

Emergency decision-making method based on multi-attribute intuitionistic fuzzy and evidence theory

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Abstract. Multi-attribute emergency decision-making problems have the characteristics of incomplete information and shortage of response time. Evidence theory can effectively express uncertain information in the decision-making process. However, evidence theory requires that the condition of evidence independence is met, and the evaluation information of experts is often vague. Therefore, an emergency decision-making method based on intuitionistic fuzzy sets and evidence theory is proposed. First, each expert gives an intuitionistic fuzzy evaluation of each emergency plan. Secondly, the proposed intuitionistic fuzzy similarity calculation method is used to obtain the similarity between experts and determine the expert weight. The attribute weight is known, the intuitionistic fuzzy evaluation is converted into a Mass function, and the evaluation of expert's decision-making plan is revised and fused using evidence theory to obtain the final decision. Finally, an example analysis proves that the model is feasible and effective.

1 Introduction

In recent years, there have been frequent emergencies in the international community, which have caused serious harm to economic and social development and the stability of social life. Scientifically and accurately responding to emergencies has become one of the current research hotspots [1]. Emergency decision-making is the core of emergency management. It not only needs to make decisions quickly for different scenarios, but also needs to use incomplete information to make uncertain multi-attribute decisions. Effective emergency decision-making can reduce casualties and property losses. Therefore, to solve the problem of incomplete information, fusion of multi-attribute decision-making is the current research focus in the field of emergency decision-making.

Evidence theory [2] has obvious advantages in dealing with uncertain information, provides a powerful tool for the representation and fusion of decision-level uncertain information. Ju [3] uses the DS/AHP method to identify focal elements from an incomplete decision matrix, and solves the problem of group multi-criteria decision-making with incomplete information. Qiao Xiaojiao et al. [4] used fuzzy evidence reasoning theory to construct a fuzzy risk analysis model for the emergency risk analysis of uncertain and incomplete information. However, the evidence theory has shortcomings, for example, the evidence is subjectivity; the independence conditions between evidences cannot be satisfied. In the emergency management, it usually requires multiple experts and departments to coordinate decision-making. Therefore, independence of evidence in the process of evaluating

information fusion is particularly prominent. In addition, the evaluation information given by experts is generally a fuzzy intuitive evaluation. Transforming vague evaluations into independent evidence can better solve emergency problems in real life. The trust function and likelihood function in the evidence theory can be used to describe the upper and lower bounds of the trust degree of a certain proposition. This has a one-to-one correspondence with the membership function in the intuitionistic fuzzy set and the remaining part except the non-membership function. The intuitionistic fuzzy set [5] can well express the expert's hesitation to the decision-making subject. It uses the degree of membership and non-membership to express the three states of support, opposition, and hesitation of the decision maker's objective things, and more accurately describes the objective things. Natural attributes. Based on the above, for the problem of multi-attribute emergency decision-making with unknown and uncertain weights, this paper proposes an emergency decision-making method based on multi-attribute intuitionistic fuzzy and evidence theory.

2 Intuition Fuzzy sets & Dempster-Shafer theory

2.1 Intuitionistic Fuzzy sets

Definition 1 Suppose X is a given universe, then an intuitionistic fuzzy set A on X is represented as $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$. Where $\mu_A(x)$ and $\nu_A(x)$ represent the membership and non-membership of

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the element x in X belonging to A , and for $\mu_A(x) \in [0,1]$, $v_A(x) \in [0,1]$, satisfy $0 \leq \mu_A(x) + v_A(x) \leq 1, x \in X$. In addition, suppose $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ represents the hesitation that the element x in X belongs to A , and also $\pi_A(x) \in [0,1]$. The entire set of intuitionistic fuzzy sets on X is denoted as $IFSs(x)$.

Definition 2 Suppose A and B are two intuitionistic fuzzy sets on the universe X , S is a mapping, and the mapping S is known: $IFSs(x) \times IFSs(x) \rightarrow [0,1]$, then $S(A,B)$ It is the similarity between the intuitionistic fuzzy set $A \in IFSs(x)$ and the intuitionistic fuzzy set $B \in IFSs(x)$, and satisfies the following properties:

- $0 \leq S(A, B) \leq 1$.
- if $A=B$, then $S(A, B) = 1$; $S(A, B) = S(B, A)$
- if $A \subseteq B \subseteq C$, $A, B, C \in IFSs(x)$, then $S(A, C) \leq S(A, B), S(A, C) \leq S(B, C)$.

2.2 Dempster-Shafer theory

Definition 3 The complete set of mutually incompatible basic propositions is called the recognition framework Θ , which represents all possible answers to a certain question, but only one answer is correct, which can be expressed as $\Theta = \{\theta_1, \theta_2, \dots, \theta_j, \dots, \theta_n\}$. The power set of the recognition frame refers to the set consisting of all the subsets of the recognition frame Θ , denoted as 2^Θ , expressed as $2^\Theta = \{\phi, \{\theta_1\}, \{\theta_2\}, \dots, \{\theta_n\}, \{\theta_1, \theta_2\}, \dots, \Theta\}$.

Definition 4 Let Θ be the recognition framework, the basic trust assignment function m is a mapping from the set 2^Θ to $[0,1]$, A represents any subset of the recognition framework Θ , denoted as $A \subseteq \Theta$, and satisfies

$$\begin{cases} m(\phi) = 0 \\ \sum_{A \subseteq \Theta} m(A) = 1 \end{cases}$$

Definition 5 Suppose m_1, m_2, \dots, m_n are n basic trust allocation functions on the same recognition framework Θ , and the focal elements are $A_i (i = 1, 2, \dots, n)$, then the synthesis rule is:

$$\begin{cases} m(\phi) = 0 \\ m(A) = \frac{\sum_{A_1 \cap \dots \cap A_n = A} m_i(A_i)}{1 - \sum_{A_1 \cap \dots \cap A_n = \phi} m_i(A_i)} \end{cases} \quad (1)$$

3 Construction of emergency decision-making model

3.1 Problem Description

For multi-attribute emergency group decision-making problems with unknown and uncertain weights, there are decision schemes $A_i = \{a_1, a_2, \dots, a_n\}$, experts $M_k = \{m_1, m_2, \dots, m_m\}$, each scheme including decision attributes $C_j = \{c_1, c_2, \dots, c_p\}$. Assuming that the evaluation of the attribute C_j of the scheme A_i by the expert M_k is expressed as $d_{ij}^k = (\mu_{ij}^k, v_{ij}^k)$, then the emergency decision-making evaluation matrix of the expert M_k 's intuitionistic fuzzy set $D^k = [d_{ij}^k]_{p \times n}$.

$$D^k = \begin{matrix} & a_1 & a_2 & \dots & a_n \\ \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{matrix} & \begin{bmatrix} (\mu_{11}^k, v_{11}^k) & (\mu_{12}^k, v_{12}^k) & \dots & (\mu_{1n}^k, v_{1n}^k) \\ (\mu_{21}^k, v_{21}^k) & (\mu_{22}^k, v_{22}^k) & \dots & (\mu_{2n}^k, v_{2n}^k) \\ \vdots & \vdots & \dots & \vdots \\ (\mu_{p1}^k, v_{p1}^k) & (\mu_{p2}^k, v_{p2}^k) & \dots & (\mu_{pn}^k, v_{pn}^k) \end{bmatrix} \end{matrix}$$

3.2 Decision-making Process

- **Step 1** Construct a basic probability distribution function. Taking the scheme set as the recognition framework $\Theta = \{a_1, a_2, \dots, a_n\}$, the expert M_k is used to evaluate the intuitionistic fuzzy number vector $d_j^k = [(\mu_{1j}^k, v_{1j}^k), \dots, (\mu_{nj}^k, v_{nj}^k)]$ is regarded as a piece of evidence, and its basic probability distribution function m_j^k [6] is expressed as:

$$m_j^k = \begin{cases} m_j^k(\emptyset) = 0 \\ m_j^k(a_i) = \frac{\mu_{ij}^k}{\sum_{i=1}^n (1 - v_{ij}^k)} \\ m_j^k(\Theta) = 1 - \sum_{i=1}^n m_j^k(a_i) \end{cases} \quad (2)$$

- **Step2 Calculate the similarity of intuitionistic fuzzy sets among experts, determine the weight of experts, and modify the evidence of expert sets.** At present, the common methods for measuring similarity of intuitionistic fuzzy sets are based on distance measurement [7,8], which is to measure the distance between the membership and non-membership functions. And the similarity measure of the intuitionistic fuzzy set containing parameters [9,10], the similarity is calculated by setting reasonable parameters, but only the membership and non-membership functions are considered, and the hesitation degree may be affected by the result. Therefore, this paper proposes a method to measure the similarity of intuitionistic fuzzy sets considering hesitation. Let $A = \{(x_i, \mu_A(x_i), v_A(x_i)) | x_i \in X\}$, $B = \{(x_i, \mu_B(x_i), v_B(x_i)) | x_i \in X\}$ is two intuitionistic fuzzy sets on $X = \{x_1, x_2, \dots, x_n\}$, let $f(x) = \frac{1}{2}(\mu(x) + 1 - v(x))$, the similarity of intuitionistic fuzzy sets is defined as: $S(A, B) = 1 - \frac{1}{n} \sum_{i=1}^n \frac{[|\mu_A(x_i) - \mu_B(x_i)| + |f_A(x_i) - f_B(x_i)| + |v_A(x_i) - v_B(x_i)|]}{MAX(|\mu_A(x_i) - \mu_B(x_i)| + |f_A(x_i) - f_B(x_i)| + |v_A(x_i) - v_B(x_i)|)}$ (3)

Using formula (3), we can get the intuitionistic fuzzy similarity $S_j^{st}(d_j^s, d_j^t)$ between expert S and T with respect to attribute J , and the weight of expert under attribute J is $\lambda_j^t = \frac{\sum_{s=1, s \neq t}^m S_j^{st}(d_j^s, d_j^t)}{\sum_{s=1}^m \sum_{t=1}^m S_j^{st}(d_j^s, d_j^t)}$. Based on this, the discount factor $\alpha_j^k = \frac{\lambda_j^k}{\lambda_{max}^k}$ can be determined. Correct the original data to get the final basic probability distribution function \hat{m}_j^k :

$$\hat{m}_j^k = \begin{cases} \hat{m}_j^k(\emptyset) = 0 \\ \hat{m}_j^k(a_i) = \frac{\lambda_j^k}{\lambda_{max}^k} m_j^k(a_i) \\ \hat{m}_j^k(\Theta) = 1 - \sum_{i=1}^n \hat{m}_j^k(a_i) \end{cases} \quad (4)$$

- **Step3 Expert decision information gathering.**
 According to the D-S synthesis formula (1), the revised evidence is merged to obtain the final reliability value M_{Total} of each plan, which is recorded as $M_{Total} = \{M(a_1), M(a_2), \dots, M(a_n)\} M(\theta)$

4 Case Analysis

This paper takes the traffic emergencies in paper [11] as an example to illustrate the feasibility and effectiveness of the decision-making method based on intuitionistic fuzzy sets and evidence theory. After a traffic accident, in order to determine the best rescue point as soon as possible, for the three emergency rescue points (A_1, A_2, A_3), a decision-making group is formed by four experts (M_1, M_2, M_3, M_4). Evaluation attributes include travel capacity (C_1), reserve capacity (C_2), dispatch capacity (C_3), logistics support capacity (C_4). The current expert decision group gives the attribute weight $w_j = (0.35, 0.2, 0.25, 0.2)$. Table 1 shows specific fuzzy evaluation of three rescue points under four attributes by different experts.

Table1. Decision-makers' Intuitionistic Fuzzy Evaluation of Emergency Rescue Points

Attribut	Exper	A_1	A_2	A_3	A_4
C_1	M_1	(0.3,0.5)	(0.6,0.1)	(0.4,0.3)	(0.1,0.6)
	M_2	(0.6,0.3)	(0.5,0.2)	(0.6,0.1)	(0.7,0.1)
	M_3	(0.4,0.4)	(0.8,0.1)	(0.5,0.1)	(0.6,0.2)
C_2	M_1	(0.2,0.4)	(0.4,0.1)	(0.9,0.1)	(0.8,0.1)
	M_2	(0.5,0.2)	(0.4,0.3)	(0.5,0.1)	(0.3,0.2)
	M_3	(0.1,0.3)	(0.2,0.3)	(0.4,0.1)	(0.2,0.3)
C_3	M_1	(0.5,0.1)	(0.6,0.2)	(0.6,0.1)	(0.7,0.1)
	M_2	(0.5,0.2)	(0.7,0.2)	(0.5,0.3)	(0.7,0.2)
	M_3	(0.5,0.2)	(0.8,0.1)	(0.5,0.2)	(0.7,0.2)
C_4	M_1	(0.5,0.3)	(0.5,0.2)	(0.7,0.3)	(0.7,0.2)
	M_2	(0.5,0.2)	(0.6,0.1)	(0.7,0.2)	(0.7,0.2)
	M_3	(0.6,0.2)	(0.6,0.2)	(0.7,0.2)	(0.6,0.2)

4.1 Step1 Construct a basic probability distribution function.

Taking the data in table 1 into formula (2) to obtain the initial basic probability distribution function, which is recorded as m_j^k . As shown in Table 2.

Table2. Initial probability distribution function

Attribut	Exper	A_1	A_2	A_3	A_4	θ
C_1	m_1^1	0.1200	0.2400	0.1600	0.0400	0.4400
	m_1^2	0.1818	0.1515	0.1818	0.2121	0.2727
	m_1^3	0.1250	0.2500	0.1563	0.1875	0.2813

C_2	m_1^1	0.0606	0.1212	0.2727	0.2424	0.3030
	m_1^2	0.1563	0.1250	0.1563	0.0938	0.4688
	m_1^3	0.0333	0.0667	0.1333	0.0667	0.7000
C_3	m_1^1	0.1429	0.1714	0.1714	0.2000	0.3143
	m_1^2	0.1613	0.2258	0.1613	0.2258	0.2258
	m_1^3	0.1515	0.2424	0.1515	0.2121	0.2424
C_4	m_1^1	0.1667	0.1667	0.2333	0.2333	0.2000
	m_1^2	0.1515	0.1818	0.2121	0.2121	0.2424
	m_1^3	0.1875	0.1875	0.2188	0.1875	0.2188

4.2 Step2 Calculate the similarity of different experts' intuitionistic fuzzy set matrices under the same attribute, determine the expert weight, and modify the initial basic probability distribution function.

According to formula (3), the similarity $S_j^{st}(d_j^s, d_j^t)$ of different expert intuitionistic fuzzy sets under the same attribute can be obtained, that is, $S_1^{st}(d_1^s, d_1^t) = [0.7125, 0.0775, 0.8500]$, $S_2^{st}(d_2^s, d_2^t) = [0.8500, 0.7105, 0.8375]$, $S_3^{st}(d_3^s, d_3^t) = [0.6000, 0.9000, 0.9563]$, $S_4^{st}(d_4^s, d_4^t) = [0.9375, 0.9188, 0.9438]$, where $s=1,2,3$ and $t=1,2,3$. After that, the obtained similarity is brought into formula (4) to obtain the revised mass function. Then synthesize the evidence of different experts under the same attribute, obtain reliability value of each emergency decision under different attributes. That is, $m_1 = \{0.1786, 0.3046, 0.2183, 0.1993\}0.0993$, $m_2 = \{0.1170, 0.1550, 0.3199, 0.2157\}0.1923$, $m_3 = \{0.1777, 0.2943, 0.1428, 0.2819\}0.1032$, $m_4 = \{0.2081, 0.1675, 0.2293, 0.3242\}0.0710$

4.3 Step3 The attribute weight is known, and the evidence is synthesized.

according to formula (1) to obtain the final reliability value M_{Total} of each scheme, namely

$$M_{Total} = \{0.1639, 0.3039, 0.2282, 0.2662\}$$

4.4 Results Comparison

The results are compared with paper [12], as shown in Table 3. The choice of the optimal scheme is both A_2 , which demonstrates the feasibility of the model proposed in this paper. However, the second option has changed, and the difference between the options is greater. This is because this article calculates the similarity between experts separately from the expert level, and uses it as the evidence correction coefficient to improve the evidence and obtain more accurate results. Therefore, using the similarity of the expert's intuition fuzzy matrix can eliminate obvious errors and satisfy the independence condition, thereby obtaining more accurate and objective decision-making results.

Table3. Comparison of the results of the two methods

Results of this article		Paper [12] results	
M_{Total}	Sort	S_{Total}	Sort
0.3039	A_2	0.3575	A_2
0.2662	A_4	0.3004	A_1
0.2282	A_3	0.1755	A_4
0.1639	A_1	0.1666	A_3

5 Conclusion

Emergency decision-making problems generally have the characteristics of incomplete information, urgency of development, and comprehensive evaluation of complex attributes. In view of these characteristics, this article uses evidence theory to integrate the evaluation of various experts on different emergency plans. However, in the actual evidence fusion process, evidence theory has the defect that evidence does not satisfy independence. Experts are due to their own abilities and knowledge backgrounds. There may be conflicting parts in the evaluation of each decision-making plan. Therefore, this paper proposes an emergency decision-making method based on multi-attribute intuitionistic fuzzy and evidence theory. First, for the emergency decision-making problem with unknown and uncertain weights, various experts give a direct fuzzy evaluation. Secondly, the proposed intuitionistic fuzzy similarity calculation method is used to determine the expert weight, and the expert weight conversion is used to obtain the correction coefficient and modify the original evidence. The intuitionistic fuzzy similarity calculation method takes into account the hesitation in the intuitionistic fuzzy number and improves the similarity. The accuracy of calculation. Evidence fusion is then carried out to obtain the choice of emergency plan under multiple attributes, and realize the ultimate goal of emergency decision-making. Finally, the application calculation process of the model is explained through calculation examples, and the feasibility and rationality of the model are verified by comparison with other methods, which effectively slows down the requirement of evidence theory for the independence of evidence, and is also the weight of emergency decision-making problems. Confirmation provides a scientific and effective method to make it more widely used in practical applications.

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