} & \frac{-1}{M} & 0 & 0 \\ 0 & \frac{-1}{T_{ch}} & \frac{1}{T_{ch}} & 0 \\ \frac{1}{RT_g} & 0 & \frac{-1}{T_g} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}$$

$$B = [0 \quad 0 \quad \frac{1}{T_g} \quad 0]^T, C = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = [\frac{-1}{M} \quad 0 \quad 0 \quad 0]^T$$

Since no power exchange of net tie-line in the one-area LFC scheme, the output of the system, namely area control error (ACE), is defined as

$$ACE = \beta \Delta f \quad (2)$$

where $\beta > 0$ is the frequency bias factor. As shown in fig. 1, the delayed ACE signal is the input of the PI controller. The control command $u(t)$ can be expressed as

$$\begin{aligned} u(t) &= K_P ACE + K_I \int ACE \\ &= Ky(t-\tau) = KCx(t-\tau) \end{aligned} \quad (3)$$

where K, K_P and K_I represent a matrix including $[K_P \quad K_i]$, integral gain and proportional gain respectively. And the closed-loop dynamic model of AGC system is obtained as

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + F\Delta P_d(t) \quad (4)$$

where $A_d = BKC$

Considering the behavior of communication delay which is described in Fig. 1, the communication delay from ACE to PI controller is represented by exponential block $e^{-\tau s}$. Due to the transcendental term $e^{-\tau s}$. AGC system becomes an infinite dimensional system, which brings difficulties to the eigenvalue calculation and controller design of the AGC system.

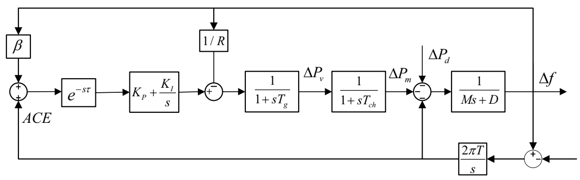

Fig. 1. Dynamic model of single-area AGC system.

2.2 State-space model based on Padé approximation

Padé approximation (PA) is an effective method to approximate the original analytic function better than truncating the power series of the original analytic function. Base on PA, the transcendental term can be transformed into finite-dimension polynomials [14]-[15].

$$e^{-\tau s} \approx P(s) = \frac{b_0 + b_1\tau s + \cdots + b_j(\tau s)^j + \cdots + b_l(\tau s)^l}{a_0 + a_1\tau s + \cdots + a_j(\tau s)^j + \cdots + a_k(\tau s)^k} \quad (5)$$

where l and k represent the order of PA. l has the same value as k in general case. a_j and b_j are corresponding coefficients.

Since fourth-order PA has high enough accuracy, this paper takes the fourth order PA as an example to demonstrate and deduce. The equation of fourth-order PA for $e^{-\tau s}$ is

$$e^{-\tau s} \approx \frac{(\tau s)^4 - 20(\tau s)^3 + 180(\tau s)^2 - 840\tau s + 1680}{(\tau s)^4 + 20(\tau s)^3 + 180(\tau s)^2 + 840\tau s + 1680} = 1 + \frac{p_3 s^3 + p_1 s}{s^4 + q_3 s^3 + q_2 s^2 + q_1 s + q_0} \quad (6)$$

It should be noted that the fourth-order PA brings four additional state variables in the time-delay block. The system diagram for the part of PA is shown in Fig. 2.

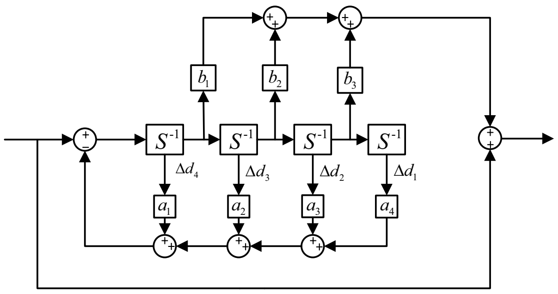

Fig. 2. Dynamic model of fourth-order PA.

With this approximation, it is possible to obtain equations of AGC system as shown in (7) which provides an appropriate depiction of a single time delay included in state variable:

$$\dot{X} = \tilde{A}X + F\Delta P_d \quad (7)$$

where

$$X = \begin{bmatrix} \Delta f & \Delta P_m & \Delta P_v & \int ACE & \Delta d_1 & \Delta d_2 & \Delta d_3 & \Delta d_4 \end{bmatrix}^T$$

$\tilde{A}$ represents the state matrix of a single-area AGC system.

The eigenvalue of the system (7) are the solutions of the characteristics equation.

3 Time margin computation and controller design based on root locus analysis

3.1 Delay margin computation

Determining whether the system delay-independent or delay dependent stable is the core of the research on stability of time-delay system. For delay-independent stability, the system is stable for all finite delays. In a delay-dependent stability analysis, there are existing a delay margin τ^* which ensure the system is asymptotically stable for $\tau \le \tau^*$ and indicate the system is unstable for $\tau > \tau^*$. From the general stability theory of dynamic system, a well-known sufficient condition is that all roots of Equation. (8) lie in the left half s-plane to be asymptotically stable. In the other words, when the Equation. (8) has roots (if any) on the imaginary axis, it is sufficient to find the delay-margin value. So, for delay margin of a fixed parameters AGC system, tracking the change of characteristic root corresponding to dominant mode is an effective method. As shown in Fig. 3, the value of time-delay margin is 4.2750. Time-domain simulations with different time delays are presented in Fig. 4. It shows the frequency response is stable if $\tau = 4.3500$ or $\tau = 4.2000$. When $\tau = 4.2750$. The curve gets divergent and the system is unstable. The time-domain simulation results verify the correctness of delay margin.

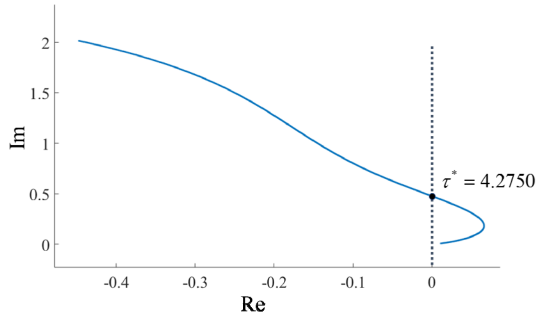

Fig. 3. Root locus of dominant mode with time delay increasing..

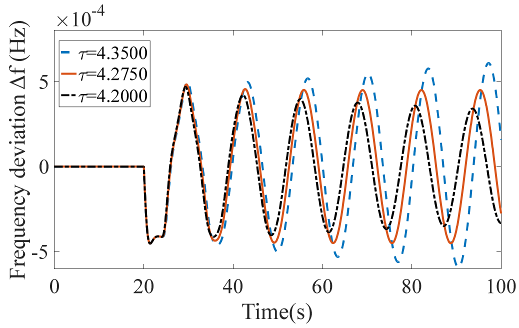

Fig. 4. Frequency response curves with different delays

3.2 PI controller design

PI controller parameters also have great influence on eigenvalues and oscillation characteristics. With the change of K_P and K_I, the delay margin will change subsequently. As shown in Tab. 1, the delay margin τ^* is 4.5910 for K_P=1.5, K_I=0.8. However, the simulation result presented in Tab. 1 indicates that the delay margin τ^* is 3.8865 for K_P=1.6, K_I=0.9. Therefore, it is essential to find the optimal PI controller parameters to derive delay margin largest and an ideal damping performance

Table 1. Time-delay margin with different PI.

PI	Delay Margin τ^*(S)
$K_P = 1.5, K_I = 0.8$	4.5910
$K_P = 1.6, K_I = 0.9$	3.8865

In order to achieve the best stability performance, this paper utilizes PSO to obtain the optimal PI controller parameters. The fitness function is selected as the delay margin, particle is selected as the proportional gain and the integral gain. The steps for determining maximal allowable transmission delay are shown in Fig. 5.

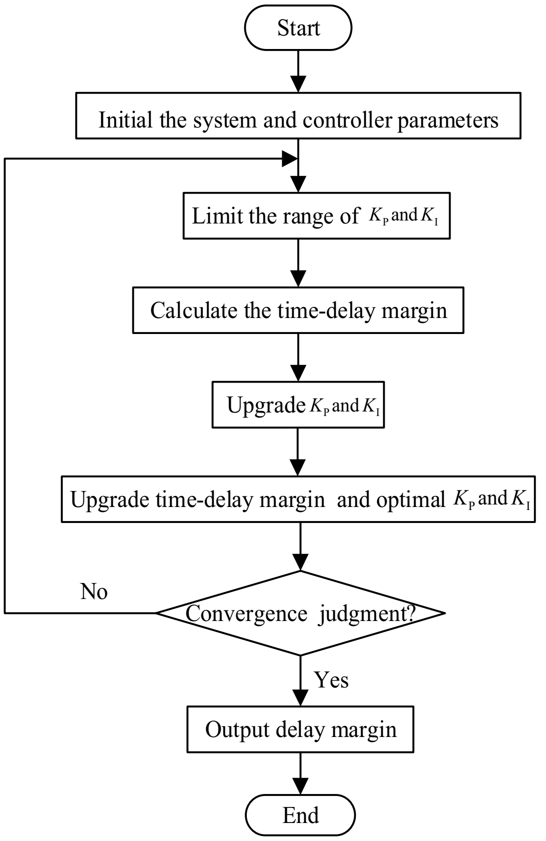

Fig. 5. The step for determining maximal allowable transmission delay

Step 1: Initial AGC system and controller parameter.
Step 2: Limit the range of K_p and K_I.
Step 3: Calculate the time-delay margin.
Step 4: Upgrade K_p and K_I for the next calculation.
Step 5: Upgrade time-delay margin and optimal K_p and K_I.
Step 6: Determine whether the system converges, if the system converges, outputting delay margin and the optimal K_p and K_I, otherwise, return Step 2.

4 Case study

Case studies are carried out based on one-area AGC system and this section covers three parts. Firstly, the eigen analysis is conducted by utilizing the proposed state-space model. Its eigenvalue analysis is verified by time-domain simulations and Prony analysis. Then the model is applied to root locus analysis and delay-margin calculation. Finally, PSO is used to design PI controller parameters for the best stability performance. The AGC system parameters are given in Tab. 2.

Table 2. Parameters of AGC system

Parameter	Value
T_g	0.2s
T_{ch}	0.2s
M	8s
D	1
K_P	1.5
K_I	0.9
β	5
R	0.09

4.1 Prony analysis and time-domain verification

Fig. 4 presents the time-domain simulation with different delays near delay margin. Based on the proposed model, eigenvalues with $\tau = 3.9$ and $\tau = 4.1$ are calculated and given in Tab. 3 respectively. It shows the one-area AGC system has 8 eigenvalues, and when $\tau = 3.9$, all eigenvalues are in left half s-plane which indicates the system is stable. As for $\tau = 4.1$, one of eigenvalues changes to the right plane and the system become unstable. The results of eigen analysis are matched with the time domain simulation.

Table 3. Eigenvalues of AGC system with (τ=3.9 s and τ=4.1 s)

λ_i ($\tau = 3.9$s)	λ_i ($\tau = 4.1$s)
-8.5561	-8.5059
-2.8941+j3.7982	-2.8107-j3.7366
-2.8941-j3.7982	-2.8107+j3.7366
-0.1894+j2.0406	-0.1759+j1.9675
-0.1894-j2.0406	-0.1759-j1.9675
-0.5221	-0.5317
-0.0037+j0.4485	0.0040+j 0.4654
-0.0037-j0.4485	0.0040-j0.4654

Prony analysis is an extended version of Fourier analysis. It can directly estimate the oscillation frequency, damping, amplitude and relative phase of the system by the response under the given input signal. Prony analysis is applied in response curves with $\tau = 3.9$s and $\tau = 4.1$s. The results are presented in Tab. 4 and Tab. 5. According to tables, Prony analysis verifies the rightness of eigen analysis including damping coefficient, frequency component and angular velocity.

Table 4. Prony analysis results (τ=3.9 s)

A_i	σ_i	f_i	ω_i
4.4314×10^{-4}	-0.0037	0.0777	0.4884
9.5314×10^{-5}	-0.2967	0	0
3.0977×10^{-5}	-0.5468	0	0

Table 5. Prony analysis results ($\tau = 4.1s$)

A_i	σ_i	f_i	ω_i
4.5385×10^{-4}	0.0040	0.	0.4654
6.8553×10^{-5}	-0.1570	0	0

A_i is the amplitude coefficient, σ_i is the damping coefficient, f_i is the frequency component and ω_i is the angular velocity.

4.2 Root locus analysis and margin computation

Based on the eigen-analysis model, root locus analysis is performed for this system. According to Fig. 6, all eigenvalues are moving from the left half s-plain to the right with the delay increasing and the system is becoming unstable. Among these eigenvalues, eigenvalue of domain mode firstly crosses imaginary axis, so that it determines the time-delay margin of one-area AGC system. For the simulation system with given parameters, the margin is 3.9947s. The frequency response with different delays near margin is presented in Fig. 7. This demonstrates the correctness of margin results.

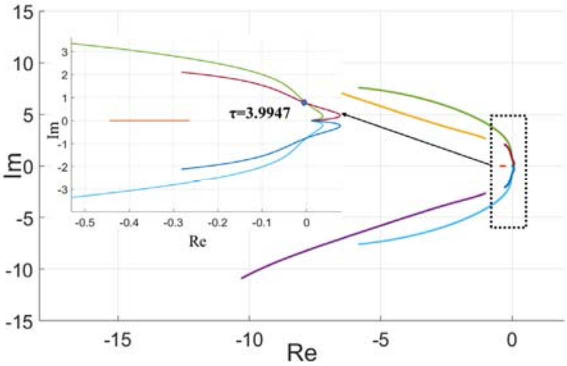

Fig. 6. Root locus with time delay.

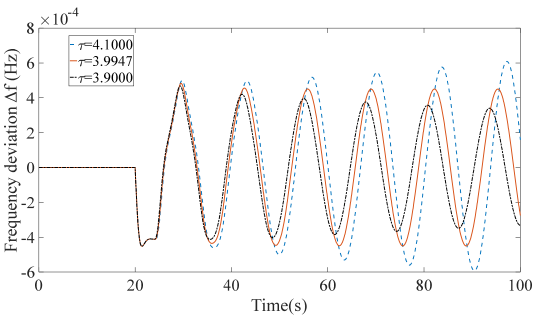

Fig. 7. Frequency response curves with different delays near delay margin

4.3 Controller parameter tuning and result analysis

As can be seen from Tab. 6, for the AGC system with time delay, with the change of PI parameters, the effect of the PI parameters on the delay margin becomes obvious. A large set of PI controller parameters, the delay margin is calculated using Equation. (8) and the root locus. The results show that τ^* decreases with the increase of K_I for a fixed K_P. This indicates that the increase of K_I causes instability in AGC system. When K_I is fixed, the effect of K_P on τ^* has two different modes. For all values of K_I, when K_P is within a certain range, τ^* increases with the increase of K_P. However, if K_P is beyond this range, τ^* decreases with the increase of K_P.

In order to make the delay margin largest and have an ideal damping performance, this paper utilizes PSO to obtain the optimal PI controller parameters. The results are shown in Tab. 7. The Fig. 8 shows that the delay margin converges gradually with the increase of iteration. The optimal PI controller parameters are $K_P = 1.0319$, $K_I = 0.1$ and delay margin is 43.400. This indicates AGC system can remain stable under large time delay. Besides, the PI controller is designed based on the standard value. When the parameters of the real system deviate from the standard value, the delay margin of AGC system also change accordingly. So, it is necessary to ensure that the controller has a greater delay stability margin than the delay of real system when designing the controller.

Table 6. Delay margin results for various values of KP and KI

τ^*(s)	K_I					
K_P	0.1	0.3	0.5	0.7	0.9	1.1
0.1	38.318	12.305	7.0950	4.8590	3.6180	2.8260
0.3	40.044	12.880	7.4430	5.1101	3.8115	2.9815
0.5	41.461	13.352	7.7262	5.3121	3.9675	3.1105
0.7	42.525	13.707	7.9388	5.4634	4.0852	3.2058
0.9	43.185	13.925	8.0686	5.5562	4.1565	3.2634
1.1	43.375	13.975	8.1056	5.5788	4.1740	3.2758
1.3	42.986	13.857	8.0250	5.5207	4.1257	3.2351

Table 7. Delay margin results for the optimal of KP and KI

PI	Delay Margin τ^*(s)
$K_P = 1.0319$ $K_I = 0.1$	43.400

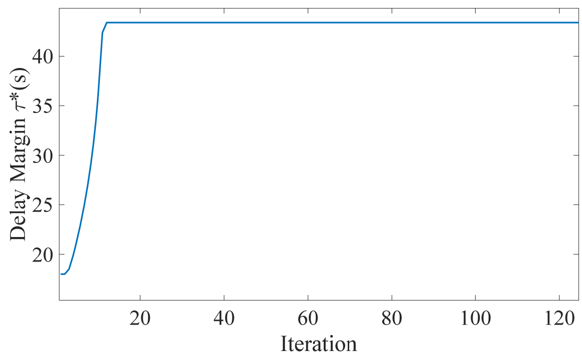

Fig. 8. Convergence curve of delay margin

5 Conclusion

In this paper, the focus is drawn on the delay-dependent stability of AGC systems. The state-space model is established based on Padé approximation to conduct eigenvalue calculation. Eigenvalue analysis is verified by time-domain simulations and Prony analysis. In order to verify the results of delay margin, a simulation study has been carried out in case study. Finally, this paper designs controller parameters for the best stability. The following observations and comments can be drawn from the results:

1) This paper establishes the model to analyze the influence of time delay on AGC system and calculate its eigenvalues. The results of time domain simulation and Prony analysis verify the correctness of the model.

2) The time-delay root locus of eigenvalues reflects the stability of AGC system with time delay increasing, so that the time-delay margin can be obtained, which is an important stability parameter of AGC system.

3) The influence of different controller parameters on the stability of the system is complex. In this paper, the controller parameters are designed based on particle swarm optimization algorithm with the goal of maximizing the delay margin. The obtained parameters can maximize the system delay margin and improve the robustness and stability of the system.

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