

# A Calculation Method of Available Transmission Capacity for Medium and Long-term Electricity Trading

Fei Shi<sup>1a\*</sup>, Shuhai Feng<sup>1b</sup>, Hengfeng Zheng<sup>1c</sup>, Liwen Wang<sup>1d</sup>, Xu Wu<sup>2e</sup>

<sup>1</sup>Electric Power Research Institute, Nanjing, China

<sup>2</sup>State Grid Anhui Electric Power Company

**Abstract:** In view of the fact that China's power dispatching agencies and trading centers are relatively independent, but the medium and long-term power transactions need physical execution, which leads to higher requirements for transaction security boundary, this paper proposes a calculation method of available transmission capacity for medium and long-term power transactions, which calculates the available transmission capacity of transmission channels through probabilistic method to improve the enforceability of transaction results. Numerical simulation results also verify the effectiveness of the algorithm.

## 1 Introduction

The calculation of available transmission capacity of power system began in the 1970s and has a history of 30 years<sup>1</sup>. In the traditional operation mode of power industry<sup>2-4</sup>, it is mainly used to evaluate the interconnection strength and compare the advantages and disadvantages of different transmission system structures. In the power market environment, the operation conditions of power system become more complex, and economic interests drive market participants to use transmission equipment to the maximum extent<sup>5-7</sup>. In this context, the calculation of available transmission capacity is very important. It can not only show the security and stability margin of power grid operation, so as to reduce the probability of congestion, but also provide market participants with detailed information of power grid usage, so as to guide them to participate in the market.

The power market model of dispatching and trading integration represented by the United States can consider the system security constraints in the clearing process through security constrained unit commitment and security constrained economic dispatch, so as to accurately coordinate the economic benefits and system security requirements<sup>8</sup>. However, China's trading institutions and dispatching institutions are relatively independent, and the safe operation boundary of the system is determined by the dispatching institutions, while the medium and long-term trading is carried out by the power trading center. Aiming at the situation that the transaction result can not be executed, the optimal reduction of the transaction result is used to make it meet the system security constraints. Transaction reduction is the last line of defense to ensure the security of the system, but frequent transaction reduction is easy to cause the

problem of market fairness<sup>9,10</sup>. Therefore, this paper mainly studies how to calculate the available transmission capacity (ATC) that can be used for further transactions under the security boundary conditions given by the dispatching agency, so as to improve the enforceability of medium and long-term transactions.

## 2 ATC Calculation method based on dispatching security boundary

In the 1990s, North American Electric Reliability Council unified the concept of transmission limit and proposed a detailed definition and calculation framework of available transmission capacity<sup>11</sup>. This work has been widely recognized in the world.

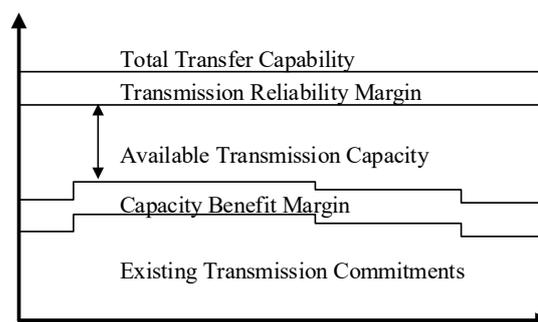


Fig 1 Definition of available transmission capacity

As shown in the figure, ATC refers to the surplus transmission capacity that can be used for further commercial activities in the actual transmission system on the basis of existing agreements. Mathematically, ATC can be expressed as total transfer capacity (TTC) minus transmission reliability margin (TRM), capacity benefit margin (CBM) and existing transmission commitment (ETC)

<sup>a</sup>email: shifei@epri.sgcc.com.cn, <sup>b</sup>email: fengshuhai@epri.sgcc.com.cn,

<sup>c</sup>email: zhenghf@epri.sgcc.com.cn, <sup>d</sup>email: wangliwen@epri.sgcc.com.cn

<sup>e</sup>email: wux201x@ah.sgcc.com.cn

$$ATC = TTC - TRM - ETC - CBM \quad (1)$$

In the formula, TTC is the core problem of traditional ATC calculation, but considering the characteristics of China's electric power medium and long-term trading, the safe operation boundary of the system is determined by the dispatching center. TTC should directly use the value given by the electric power dispatching organization, and the electric power trading center does not need and cannot carry out this calculation. In addition, the calculation of TRM and CBM mostly adopts fixed proportion, so the core problem of ATC calculation is transformed into the calculation of ETC.

The calculation of ETC, that is, the occupancy of existing transaction to the transaction channel, can be calculated by comprehensive sensitivity analysis and transaction power decomposition results. The sensitivity calculation method is as follows.

The deterministic load flow equation in power system can be summarized as:

$$W = f(X) \quad (2)$$

$W$  is the node injection vector, including the active power and reactive power,  $X$  is the state variable, including the node voltage amplitude and phase angle. The state variable  $X_0$  can be solved by Newton method under certain injection  $W_0$ .

When the node injection changes on the basis of the ground state power flow, it can be expressed as:

$$W = W_0 + \Delta W \quad (3)$$

where  $W_0$  is the value of expectation,  $\Delta W$  is the stochastic disturbance.

By expanding formula(3) with Taylor series, we can get

$$\begin{aligned} W &= W_0 + \Delta W = f(X_0 + \Delta X) \\ &= f(X_0) + J_0 \Delta X + \dots \end{aligned} \quad (4)$$

$J_0$  is the Jacobian matrix used in the last iteration of Newton method power flow calculation. Ignoring the higher order term of equation(4), we can get the following results:

$$\Delta W = J_0 \Delta X \quad (5)$$

Thus we'll get:

$$\Delta X = J_0^{-1} \Delta W = S_0 \Delta W \quad (6)$$

where  $S_0$  is the inverse matrix of  $J_0$ , generally it is called sensitivity matrix.

Since active power flow is the main concern in medium and long-term transactions, the branch power flow equation can be linearized. When the state variables of each node are known, the branch power flow equation can be written as follows:

$$Z = g(X) \quad (7)$$

The following formula can be obtained by expanding the equation according to Taylor series:

$$\begin{aligned} Z &= Z_0 + \Delta Z = g(X_0 + \Delta X) \\ &= g(X_0) + G_0 \Delta X + \dots \end{aligned} \quad (8)$$

where:

$$G_0 = \left. \frac{\partial Z}{\partial X} \right|_{X=X_0} \quad (9)$$

Ignoring the higher order term of equation(8), we can get

$$\Delta Z = G_0 S_0 \Delta W = T_0 \Delta W \quad (10)$$

$$\Delta z^k = T_0^k \Delta W^k \quad (11)$$

Through the above formula, the sensitivity relationship between any branch and node in the network can be calculated.

For cross provincial and cross regional transactions, the sensitivity calculation of DC tie line should also be considered. Based on the actual power control mode of the DC tie line, the DC transmission power is only related to the sending end node and the receiving end node connected to the DC tie line, and their sensitivities are 1 and -1, respectively. The sensitivity distribution factors of the other nodes not connected to the DC tie line are all 0.

$$Z_i = \sum_{i=1}^I G_0 S_0 W_{ii} \quad (12)$$

Where  $W_{ii}$  is the power decomposition value of the  $i$ th transaction at time  $t$ .

It is worth noting that the medium and long-term trading power curve has the characteristics of large time span and strong uncertainty<sup>12</sup>. In view of this feature, the uncertainty of node power decomposition should be considered in ATC calculation process summary, so as to accurately judge the distribution range of available transmission capacity of actual load of section.

When  $W_{ii}$  is no longer a constant, but a probability distribution function, the calculation of equation(12) requires convolution calculation of random variables, and the amount of calculation is large. In order to simplify the calculation process, the probability distribution of random variables can be obtained by introducing the cumulant and Gram-Charlier series expansion method and using the simple algebraic operation of the cumulant.

The k-order semi invariants of random variables  $\gamma_{k+1}$  can be represented by polynomials which can be expressed by the center moments of random variables. The recurrence relation is as follows:

$$\begin{aligned} \gamma_1 &= \alpha_1 \\ \gamma_{k+1} &= \alpha_{k+1} - \sum_{j=1}^k C_k^j \alpha_j \gamma_{k-j+1} \end{aligned} \quad (13)$$

On the other hand, from the cumulants of each order, the recurrence relation of the central moment of each order is obtained as follows:

$$\begin{aligned} \alpha_1 &= \gamma_1 \\ \alpha_{1+1} &= \gamma_{v+1} + \sum_{j=1}^v C_v^j \alpha_j \gamma_{v-j+1} \end{aligned} \quad (14)$$

The mathematic definitions and descriptions of cumulant method are related to many equations<sup>13</sup>, here we won't list in details. ATC calculation based on cumulant method mainly applies its two characteristics:

(1) The cumulant of the sum of independent random variables is equal to the sum of each random variable's cumulant.

(2) k-order cumulant of  $\alpha$  times of random variables is equal to  $\alpha^k$  times of its k-order cumulant.

According to these, we can obtain the approaches of getting node power injection's each order cumulant  $\Delta w^k$  and unknown variable's each order cumulant  $\Delta x^k$ :

$$\Delta w^k = \Delta w_G^k + \Delta w_L^k \quad (15)$$

$$\Delta x^k = S_0^k \cdot \Delta w^k \quad (16)$$

In the equations,  $\Delta w_G^k$  and  $\Delta w_L^k$  stand for k-order cumulants of seller's and the buyer's injection power.

After obtaining  $\Delta Z_i^k$  of branch power, the distribution function of  $\Delta Z_i$  can be obtained by the relationship between the cumulant and the center distance and the method of Gram-Charlier series expansion<sup>14</sup>. The random distribution of branch power can be obtained by formula(17).

$$f(x) = \varphi(x) + \frac{A_1 \varphi'(x)}{1!} + \frac{A_2 \varphi''(x)}{2!} + \frac{A_3 \varphi^{(3)}(x)}{3!} + \frac{A_4 \varphi^{(4)}(x)}{4!} + \dots \quad (17)$$

In this equation,  $\varphi(x)$  is the probability density function of the standard normal distribution, and the coefficient  $A_i$  can be obtained from formula(18), where  $\delta$  is the standard deviation of the random variable.  $\varphi^{(k)}(x)$  can be obtained by Hermite polynomial formula(19)<sup>15</sup>.

$$\begin{aligned} A_1 &= 0, \quad A_2 = 0, \quad A_3 = -\frac{\Delta Z_i^3}{\delta^3}, \quad A_4 = \frac{\Delta Z_i^4}{\delta^4}, \quad A_5 = -\frac{\Delta Z_i^5}{\delta^5}, \\ A_6 &= \frac{\Delta Z_i^6}{\delta^6} + 10 \left( \frac{\Delta Z_i^3}{\delta^3} \right)^2, \quad A_7 = -\frac{\Delta Z_i^7}{\delta^7} + 35 \frac{\Delta Z_i^3}{\delta^3} \frac{\Delta Z_i^4}{\delta^4} \\ A_8 &= \frac{\Delta Z_i^8}{\delta^8} + 56 \frac{\Delta Z_i^3}{\delta^3} \frac{\Delta Z_i^5}{\delta^5} + 35 \left( \frac{\Delta Z_i^4}{\delta^4} \right)^2 \end{aligned} \quad (18)$$

$$\varphi^{(k)}(x) = (-1)^k \varphi(x) H_k(x), k = 0, 1, \dots \quad (19)$$

Where, the coefficient H is calculated as follows:

$$\begin{aligned} H_0(x) &= 1, \quad H_1(x) = x, \quad H_2(x) = x^2 - 1, \quad H_3(x) = x^3 - 3x \\ H_4(x) &= x^4 - 6x^2 + 3, \quad H_5(x) = x^5 - 10x^3 + 15x \\ H_6(x) &= x^6 - 15x^4 + 45x^2 - 15 \\ H_7(x) &= x^7 - 21x^5 + 105x^3 - 105x \\ H_8(x) &= x^8 - 28x^6 + 210x^4 - 420x^2 + 105n \end{aligned} \quad (20)$$

Thus, the probability distribution function of ETC on each branch or transmission channel can be calculated.

### 3Case studies

Taking IEEE118 system as an example, there are 19 nodes generating output in this example. For simplicity, the power decomposition result of all nodes is 230 MW, the upper limit of branch active power is 350 MW, and the total power of TRM and CBM is 50 MW. When the uncertainty of trading capacity is not considered, the order of available transfer capacity of each branch is as follows:

**Table 1** ATC calculation results without considering the uncertainty of node injection power

Branch number	Begin Node	End Node	Active Power	ATC
9	9	10	228.76	71.24
7	8	9	227.35	72.65
176	110	111	217.60	82.40
134	86	87	214.43	85.57
38	26	30	213.30	86.70
51	38	37	200.78	99.22
8	8	5	197.62	102.38
104	65	68	186.83	113.17
183	68	116	184.13	115.87
133	85	86	177.24	122.76

Table 1 lists the 10 branches with the smallest ATC value, which are the main bottleneck channels affecting the transaction.

If we further consider the uncertainty of the decomposition of the trading power, and assume that the

power injection of the generating nodes obeys the normal distribution of N (230,46), the probability of branch power flow out of limit is considered with 90% confidence. On this basis, the available transmission capacity of each branch is calculated as follows.

**Table 2** ATC calculation results considering the uncertainty of node injection power

Branch number	Begin Node	End Node	Active Power Expectation	Active Power Variance	ATC
104	65	68	186.83	100.45	-15.40
9	9	10	228.76	45.51	12.99
7	8	9	227.35	45.06	14.98

176	110	111	217.60	41.08	29.81
134	86	87	214.43	39.08	35.55
38	26	30	213.30	33.91	43.30
8	8	5	197.62	26.59	68.34
133	85	86	177.24	31.25	82.76
51	38	37	200.78	6.03	91.50
163	100	103	143.08	41.75	103.49

Based on the calculation results in Table 1 and table 2, it can be seen that whether the uncertainty of the decomposition result of trading power is considered has a great influence on the calculation result of ATC in trading channel.

In the process of actual transaction execution, in order to take into account the impact of the uncertainty of long-term electricity trading, the limit value is generally discounted according to a certain proportion. According to

the basic rules for medium and long term electricity trading, the annual trading limit shall be no less than 80% of the available transmission capacity of key channels, while the monthly trading and intra month trading shall be 90% and 95% respectively. This method is simple and clear. It can deal with the uncertainty of all nodes and branches according to a unified standard, but it may not be able to accurately consider the different distribution characteristics of different channels.

**Table 3** difference of ATC calculation results of two different treatment methods

Begin Node	End Node	Deterministic calculation method			Probability calculation method		
		95%Limit	90% Limit	80% Limit	5%Variance	10%Variance	20%Variance
9	10	53.74	36.24	1.24	56.68	42.12	12.99
8	9	55.15	37.65	2.65	58.23	43.82	14.98
110	111	64.9	47.4	12.4	69.25	56.10	29.81
86	87	68.07	50.57	15.57	73.07	60.56	35.55
26	30	69.2	51.7	16.7	75.85	65.00	43.30
38	37	81.72	64.22	29.22	97.29	95.36	91.50
8	5	84.88	67.38	32.38	93.87	85.36	68.34
65	68	95.67	78.17	43.17	81.03	48.89	-15.40
68	116	98.37	80.87	45.87	115.87	115.87	115.87
85	86	105.26	87.76	52.76	112.76	102.76	82.76

It can be seen from the calculation results in Table 3 that the traditional calculation method of limit discount according to the unified standard for most branches is more consistent with the method proposed in this paper, but the uncertainty characteristics of different branches are quite different, and the traditional calculation method can not well understand the probability distribution characteristics of all branches, which may cause some transactions to be unable to be executed and may need to be implemented Make adjustments.

#### 4Conclusion

China's annual planned electricity and bilateral trading electricity need physical delivery, and the power dispatching agency and trading center are relatively independent, so it is difficult for trading center to carry out medium and long-term trading. In this paper, a calculation method of available transmission capacity is proposed, which is suitable for medium and long-term electricity trading. The effectiveness of the algorithm is verified by numerical simulation. It is important to note that the probability calculation method is slightly deviated from

the traditional Chinese power dispatching control concept. How to apply the probabilistic analysis control method in the field of power transaction and dispatching needs further exploration in the follow-up research process.

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