Wave-induced residual pore pressure around a buried pipeline

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Abstract. Seabed instability caused by cyclic wave loading is one of the main factors affecting the foundation instability of marine structures, and it is a key problem that needs to be paid attention in the design of marine structures. Based on Biot's consolidation theory and a semi-empirical formula for calculating residual pore water pressure, the wave-induced residual seabed response around a buried pipeline was investigated by numerical simulation. The correctness of the numerical results is verified by comparing with experimental results. Effect of the self-weight of the buried pipeline on residual pore pressure development and the characteristics of residual pore pressure near the pipeline are discussed.

1 Introduction

For the free seabed, the stress field, pore pressure field and soil displacements under wave action has been studied extensively. The problem becomes more complicated when subsea pipelines are presented, and the interaction between waves, soil, and pipelines needs to be studied as an integrated system.

Based on Biot's consolidation theory, the interaction process between porous media and pore fluid can be described reasonably. Cheng and Liu¹ studied the transient pore pressure response of the soil around the buried pipeline and obtained the distribution of transient pore pressure amplitude along the pipeline. Gao and Wu² assumed that the wave was nonlinear and studied the pore pressure response in the area around a buried pipeline under a single seabed. Jeng et al.³ regarded the pipeline as an elastomer and analysed the change of internal stress during loading.

All the above studies assumed that the seabed was elastic, and only the transient pore pressure response was analysed. The seabed soil particles rearrange under the action of cyclic load, showing a decreasing trend in volume. If the increasing rate of pore water pressure during each cyclic loading exceeds the dissipating rate, the residual pore pressure will increase. Dunn et al.⁴ analysed the development of residual pore water pressure under fixed and free pipelines. The pore water pressure below the pipe developed fastest. Zhao et al.⁵ considered the contact effect between seabed soil and pipeline and integrated the model into the finite-difference analysis program with explicit time matching. Based on establishing a relationship between soil deformation and force, the development of pore pressure under linear and nonlinear waves was analysed respectively. Zhang et al.⁶ proposed a three-dimensional elasto-plastic model to analyse the influence of soil elasto-plastic on the pore

pressure response of seabed with a pipe. Cheng et al.⁷ conducted a numerical study on the pipe-soil system. The model could capture the cyclic migration behaviour of soil under loading, and the stability of the pipeline was analysed through the degradation factor model.

Gravity of the pipeline could have influence on soil residual pore pressure under waves. In this paper, based on Biot's consolidation theory and a semi-empirical formula of residual pore water pressure, using numerical simulation, the influence of pipeline weight on the initial mean effective stress in the seabed is considered for computing the residual pore water pressure, and the characteristics of residual seabed response around a buried pipeline are analysed.

2 Theory background

As shown in Figure 1, waves propagate along the x-axis on the seabed surface perpendicular to the pipe. In order to analyse the pipe-seabed interaction, it is assumed that the soil deformation is small. The seabed soil is isotropic and homogeneous. The soil permeability is constant.

Fig. 1 Schematic of the calculation model

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2.1 Wave model

Waves in marine environment are usually somewhat nonlinear. In this paper, the wave is a nonlinear progressive wave, and the wave pressure on seabed surface is obtained according to the formula proposed by Jeng and Liu⁸.

2.2 Seabed model

2.2.1 Biot's consolidation equations

Biot's consolidation equation describes the relationship between compression and seepage of soil skeleton, which is widely used to solve the problem of seabed response under wave action. Based on Biot's consolidation equation, the seabed stress balance equation and the seepage continuity equation are obtained.

The stress balance equations are:

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma'_z}{\partial z} = \frac{\partial p}{\partial z} \quad (2)$$

Where (σ'_x, σ'_z) represent the effective normal stress of soil in horizontal and vertical directions respectively. τ_{xz} is the shear stress of soil. p is the pore water pressure.

According to the linear geometric equation and the elastic stress-strain relationship, the relationship between stress and soil displacement can be obtained, which are respectively expressed as follows:

$$\sigma'_x = 2G \left[\frac{\partial u}{\partial x} + \frac{\mu}{1-2\mu} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \right] \quad (3)$$

$$\sigma'_z = 2G \left[\frac{\partial w}{\partial z} + \frac{\mu}{1-2\mu} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \right] \quad (4)$$

$$\tau_{xz} = G \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \quad (5)$$

Where (u, w) represents the horizontal and vertical displacements of soil respectively; μ is Poisson's ratio; G is the shear modulus of seabed soil.

The seepage equation can be expressed as:

$$\frac{K}{\gamma_w} \left(\frac{\partial^2 p_{osc}}{\partial x^2} + \frac{\partial^2 p_{osc}}{\partial z^2} \right) - n' \beta_s \frac{\partial p_{osc}}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \quad (6)$$

Where K represents the permeability coefficient of seabed soil, γ_w is the bulk density of seawater, n' is the porosity of seabed soil, p_{osc} is the transient part of the pore water pressure, and β_s is used to represent the compression of fluid in pores, which can be expressed as.

$$\beta_s = \frac{1}{K_f} + \frac{1-S_r}{P_{ow}} \quad (7)$$

Where, K_f is the actual volume modulus of water, 2×10^9 N/m², saturation $S_r = 1$, and P_{ow} is the absolute water pressure.

2.2.2 Governing equations of residual pore pressure

Based on Biot's consolidation equation and measured pore water pressure accumulation data, McDougal et al.^[9]

proposed the following calculation equation for residual pore pressure. The two-dimensional form is as follows:

$$\frac{\partial p_{res}}{\partial t} = c_v \left(\frac{\partial^2 p_{res}}{\partial x^2} + \frac{\partial^2 p_{res}}{\partial z^2} \right) + f(x, z) \quad (8)$$

Where p_{res} represents the residual pore pressure. c_v is the consolidation coefficient. The source term of pore water pressure growth can be expressed as:

$$f(x, z) = \frac{\sigma'_0}{T} \left(\frac{|\tau_{max}|}{\alpha \sigma'_0} \right)^{-1/\beta} \quad (9)$$

α and β are dimensionless curve fitting coefficients related to the type and relative density of seabed soil. τ_{max} is the maximum cyclic shear stress in the seabed. σ'_0 is the initial average effective pressure.

2.3 Governing equations for pipeline

According to the horizontal and vertical stress state of the pipeline, the stress balance equation is established.

$$\frac{\partial \sigma'_{px}}{\partial x} + \frac{\partial \tau_{pxz}}{\partial z} = 0 \quad (10)$$

$$\frac{\partial \tau_{pxz}}{\partial x} + \frac{\partial \sigma'_{pz}}{\partial z} = 0 \quad (11)$$

$(\sigma'_{px}, \sigma'_{pz})$ represent the effective normal stress in horizontal and vertical directions respectively; τ_{pxz} is the shear stress of pipeline; Based on the linear geometric equation and the stress-strain relationship, the relationship between stress and pipe displacement is obtained, which is respectively expressed as follows:

$$\sigma'_{px} = 2G_p \left[\frac{\partial w_p}{\partial z} + \frac{\mu_p}{1-2\mu_p} \left(\frac{\partial u_p}{\partial x} + \frac{\partial w_p}{\partial z} \right) \right] \quad (12)$$

$$\sigma'_{pz} = 2G_p \left[\frac{\partial w_p}{\partial z} + \frac{\mu_p}{1-2\mu_p} \left(\frac{\partial u_p}{\partial x} + \frac{\partial w_p}{\partial z} \right) \right] \quad (13)$$

$$\tau_{pxz} = G_p \left(\frac{\partial u_p}{\partial z} + \frac{\partial w_p}{\partial x} \right) \quad (14)$$

Where (u_p, w_p) represent horizontal and vertical displacements of the pipeline respectively; μ_p is the pipeline Poisson's ratio; G_p is the shear modulus of the pipe.

2.4 Boundary conditions

At the upper surface of the seabed, the pore water pressure is equal to the wave pressure of the seabed surface, and the shear stress and the vertical effective normal stress are zero. Periodic boundaries are used on both sides of the seabed. There is no seepage at the bottom of the seabed and the outer wall of the pipeline, the pore pressure gradient is zero.

3 Validation

Before analyzing the response characteristics of the seabed containing buried pipelines, the numerical results are compared with the experimental results.

Turcotte¹⁰ studied the oscillatory pore pressure response around buried pipelines through flume

experiments. Cheng and Liu¹ used the same parameters as the test to calculate the amplitude of transient pore pressure around the pipeline through the numerical model. The numerical results of transient seabed response in this paper are compared with the experimental results and the existing numerical research results, as shown in Figure 2.

As can be seen from the Figure 2, the results calculated by the numerical model used in this paper are consistent with experimental results and those obtained by Cheng and Liu¹, which ensures the correctness of the calculation.

Fig. 2 Verification of numerical results

4 Results and discussion

The calculation parameters are shown in Table 1.

Table 1 Parameters used in numerical simulation

Parameters	values
Wave	
Wave height(H)	3.8 (m)
Water depth(d)	15 (m)
Wave period(T)	10 (s)
Seabed	
Poisson's ratio(μ)	0.33
Shear modulus(G)	8×10^6 (Pa)
Soil permeability(K)	5×10^{-5} (m/s)
Soil porosity(n)	0.42
Relative density(D_r)	0.3
Thickness(h)	40 (m)
Degree of saturation(S_r)	1.0
Pipe	
Pipe diameter(D)	1.2 (m)
Buried depth(e)	2.0 (m)
Poisson's ratio(μ_p)	0.32
Unit weight(γ'_p)	23000 (N/m ³)

4.1 Seabed consolidation under pipeline gravity

The physical properties of pipelines are obviously different from those of the sea bed, and the dead weight of pipelines will have a significant effect on the stress distribution of the sea floor, especially in the area near the pipelines. The distribution of initial effective stress in the seabed will affect the calculation of residual pore pressure, so it is very important to determine the distribution of initial effective stress at each location. For free seabed, the initial effective stress is same at the same depth for

different horizontal positions. For the seabed with embedded pipes, the distribution of gravity-induced initial effective stress is obviously affected by pipes. As shown in Figure 3, the gravity-induced mean effective stress in seabed increases at the bottom of the pipe. At the top of the pipe, the effective stress is obviously less than that at other horizontal locations of the same depth.

Fig. 3 Distribution of the initial mean effective stress (Unit: Pa)

4.2 Effects of the pipeline self-gravity

Due to the obvious difference between the density of pipeline and seabed soil, the distribution of initial effective stress will be significantly affected after the pipeline is buried in the seabed. The initial effective stress will affect the source term of pore pressure accumulation, and then affect the development of pore water pressure. Figure 4 shows the development of pore water pressure at the top and bottom of the pipeline.

(a) At the top of the pipe

(b) At the bottom of the pipe

Fig. 4 Development of pore water pressure

From Figure 4, it can be seen that the development rate of residual pore pressure increases when considering the self-weight of the buried pipeline. The dead weight of the pipe has a greater effect on the development of residual pore pressure at the bottom than at the top of the pipe. For the bottom of the pipe, from 50-200s, the pore pressure develops significantly faster when considering the dead weight, and then the difference decreases. At 400s, the residual pore pressure at the bottom of the pipe without considering the dead weight of the pipeline is about 23.6kPa, while the residual pore pressure at the bottom of the pipe with the dead weight of the pipeline is about

26.4kPa, and the residual pore pressure increases by about 12% when considering the dead weight of the pipeline.

4.3 Residual pore pressure around the pipeline

Figure 5 shows the distribution of residual pore water pressure along the pipeline with different periods. The residual pore water pressure develops fastest at the bottom of the pipeline, but slowly at the top of the pipeline. The overall accumulated pore water pressure in the lower half of the pipeline develops faster than that in the upper half of the pipeline.

Fig. 5 Wave-induced residual pore pressure around the pipeline

5 Conclusions

Based on Biot's consolidation theory and semi-empirical formula for calculating residual pore water pressure, the seabed residual response near the buried pipeline under wave action is investigated using numerical simulation. By comparing with the experimental results, the correctness of the simulation is ensured.

After the pipeline is buried in the seabed, the source term of accumulated pore pressure is affected by changing the distribution of initial effective stress of the seabed, and the developing process of accumulated pore pressure is affected. The residual pore water pressure develops faster with considering the self-weight of the pipeline. The residual pore pressure in the lower half of the pipe develops faster than that in the upper half.

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