The Invariance Determination of the Complex Electrical System Output of

*Tokhir Makhmudov**

Tashkent State Technical University, Tashkent, Uzbekistan

Abstract. This article discusses the issues of applying the technology of embedding systems to study the invariance of the output of complex controlled electrical systems under small disturbances as stationary deterministic multidimensional dynamic systems. The technique of controller synthesis based on the modern matrix theory is presented. The synthesis of the regulator of the model of a multi-machine electrical system is obtained, which makes it possible to analyze the influence of the parameters of the electrical system mode.

1 Introduction

One of the most important properties of a dynamical system is its invariance. As noted in [1], "the problem of invariance is the problem of determining such structures and parameters of control systems, under which the influence of arbitrarily changing external disturbances and eigen parameters of systems on the dynamic characteristics of control processes can be partially or completely compensated." The automatic control systems created in this way have very high levels of accuracy and quality and are also less susceptible to the influence of various kinds of noise or interference.

This problem is quite extensive and was first posed by Professor G.V. Shchipanov, and to this day, extensive discussions of its application continue [2-4].

It should be noted that there are various types of invariant systems, differing both in design principles and in functional capabilities.

This is because the input-output sets of complex electric power systems (EPS) are subject to ambiguous transformations associated with the presence of zero divisors and non-commutativity of operators, i.e., algebraic features of the system under study, characteristic only for multidimensional systems.

The invariance problem is posed as follows. In the case of representing the system under study in the state space [5]:

$$\begin{aligned} u &= -Kx \\ \dot{x} &= Ax + Bu + Sw \quad y = Cx, \end{aligned} \tag{1}$$

*Corresponding author: tox-05@yandex.com

where x , u , y , w are the vectors of state, control, output, and disturbance of the system, respectively; A , B , C , S are matrices with constant numerical elements of the corresponding sizes; K is a matrix with constant numerical elements, called a regulator, for the invariance of the output of the controlled system under study, the transfer matrix from the disturbance $w(p)$ to the output of the system $y(p)$ with a model in the state space must be identically equal to zero:

$$F_y^w(p) = C(pI_n - A_y)^{-1}S = 0 \quad (2)$$

where $A_y = A + BK$ is the dynamics matrix of the system with the controller. The main task is to find a regulator (synthesis) that ensures the fulfillment of condition (2). However, as noted in [3], certain difficulties arise in solving this problem, since in (2) there is an operation of matrix inversion and, as a rule, polynomial.

2 Methods

Necessary and sufficient conditions under which identity (2) is valid are provided if the requirements of the theorem are satisfied [6], which asserts that system (1) for given matrices A , B , C , and S possesses invariance to perturbations in the sense that identity (2) holds if and only if the condition is satisfied:

$$\overline{\overline{R}}^L \pi S = 0 \quad (3)$$

where π is the matrix of the maximum column rank corresponding to the condition:

$$\overline{\overline{R}}^L \pi A_y \overline{C}^R \pi = 0 \quad (4)$$

in which the identity is satisfied:

$$\overline{\overline{R}}^L \pi B \overline{\overline{R}}^L \pi A \overline{C}^R \pi = 0 \quad (5)$$

and the system is closed by any regulator from the set:

$$\{K\}_{\gamma, \chi} = - \left(\overline{\overline{R}}^L \pi B \right)^{\sim} \overline{\overline{R}}^L \pi A \overline{C}^R \pi \left(\overline{\overline{R}}^L \pi \right)^{\sim} + \overline{\overline{R}}^L \pi B \chi + \gamma \overline{\overline{R}}^L \pi \quad (6)$$

where χ and γ are matrices of given sizes with arbitrary numerical elements, is the right divisor of the zero of the matrix C , $\overline{\overline{R}}^L \pi$ is left zero divisor $\overline{C}^R \pi$, matrices with upper notation ($\sim$) are the summary canonizers of the corresponding matrices, double and triple dashes above matrices denote the repeated determination of the corresponding divisor of zero of the maximum rank from the combination of matrices under this bar.

The algorithm allowing to form a matrix p of the maximum rank satisfying condition (4) in a finite number of steps is as follows [7].

1. The condition is checked:

$$\overline{CB}^L \overline{CAC}^R = 0. \quad (7)$$

If this condition is satisfied, then $\pi = \pi_0 = I_{(n-\text{rank}C)}$ is taken.

2. If condition (7) is not satisfied, then the matrix p_1 is determined by the formula:

$$\overline{CB}^L \overline{CAC}^R = 0. \quad (8)$$

If $\pi_1 = 0$, then the system has no invariance, the algorithm stops. Otherwise, condition (4) is checked for $\pi = \pi_1$.

3. The matrix π_i for $i > 1$ is determined by the formula:

$$\pi_i = \overline{C}^R \pi_{i-1} B \overline{C}^R \pi_{i-1} A \overline{C}^R \quad (9)$$

and the fulfillment of condition (5) is checked.

4. The algorithm stops at the k -th step when condition (7) is first satisfied. The maximum rank matrix π has the value π_k .

Therefore, checking the output invariance of the system (1) is reduced to the given iterative process of determining π , and the calculation of the transfer matrix $F_y^w(p)$ is not required.

3 Results and Discussion

Let us apply the above method for determining the invariance of a dynamic system in terms of output using the example of a model of a controlled electrical system (Figure 1), the parameters of which are given in [8], excluding the damper coefficients of the generators. The matrix canonization method is used to solve the invariance problem.

Fig. 1. Three-generator electrical system diagram.

The matrices of the own dynamics of the model studied by the EPS have the form [9]:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -62.3 & 18.54 & 42.12 & 0 & 0 & 0 \\ 45.31 & -86.66 & 68.9 & 0 & 0 & 0 \\ 17.3 & 58.41 & -88.7 & 0 & 0 & 0 \end{bmatrix}, \quad (10)$$

$$C = [\Delta\delta_1 \ 0 \ 0 \ 0 \ 0 \ 0] = [1 \ 0 \ 0 \ 0 \ 0 \ 0], \quad S = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2.0837 & 0 & 0 \\ 0 & -2.0678 & 0 \\ 0 & 0 & 0.97 \end{bmatrix}. \quad (12)$$

The calculation shows that with the accepted parameters of the mode (basic version), the system is unstable, which can be seen from the spectrum of the matrix (10) of the intrinsic dynamics of the studied EPS: $0 \pm 12.136i$, $0 \pm 9.6697i$, $\pm 1.7685i$.

To check the required conditions for the invariance of the EPS output and as a final result of determining the parameters of the controller (6), we will find the corresponding matrices successively.

Condition (7) requires the definition of the right divisor of the matrix C and the left divisor of the matrix CB, which we obtain as a result of the canonization of these matrices:

$$\begin{aligned} \overset{-R}{C} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad CB = [0 \ 0 \ 0], \quad \overset{-L}{CB} = \overset{-L}{[0 \ 0 \ 0]} \overset{-L}{=} 1 \\ \overset{-R}{CAC} &= [0 \ 0 \ 1 \ 0 \ 0], \quad \overset{-L}{CB} \overset{-R}{CAC} = [0 \ 0 \ 1 \ 0 \ 0] \neq 0. \end{aligned}$$

Condition (7) is not satisfied; therefore, using formula (8), we determine the matrix π_1 :

$$\pi_1 = \overline{\overline{L}}^R \overline{CAC}^R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

We check the fulfillment condition (5) with the equality $\pi = \pi_1$:

$$\begin{aligned} \overline{\overline{C}}^R \pi_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \overline{\overline{C}}^R \pi_1^L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ \overline{\overline{C}}^R \pi_1^L B &= \begin{bmatrix} 0 & 0 & 0 \\ -3.54 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{C}}^R \pi_1^L B^L = [1 \ 0], \\ \overline{\overline{C}}^R \pi_1^L A \overline{\overline{C}}^R \pi_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 42.12 & 18.54 & 0 & 0 \end{bmatrix}, \quad \overline{\overline{C}}^R \pi_1^L B^L \overline{\overline{C}}^R \pi_1^L A \overline{\overline{C}}^R \pi_1 = [0 \ 0 \ 0 \ 0]. \end{aligned}$$

Thus, condition (5) is satisfied. Next, we check the fulfillment of condition (3).

$$\overline{\overline{C}}^R \pi_1^L S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Therefore, since condition (3) is satisfied, it is possible to form the matrix of the coefficients of the controller (6), for which it is necessary to determine the matrices included in this formula:

$$\left(\overline{\overline{C}}^R \pi_1^L B \right)^{\sim} = \begin{bmatrix} 0 & -0.2825 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \left(\overline{\overline{C}}^R \pi_1 \right)^{\sim} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \overline{\overline{C}}^R \pi_1^L B^R = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Substituting the obtained numerical values of the matrices into (6), we obtain:

$$\begin{aligned} \{K\}_{\gamma, \chi} &= \left(\overline{C^R}^L \pi B \right)^{\sim} \overline{C^R}^L \pi A \overline{C^R}^L \pi \left(\overline{C^R}^L \pi \right)^{\sim} + \overline{C^R}^L \pi B \chi + \gamma \overline{C^R}^L \pi = \\ &= \begin{bmatrix} \gamma_{11} & -5.23 & -11.89 & \gamma_{12} & 0 & 0 \\ (\gamma_{21} + \chi_1) & \chi_2 & \chi_3 & (\gamma_{22} + \chi_4) & \chi_5 & \chi_6 \\ \gamma_{31} & 0 & 0 & \gamma_{32} & 0 & 0 \end{bmatrix}, \end{aligned} \quad (13)$$

where the forming matrices χ and γ , with arbitrary numerical values, are taken in the form:

$$\chi = [\chi_1 \quad \chi_2 \quad \chi_3 \quad \chi_4 \quad \chi_5 \quad \chi_6] \text{ and } \gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{bmatrix}.$$

In [5, 9], a regulator was justified, which for a three-generator electrical system has the form:

$$K = \begin{bmatrix} k_{E_{q1}}^{\Delta\delta_1} & 0 & 0 & k_{E_{q1}}^{\Delta s_1} & 0 & 0 \\ 0 & k_{E_{q2}}^{\Delta\delta_2} & 0 & 0 & k_{E_{q2}}^{\Delta s_2} & 0 \\ 0 & 0 & k_{E_{q3}}^{\Delta\delta_3} & 0 & 0 & k_{E_{q3}}^{\Delta s_3} \end{bmatrix}, \quad (14)$$

with coefficients for the parameters of the electrical system mode.

The synthesized controller (14) with matrices c and g with arbitrary numerical values of the elements must be designed so that the necessary technical requirements are provided in the dynamic system: stability, damping of low-frequency oscillations [10-15].

Note that (13) must be consistent with (14), namely, we can assume $k_{E_{q1}}^{\Delta\delta_1} = \gamma_{11}$, $k_{E_{q1}}^{\Delta s_1} = \gamma_{12}$, $k_{E_{q2}}^{\Delta\delta_2} = \chi_2$, $k_{E_{q2}}^{\Delta s_2} = \chi_5$, and the remaining elements are equal to zero, and finally, the matrix of the coefficients of the controller (13) will have the form:

$$\{K\}_{\gamma, \chi} = \begin{bmatrix} \gamma_{11} & 0 & 0 & \gamma_{12} & 0 & 0 \\ 0 & \chi_2 & 0 & 0 & \chi_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

It is characteristic that for the selected output matrix of the system under study C and the matrix of disturbances S , the third controller does not participate in regulating the mode of a complex electrical system.

Now let us check the influence of the regulator (15) on the spectrum of the intrinsic dynamics of a controlled electrical system with a matrix

$$A_y = A + BK. \quad (16)$$

Let us choose $k_{E_{q1}}^{\Delta\delta_1} = \gamma_{11} = 15$, $k_{E_{q1}}^{\Delta s_1} = \gamma_{12} = 2$, $k_{E_{q2}}^{\Delta\delta_2} = \chi_2 = 4$, $k_{E_{q2}}^{\Delta s_2} = \chi_5 = 4$, for which matrix (16) is equal to:

$$A_y = A + BK = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -115.4 & 37.08 & 84.24 & -7.08 & 0 & 0 \\ 45.31 & -112.26 & 68.9 & 0 & -3.2 & 0 \\ 17.3 & 58.41 & -88.7 & 0 & 0 & 0 \end{bmatrix},$$

with spectrum: $-3.1054 \pm 11.759i$; $-0.6453 \pm 12.5429i$; $-1.38 \pm 1.6i$. With the selected parameters of the controller (15), the electrical system became stable (Figure 2).

Fig.2. 3D visualization of the roots of the electrical system under study with a spectrum: $-1.0790 \pm 4.2951i$; $-0.6007 \pm 12.2361i$; $-1.4379 \pm 10.3156i$.

Obviously, in the presence of A_y , it is possible to comprehensively investigate the dynamic properties of a complex controlled electrical system by varying the parameters of the controller (15) and, among other things, to determine the conditions for the invariance of the output of the system under study to disturbances arising in the system under study. The horizontal axes in Figure 2 correspond to the axes of the complex plane. The logarithm of the norm of the resolvent function is plotted along the vertical axis. The peaks localize the eigenvalues of the matrix.

Figure 3 shows the characteristics of the change in the deviation of the angle of the first generator $\Delta\delta_1=f(t)$, a stable, controlled (16) electrical system (Figure 3, A), with the synthesized parameters of the controller (15), and an unstable, unregulated EPS (10), (Figure 3, B).

Fig. 3. Characteristics of changing the angle of the first generator $\Delta\delta_1 = f(t)$ of a three-generator electrical system.

4 Conclusions

The given method of synthesis of an electric system controller is quite simple since it is based on the modern theory of matrices adapted for computer use. Therefore it is computationally efficient and can be recommended to study complex, automatically controlled complex EPS.

Thus, it can be noted that based on the matrix canonization method, which is the basis of the system embedding technology, the conditions for the invariance of the output of the electrical system to arbitrary external disturbances are substantiated. To solve this problem, a corresponding regulator has been synthesized.

The main difference in applying the technology of nesting systems is the reduction in the number of computational costs since this technology is based on matrix analysis, for which rich software systems have been developed. In addition, an analytical description of the class of controllers that provide the required dynamic properties of the systems under study, including invariance, is also very important.

References

1. Misrihanov M Sh, Klassicheskie i novye metody analiza mnogomernykh dinamicheskikh system [Classical and new methods of analysis of multidimensional dynamic systems]. Energoatom Publ. Moscow (in Russian) p 566. (2004)
2. Abdellatif B M, Stability with respect to part of the variables of nonlinear Caputo fractional differential equations, Mathematical Communications. **23** pp 119–126. (2018).
3. Klos A, Mathematical Models of Electrical Network Systems: Theory and Applications - An Introduction. Springer International Publishing AG p 108. (2017).<https://doi.org/10.1007/978-3-319-52178-7>
4. Kothari D P, Nagrath I J, Modern Power System Analysis. McGraw Hill Education p 694.(2003)
5. Allaev K R, Mirzabaev A M, Matrichnye metody analiza mal'kh kolebaniy elektricheskikh system [Matrix methods for the analysis of small oscillations of electrical systems], Fan va texnologiya Publ., Tashkent (in Russian) p 431. (2016).

6. Gotman V I, Common algorithm of static stability estimation and computation of steady states of power systems *Power engineering* **311** (4) pp 127–130. (2007), http://www.lib.tpu.ru/fulltext/v/Bulletin_TPU/2007/v311eng/i4/30.pdf
7. Allaev K, Makhmudov T, Research of small oscillations of electrical power systems using the technology of embedding systems *Electrical Engineering*. **102** (1) pp 309-319. (2020), <https://doi.org/10.1007/s00202-019-00876-9>
8. Kovalenko S, Sauhats A, Zicmane I, Utans A, New Methods and Approaches for Monitoring and Control of Complex Electrical Power Systems Stability IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC 2016), Italy, Florence, 7-10 June 2016 pp 270-275. (2016), <https://doi.org/10.1109/EEEIC.2016.7555882>
9. Allaev K, Makhmudov T, Analysis of small oscillations of complex electrical systems, E3S Web Conf., Rudenko International Conference "Methodological problems in reliability study of large energy systems" (RSES 2020) . **216** pp 1-4. (2020), <https://doi.org/10.1051/e3sconf/202021601097>
10. Irwanto M et al. Improvement of Dynamic Electrical Power System Stability Using Riccati Matrix Method Applied Mechanics and Materials, **793** pp 29–33. (2015). <https://doi.org/10.4028/www.scientific.net/AMM.793.29>
11. Anderson P M, Fouad A A, Power system control and stability, Second edition, Willey-Interscience A John Wiley and Sons Inc, USA p 672. (2003)
12. Allaev K, Makhmudov T 2019 Prospects of diversification and ensuring energy safety of Uzbekistan E3S Web Conf., Rudenko International Conference "Methodological problems in reliability study of large energy systems" (RSES 2019). **139** pp 1-5. (2019).
13. Albertos A P, Sala A, Multivariable control systems, Springer, London p 340. (2007), [dx.doi.org/10.1007/b97506](https://doi.org/10.1007/b97506)
14. Kunder P, Power system stability and control, McGraw-Hill, Inc., USA; p 1196. (1994).
15. Mirzabaev A, Makhkamov T, The investigation of invariance of the output of complex electric power system with application system's embedding approach *European science review* **7-8** pp 211-214. (2016).