Stress-strain state of spherical shells with two unequal holes

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Abstract. This article presents the results of a numerical study of the stress concentration around two equal and unequal holes in an orthotropic spherical shell made of composite materials under the action of internal pressure. The influence of geometric (hole radii, shell thickness, distance between holes) characteristics, as well as material orthotropy and shear stiffness, on the stress state of spherical shells made of composite materials is studied. A numerical algorithm based on the finite element method has been developed and a software package has been implemented on a computer that allows solving the problem of stress concentration near two unequal holes in spherical shells made of composite materials.

1 Introduction

A fairly large number of works are devoted to the methods of calculating shell structures made of composite materials and the study of stress concentration near holes. However, most of the studies were carried out within the framework of the classical Kirchhoff-Love hypothesis, which does not take into account interlayer and transverse shears characteristic of composite materials [1, 2].

Within the framework of the improved theory of the Timoshenko type, the application of the finite element method is proposed for solving problems of orthotropic shells made of composite materials, weakened by several holes [3, 4].

Multiply connected shells made of composite materials have been considered by a few authors. The main results are given in [2, 7-9]. For an orthotropic cylindrical shell with several holes, a solution was constructed in the work of K I Shnerenko, N Kh Noraliev [10-14]. Moreover, the distribution of stresses around two equal circular holes in an orthotropic spherical shell made of a composite material was investigated taking into account shear stiffness [5, 6]. As follows from the above review of works, at present, the study of the stress-strain state of shells made of composite materials with two unequal holes is insufficient.

Therefore, in this paper, we study the stress distribution around two unequal circular holes in an orthotropic spherical shell made of a composite material. Using the refined theory of Timoshenko-type shells, we will take into account the effect of transverse shear deformations for the entire package of the shell as a whole. In this case, the composite material will be considered as homogeneous with the given characteristics.

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2 Materials and methods

A spherical shell with two unequal circular holes of radii r_{01} and r_{02} (Fig. 1) and intensity p_0 under the influence of internal pressure e is considered. The study area is divided into finite elements using the finite element method [5, 6].

Fig. 1. Spherical shell with two unequal circular holes.

In the middle surface of the shell, choose an orthogonal curvilinear coordinate system (x_1, x_2) ; the third coordinate is reported in the direction of the outer normal to the middle surface of the shell.

We will proceed from the variational equation [1]:

$$\iint \dot{\Omega} (T_1 \delta \varepsilon_1 + T_2 \delta \varepsilon_{12} + S_{12} \delta \varepsilon_{12} G_1 \delta N_1 + G_2 \delta N_2 + 2H_{12} \delta N_{12} + Q_1 \delta \varepsilon_{13} Q_2 \delta \varepsilon_{23}) d\Omega + \int_{\Gamma} [T_{\rho}^0 \delta u_{\rho} + s_{\rho 0}^0 \rho u_0 + G_{\rho}^0 \delta \gamma_{\alpha} + H_{\rho \theta}^0 \delta \gamma_{\theta} + Q_{\rho}^0 \delta \omega] d\Gamma = 0, \quad (1)$$

where $\Gamma = \Gamma_1 \Gamma_2$; Γ_1, Γ_2 -outlines of holes; Ω - area, the boundary of which is far enough from the contours of the holes, and on it the damping conditions are satisfied; $T_1, \dots, Q_2$ -components of the disturbed stress state; $T_{\rho}^0, \dots, Q_{\rho}^0$ -components of the main stress state.

3 Results and discussion

Table 1 shows the results of the study of the concentration coefficients of the annular forces k_{θ}^T (upper row) and k_{θ}^M (lower row) at various points of the hole contour, depending on the distance between the holes $l/\sqrt{Rh}$. The shell parameters were chosen as follows:

$$\frac{r_{01}}{R} = 0.1, \quad \frac{h}{r_{01}} = 0.1, \quad \frac{r_{02}}{R} = 0.075, \quad \vartheta_1 = 0.3, \quad \frac{E_1}{E_2} = 0.8, \quad \frac{G_{13}}{E_1} = \frac{G_{23}}{E_1} = 0.1, \quad \frac{G_{12}}{E_1} = 0.23$$

As can be seen from the table, the stresses on the contour of the larger hole at the point $\theta = 0$ up to the distance $l/\sqrt{Rh} = 1.5$ hardly change. With further approach of the holes to $l/\sqrt{Rh} = 0.25$ at the points of the contour of the larger hole, the coefficient k_{θ}^T increases, and the coefficient k_{θ}^M at the points $\theta = 0, \pi/2$ decreases, and at the point $\theta = \pi$, it increases. Of interest is the influence of the radius of the larger holes and the value of the jumper d on the stresses at the point $\theta = 0$ of the contour of the small holes. This can be seen from the first column of the table. When the holes approach each other at the point $\theta = \pi$ on the contour of the larger hole, the stress concentration k_{θ}^T and k_{θ}^M increase by 131% and 102%, respectively. Note that when the holes approach each other in the considered example, a non-monotonic character of the change in the stress concentration coefficients on the hole contours is also observed. A similar phenomenon was previously found for an isotropic shell with equal holes [7, 8].

Table 1. Concentration coefficients of the annular forces at various points of the hole contour.

θ	$l/\sqrt{Rh}$, within $r_{02}/\sqrt{Rh} = 0.75$								
	0.25	0.50	0.75	1.0	1.5	2.0	2.5	3.0	3.5
0	5.033	4.930	4.905	4.905	4.932	4.959	4.974	4.978	4.985
	1.670	1.790	1.822	1.826	1.803	1.777	1.761	1.755	1.746
$\frac{\pi}{4}$	3.319	3.264	3.250	3.247	3.255	3.267	3.273	3.276	3.278
	0.727	0.786	0.805	0.812	0.809	0.802	0.796	0.793	0.791
$\frac{\pi}{2}$	4.578	4.560	4.548	4.540	4.525	4.515	4.507	4.499	4.498
	1.286	1.330	1.372	1.408	1.467	1.500	1.512	1.512	1.512
$\frac{3\pi}{4}$	2.276	2.291	2.852	2.927	3.068	3.178	3.264	3.339	3.340
	0.407	0.466	0.525	0.579	0.654	0.670	0.641	0.592	0.591
π	9.447	6.313	5.374	4.943	4.720	4.819	4.953	5.050	5.050
	3.190	2.260	1.932	1.774	1.664	1.658	1.646	1.615	1.601

In Fig. 2 and 3 show the results of the study of stress concentration k_{θ}^T and k_{θ}^M at different values of the shear parameter G_{23}/G_{13} . The shell parameters were chosen as in the case of Table 1. Curves 1, 2, and 3 respectively refer to the points $\theta = 0, \pi/2, \pi$. It can be seen that the influence of G_{23}/G_{13} on the stress concentration is most pronounced at the points $\theta = 0$ and $\theta = \pi$.

The coefficients k_{θ}^T and k_{θ}^M with a change in the parameter G_{23}/G_{13} , from 0 to 0.1 at the point $\theta = 0$ on the contour of the larger hole (r_{01}) they increase by 10%, and at the point $\theta = \pi$ by 7%. At the point $\theta = \pi/2$ on both contours of the holes, the influence of G_{23}/G_{13} is insignificant. On the contour of the small hole at the points $\theta = 0$ and $\theta = \pi$ with a change in G_{23}/G_{13} , the coefficient k_{θ}^T increases by 11% and 5%, respectively. Solid curves refer to hole r_{01} , and dashed curves to hole r_{02} .

Fig. 2. Stress concentration k_{θ}^T .

Fig. 3. Stress concentration k_{θ}^M .

The effect of orthotropy of the material on the stress concentration k_{θ}^T (top line) and k_{θ}^M (bottom line) at various points of the hole contour is given in Table 2. The radius of one hole was fixed $r_{01}/R = 0.1$, and the second hole $r_{02}/R = 0.075$. It is seen that the orthotropy of the material significantly affects the stress concentration along both contours of the holes. With a change in the orthotropy of the material E_1/E_2 from 2.5 to 0.8 in points $\theta = 0$ and $\theta = \pi$ on the contour of the larger hole, the coefficient k_{θ}^T increases by 41% and 25%, respectively, and k_{θ}^M by 61% and 40%.

Table 2. Stress concentration k_{θ}^T (top line) and k_{θ}^M (bottom line) at various points of the hole contour.

θ	$l/\sqrt{Rh}$, within $r_{01}/\sqrt{Rh} = 1.0$								
	0.25	0.50	0.75	1.0	1.5	2.0	2.5	3.0	3.5
0	4.384	4.092	4.019	4.024	4.108	4.186	4.204	4.236	4.245
	1.406	1.653	1.723	1.728	1.676	1.620	1.601	1.577	1.565
$\frac{\pi}{4}$	2.902	2.762	2.720	2.713	2.737	2.766	2.783	2.790	2.801
	0.596	0.778	0.773	0.794	0.796	0.783	0.771	0.766	0.761
$\frac{\pi}{2}$	3.808	3.857	3.881	3.888	3.887	3.860	3.845	3.828	3.819
	0.695	0.804	0.931	1.054	1.239	1.332	1.350	1.354	1.352
$\frac{3\pi}{4}$	1.970	2.079	2.183	2.292	2.478	2.613	2.714	2.796	2.800
	0.02	0.03	0.170	0.318	0.550	0.661	0.686	0.668	0.664
π	9.835	5.885	4.550	4.003	3.805	3.991	4.189	4.312	4.320
	3.066	1.988	1.607	1.462	1.431	1.484	1.511	1.502	1.500

This increase on the contour of the small hole at the points $\theta = 0$ and $\theta = \pi$ for the coefficient k_{θ}^T is 35% and 15%, and for k_{θ}^M 140% and 32%. At the point $\theta = \pi/2$ with a change in E_1/E_2 , the coefficients k_{θ}^T and k_{θ}^M decrease.

Table 3 shows the results of studying the stress concentration factors k_{θ}^T and k_{θ}^M depending on the change in the radii of the holes r_{02}/r_{01} . In this case, the radius of one hole was fixed $r_{01}/R = 0.1$, and the radius of the other hole was changed $r_{02}/r_{01} = 0.5, 1.0, 1.5, 2.0, 2.5$. As the radius of the second hole increases, the stress concentration increases on the contour of both holes. When the parameter $r_{02}/r_{01} = 0.5, 1.0, 1.5, 2.0, 2.5$ changes at the points $\theta = \pi/2$ and $\theta = \pi$ on the contour of the first hole (r_{01}), the coefficient k_{θ}^T increases by 6% and 21%, respectively, and at the point $\theta = 0$ decreases by 10%.

Table 3. Stress concentration factors k_{θ}^T and k_{θ}^M depending on the change in the radii of the holes.

θ	E_1/E_2							
	$r_{01}/R=0.125$				$r_{02}/R=0.10$			
	0.8	1.0	1.3	2.5	0.8	1.0	1.3	2.5
0	4.930	4.556	4.118	3.441	4.092	3.806	3.479	2.973
	1.390	1.559	1.303	0.836	1.653	1.414	1.154	0.695
$\frac{\pi}{6}$	3.634	3.666	3.681	3.709	3.084	3.107	3.175	3.242
$\frac{\pi}{4}$	1.061	1.032	0.998	0.890	0.897	0.935	0.870	0.710
$\frac{\pi}{3}$	3.264	3.412	3.531	3.755	2.762	2.896	3.010	3.263
$\frac{\pi}{2}$	0.786	0.821	0.881	0.983	0.723	0.741	0.754	0.743
$\frac{2\pi}{3}$	3.444	3.601	3.776	4.176	2.924	3.086	3.246	3.637
$\frac{3\pi}{4}$	0.926	0.977	1.049	1.210	0.778	0.805	0.824	0.850
$\frac{5\pi}{6}$	4.560	4.661	4.760	4.934	3.857	3.947	4.019	4.153
π	1.330	1.351	1.397	1.524	0.804	0.774	0.749	0.700
$\frac{7\pi}{6}$	3.331	3.486	3.674	4.156	2.531	2.666	2.816	3.204
$\frac{5\pi}{4}$	0.680	0.700	0.735	0.836	0.067	0.035	0.009	0.001
$\frac{3\pi}{2}$	2.791	2.951	3.167	3.672	2.079	2.227	2.442	2.942
$\frac{7\pi}{4}$	0.466	0.487	0.510	0.547	0.035	0.028	0.029	0.032
$\frac{5\pi}{3}$	3.244	3.380	3.545	3.838	2.279	2.958	3.172	3.553
$\frac{11\pi}{6}$	0.694	0.715	0.727	0.687	0.393	0.425	0.458	0.486
$\frac{3\pi}{2}$	6.313	5.990	5.616	4.986	5.985	5.722	5.534	5.206
$\frac{7\pi}{4}$	2.260	2.111	1.919	1.550	2.005	1.878	1.733	1.444

The coefficient k_{θ}^M on the contour of the first hole r_{01} at the points $\theta = \pi/2$ and $\theta = \pi$ decreases, and at the point $\theta = 0$ it increases. At the points $\theta = 0, \pi/2, \pi$ on the contour of the second hole r_{02} with an increase in the radius of the holes, the coefficient k_{θ}^T increases by 200%, 170%, 105%, respectively, and the coefficient k_{θ}^M by 60%, 300%, and 100%.

Fig. 4-5 shows the distribution of stress concentration k_{θ}^T and k_{θ}^M at the points of the contour of the holes, depending on the effect of shear stiffness G_{13}/E_1 . In this case, the radius of one hole was chosen $r_{01}/R = 0.10$, and the other $r_{02}/R = 0.15$.

Fig. 4. Stress concentration k_{θ}^T at the points of the contour of the holes.

With a change in the shear stiffness G_{13}/E_1 from 1.0 to 0.01 at the points $\theta = 0, \pi/2, \pi$ on the contour r_{01} , the coefficient k_{θ}^T increases by 7%, 40%, 3%, respectively, and the coefficient k_{θ}^M - by 8%, 4%, 30%. At points $\theta = 0, \pi/2, \pi$ on the contour r_{02} with a change in G_{13}/E_1 , the coefficient k_{θ}^T increases, respectively, by 6%, 10%, 4%, and the coefficient k_{θ}^M by 16%, 31%, 10%.

Fig. 5. Stress concentration k_{θ}^M at the points of the contour of the holes.

Table 4 shows the results of a study of the concentration of annular forces K_{θ}^T and annular moments K_{θ}^M depending on the shell thickness h/R . The shell parameters were chosen as in the case of Table 1. It can be seen that with decreasing shell thickness, the coefficients k_{θ}^T and k_{θ}^M on the contours of the holes r_{01} and r_{02} increase. At the points $\theta = 0, \pi/2, \pi$ on the contour of the larger hole r_{01} with a decrease in thickness from 0.003 to 0.01, the coefficient k_{θ}^T increases by 27%, 31%, 11%, respectively, and the coefficient k_{θ}^M - by 32%, 24 %, 3%. By decreasing the shell thickness at the indicated points of the contour of the small hole r_{02} , the coefficient k_{θ}^T increases by 10%, 32%, 7%, and the coefficient k_{θ}^M - by 50%, 42%, 7%.

Table 4. Concentration of annular forces k_{θ}^T and annular moments k_{θ}^M .

θ	h/R					
	$r_{01}/R = 0.125$			$r_{02}/R = 0.10$		
	0.01	0.02	0.03	0.01	0.02	0.03
0	4.930	4.153	3841	4.092	3.767	3690
	1.790	1.519	1.367	1.653	1.102	1.081
$\frac{\pi}{6}$	3.634	3.059	2.818	3.084	2.827	2.738
	1.061	0.920	0.835	0.987	0.765	0.667
$\frac{\pi}{4}$	3.264	2.717	2.491	2.762	2.472	2.355
	0.786	0.694	0.636	0.723	0.532	0.463
$\frac{\pi}{3}$	3.444	2.861	2.617	2.924	2.579	2.424
	0.926	0.783	0.705	0.778	0.570	0.500
$\frac{\pi}{2}$	4.560	3.792	3.424	3.857	3.198	2.885
	1.330	1.162	1.067	0.804	0.623	0.561
$\frac{2\pi}{3}$	3.331	2.676	2.378	2.531	2.003	1.770
	0.680	0.650	0.624	0.067	0.138	0.173
$\frac{3\pi}{4}$	2.791	2.286	2.071	2.079	1.788	1.664
	0.466	0.498	0.503	0.035	0.154	0.210
$\frac{5\pi}{6}$	3.244	2.861	2.676	2.279	2.702	2.620
	0.694	0.747	0.773	0.393	0.576	0.655
π	6.313	5.874	5.659	5.985	5.698	5.608
	2.260	2.239	2.190	2.075	2.160	2.010

For the same shell, Table 5 shows the nature of changes in the coefficients k_{θ}^T and k_{θ}^M from the shell radius. It is seen that with an increase in the shell radius R/r_{01} , the stress concentration factor decreases along both contours of the holes. With an increase in the shell radius R/r_{01} from 5 to 20 at the points $\theta = 0, \pi/2, \pi$ on the contour of the larger hole r_{01} , the coefficient k_{θ}^T decreases by 16%, 36%, 84%, respectively, and the coefficient k_{θ}^M - by 27%, 26%, 14%. With an increase in the radius of the shell at the indicated points on the contour of a small hole r_{02} , the coefficient k_{θ}^T decreases by 26%, 36%, 10%, and the coefficient k_{θ}^M decreases by 47%, 43%, 7%.

Table 5. Nature of changes in the coefficients k_{θ}^T and k_{θ}^M .

θ	R/r_{01}							
	$r_{01}/R=0.1$				$r_{02}/R=0.075$			
	0.2	0.1	0.06	0.05	0.2	0.1	0.06	0.05
0	6.290	4.930	4.419	4.125	5.034	4.092	3.852	3.749
	1.950	1.790	1.595	1.446	1.996	1.653	1.360	1.064
$\frac{\pi}{6}$	4.836	3.634	3.262	3.029	3.915	3.084	2.908	2.805
	0.996	1.061	0.991	0.939	1.061	0.987	0.856	0.772
$\frac{\pi}{4}$	4.416	3.264	2.901	2.683	3.558	2.762	2.565	2.445
	0.636	0.786	0.754	0.728	0.766	0.723	0.624	0.557
$\frac{\pi}{3}$	4.551	3.444	3.051	2.822	3.721	2.924	2.693	2.545
	0.926	0.926	0.867	0.819	0.941	0.778	0.665	0.587
$\frac{\pi}{2}$	5.792	4.560	4.048	3.730	4.918	3.857	3.421	4.142
	1.493	1.331	2.202	1.123	1.065	0.804	0.667	0.616
$\frac{2\pi}{3}$	4.498	3.331	2.887	2.618	3.530	2.531	2.135	1.958
	0.739	0.680	0.665	0.664	0.060	0.067	0.127	0.182
$\frac{3\pi}{4}$	3.783	2.791	2.438	2.240	2.671	2.079	1.8712	1.758
	0.428	0.466	0.500	0.527	0.092	0.035	0.132	0.201
$\frac{5\pi}{6}$	4.044	3.244	2.976	2.817	2.997	2.279	2.728	2.673
	0.579	0.694	0.747	0.781	0.198	0.393	0.5666	0.576
π	6.894	6.313	6.034	5.811	6.390	5.985	5.861	5.768
	2.479	2.260	2.181	2.144	2.136	2.005	1.990	2.001

4 Conclusions

In this paper, we studied the stress distribution around two unequal circular holes in an orthotropic spherical shell made of a composite material. Using the refined theory of Timoshenko-type shells, we considered the effect of transverse shear deformations for the entire package of the shell as a whole. In this case, the composite material is considered as homogeneous with the given characteristics.

Thus, the developed technique can be used to calculate the elements of shell structures made of composite materials with several holes.

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