Engineering networks simulation and assessment of the mathematical model accuracy

Umarkhodzha Baxramov*, *Uchqun* Umarov, and *Axror* Obidzhonov

Tashkent State Transport University, Tashkent, Uzbekistan

Abstract. The article shows the way to implement a quasilinear mathematical model of flow distribution in pipeline engineering networks that is effective in a wide range of changes in the multidimensional random vector of loads at network nodes and provides reliable determination of the parameters of the probability distribution functions of flows in active and passive network elements. The proposed model consists of determining the matrix of generalized network parameters-the load distribution coefficients along the branches of the circuit, calculated at the point corresponding to the mathematical expectation of the node loads. Based on the obtained model, the convergence of the results obtained with the results of simulation of engineering networks is proved using a numerical experiment on an electronic computer. The effectiveness of the developed model, the corresponding algorithms and a set of programs for an electronic computer is shown - the value of the criterion of reduced costs for parametric optimization of engineering networks can be reduced by 5-7% compared to the methods used in practice. The possibility of obtaining at the design stage the equivalent hydraulic characteristics of engineering networks in the form corresponding to the data of experimental measurements of pressures at the nodes of real complex engineering networks is proved.

1 Introduction

The main task of mathematical modelling of engineering networks in this work is to assess the reliability of the mathematical model of stochastic flow distribution proposed in [1, 2]. For simulation, three calculation schemes of the utility network are used, shown in Figures 1, 2 and 3. It is easy to see that these calculation schemes differ in their dimensionality, the number of nodes and branches, which allows us to objectively reveal the advantages of the mathematical model on networks of varying complexity.

2 Materials and Methods

*Corresponding author: umarxodja@bk.ru

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Simulation is reduced to carrying out a large number of calculations of the steady-state flow distribution at various values of the loads in the nodes of the network circuit. With the accumulation of data from such calculations, it becomes possible to estimate the parameters of the following probability distribution functions:

1. Flow values in each passive circuit element (network section) qi.
2. The values of the head loss in each passive element - hi.
3. The values of the total feeds of the target product to all nodes of the scheme - ΣQi.
4. The pressure difference values at the active sources and at the dictating point of the circuit - HΔ (these values correspond to the largest values in the matrix [1]).

To simplify the analysis, a normal distribution of all random variables is assumed. Therefore, only two non-random parameters are determined for each of the distributions - the mathematical expectation and variance. Also, for active sources, the covariance values of the dependent random variables are determined – h0 and ΣQj, which, as shown below, is necessary to calculate the total energy consumption for the transportation of the target product.

A general algorithm for simulation (figure - 4) consists of three blocks – A1 in which random load values are generated for all consumption nodes of the target product; A2 providing the calculation of the steady-state flow distribution, and A3, designed for statistical processing of the results obtained.

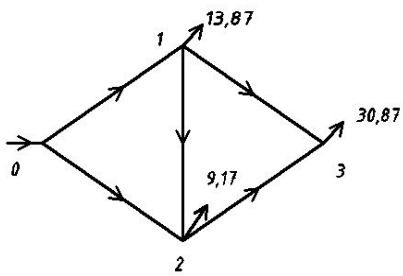

Fig. 1. The first design scheme of a utility network with 2 rings

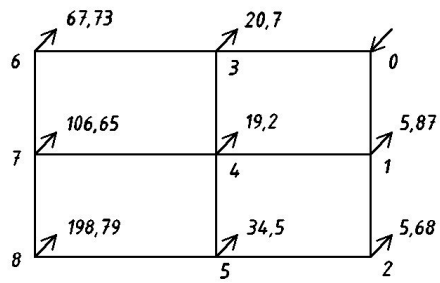

Fig. 2. The second design scheme of the utility network with 4 rings

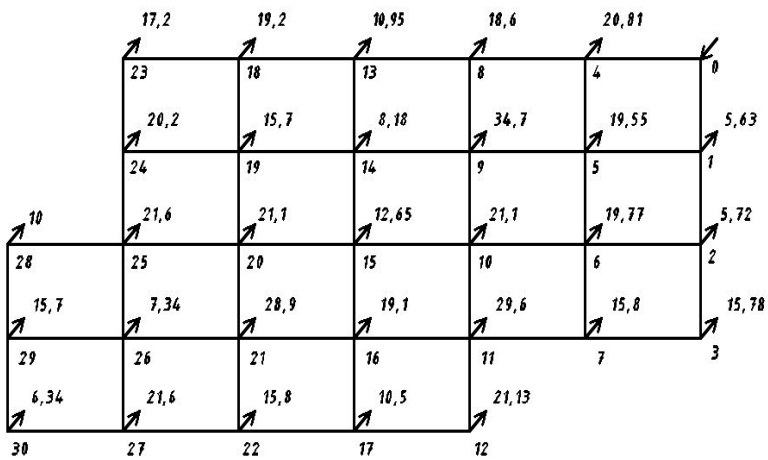

Fig. 3. The third design scheme of the utility network with 20 rings

Block A1 built according to section data 2.1 [2]. Since the parameters of this model, in the general case, change every day of operation of the utility network, for the operation of the block, the initial information contains not only the values of the mathematical expectations of the amplitude and phase shift for each of the two harmonies but also the values of the coefficients of their variation. Also, the mathematical expectation and the coefficient of variation are also characterized by the value in (2, 3) [2]. Such a volume of initial information makes it possible to fairly reliably simulate the process of consumption of the target product at any node of the calculation scheme. In this work, the process of consuming a target product in water supply systems is modelled, based on the results, the study of which is used for the initial information necessary for modelling. In the block A1, a sensor of pseudo-random numbers distributed according to the normal law with zero mathematical expectation and unit variance is provided [6].

Considering the block A2, it should be noted that almost any of the known algorithms and programs for calculating the steady-state flow distribution can be used here [7, 8]. The only requirement for them from the standpoint of the features of simulation is the need for a fairly convenient software replacement of the values of nodal loads based on the results of the block A1. The block algorithm used in this work A2 detailed in [2] section 3.3

Block A3 the simulation algorithm is quite simple, and its essence boils down to the fact that for all elements of the network design scheme, including active elements, mathematical expectations are calculated, variance, standard deviations and coefficients of variation for each of the distributions of interest of random variables (P) by well-known [6] formulas:

$$M(P)=\frac{\Sigma P}{N};\quad D(P)=\frac{\Sigma P^2}{N}-[M(P)]^2;$$
$$\sigma(P)=\sqrt{D(P)};\quad \upsilon_p=\frac{\sigma(P)}{M(P)}; \tag{1}$$

Initial information used to operate the unit A1 is presented below. The algorithm for simulation modelling of the utility network Figure 3, described in [9], is given in Table 3.1. When modelling a network, Figure 1 used data for nodes 1, 2, 3 from table 3.1, but for the network Figure 2 received data corresponding to nodes 1 9 from table 1. In table 3.1, amplitude values harmonics and standard deviation are given in relative units – in shares for each of the nodes of the design scheme.

3 Results and Discussion

The results of the simulation modelling of three engineering networks are presented in tables 3.2÷3.4. Figure 4 shows the relationship between the coefficients of variation of flows in the lines of networks (υ_q) and head losses (υ_h), obtained from modelling. Also shown here is the line corresponding to the above [1]) the relationship between these coefficients. Good agreement between the experimental and theoretical data confirms the correctness of the latter and the possibility of calculating the parameters of the distribution functions of the pressure delivery in passive elements from the data on the parameters of the flow distribution functions.

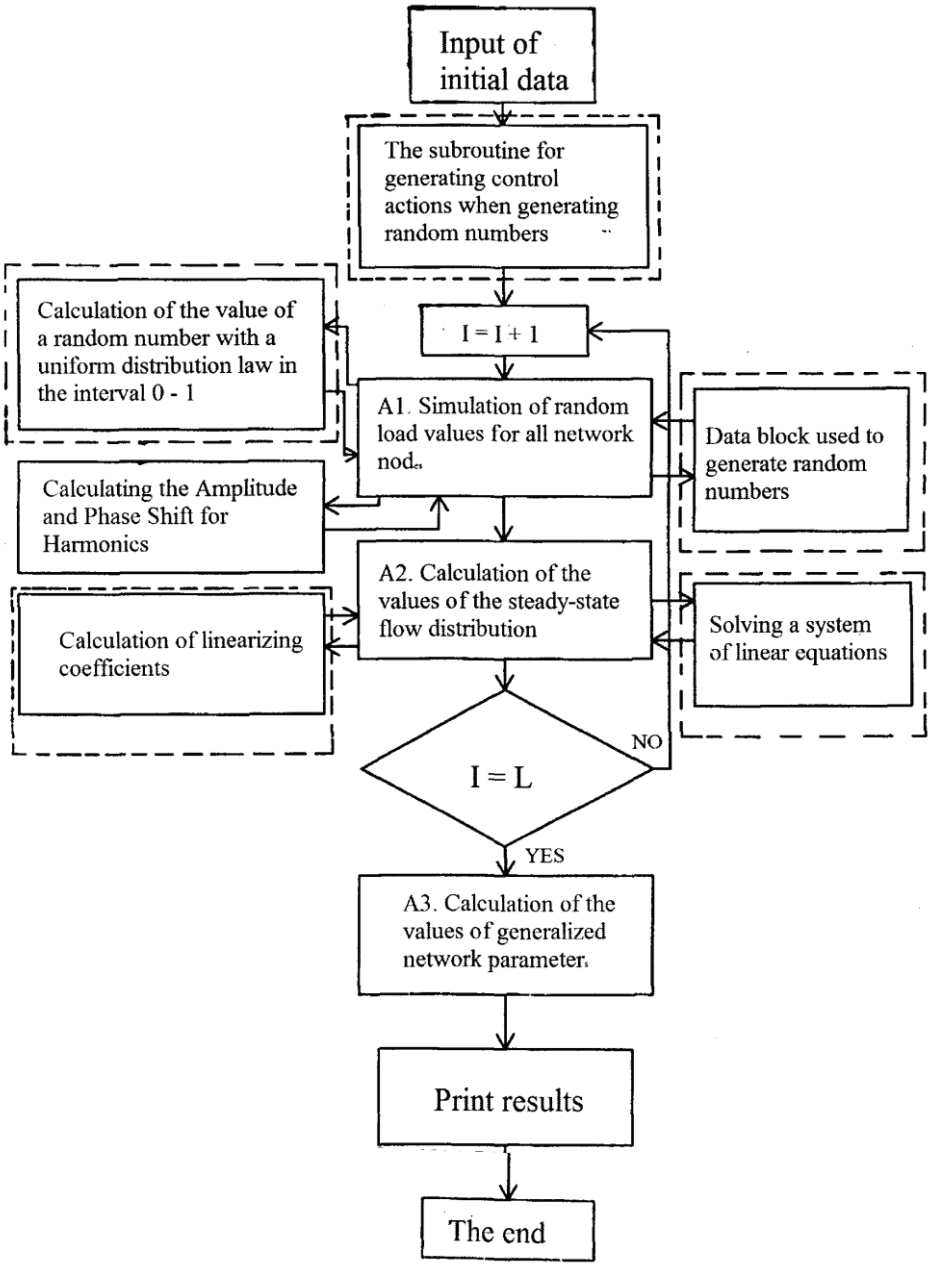

Fig. 4. Block diagram of the simulation algorithm

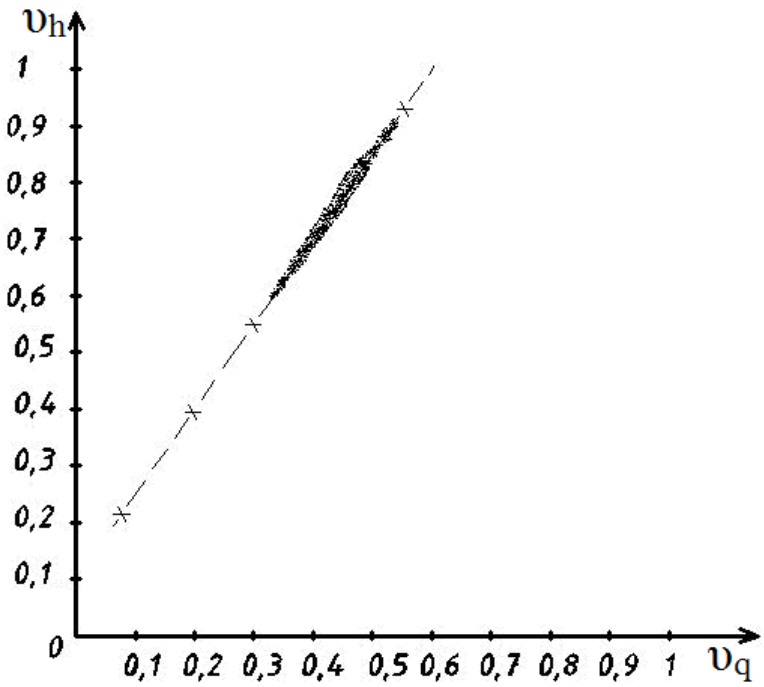

Fig.-5. $\upsilon_q - \upsilon_h$ calculation line according to (3.30) [2]

Comparison of the parameters of the distribution functions of flows in passive elements obtained during simulation and calculation by [1] and [2] (see table 2 ÷ 4), shows that the proposed mathematical model of stochastic flow distribution in nonlinear pipeline networks provides an accuracy sufficient for practical purposes – calculation error q_i less than -8% , and for υ_{q_i} -10%.

When calculating the parameters of the distribution function of the total loads in the network (1) and head losses in the network (1) by formulas [1] and [2], [1] a single value of the correlation coefficient between the process of consumption of the target product in the nodes of the utility network was adopted r_{ij}=0,25. The quantity r_{ij} obtained from the graph of figure 3.5, where the change in the variance of the total network load is shown depending on the value r_{ij} in [1]. For all three considered networks, the value r_{ij} at which the calculated value of the variance of the total load (in simulation modelling) coincides with the value obtained from [1]) approximately equals 0.25. The same meaning r_{ij} is also used in calculating the dispersion of head losses in the network, which is quite acceptable since the discrepancy between the simulation data and the calculation by the mathematical model of the stochastic flow distribution does not exceed 10% (see tables 2 - 4).

Calculation of the parameters of stochastic flow distribution for networks figures 3, 2 and 3, 3 very cumbersome due to the large dimension of the matrix of load distribution coefficients C_{ij} and are performed only using an electronic computer.

According to the results of the calculation (network 3.3) on the graph (figure 3.6), the field of possible changes in the pressure losses in the network was constructed by H_Δ , and total load $\sum Q_j$, two points of which (A and B) correspond to the limiting (smallest and largest) values of the head losses in the network at the minimum and maximum values of the total network load.

For the correct selection of pumping equipment and tormented points, it is necessary to find the limits of the possible change in head losses in the network at different values of the total load $\sum Q_j$. This can be done by considering a system of two random variables H_Δ and $\sum Q_j$ assuming for each of them the normal probability distribution law. Let's consider the correlation coefficient between the values of these random variables as known, for example, take it equally as before 0.25. We can find the so-called conditional distribution H_Δ, that is, the laws of its distribution for various fixed values $\sum Q_j$, known [6, 9, 10], that the density of the conditional distribution of two correlation normally distributed random variables is determined by the expression:

$$\rho(H_\Delta, \Sigma Q_j = \mathrm{E}) = \frac{1}{2\pi\sigma(H_\Delta)\sigma(\Sigma Q_j)\sqrt{1-r^2}} \exp \frac{1}{2}\left[\frac{(H_\Delta - \overline{H_\Delta})^2}{\sigma^2(H_\Delta)} - \frac{2r(H_\Delta - \overline{H_\Delta})^2(E-\underline{E})^2}{\sigma(H_\Delta)\sigma(\Sigma Q_j)} + \frac{(E-\overline{E})^2}{\sigma^2(\Sigma Q_j)}\right] \quad (2)$$

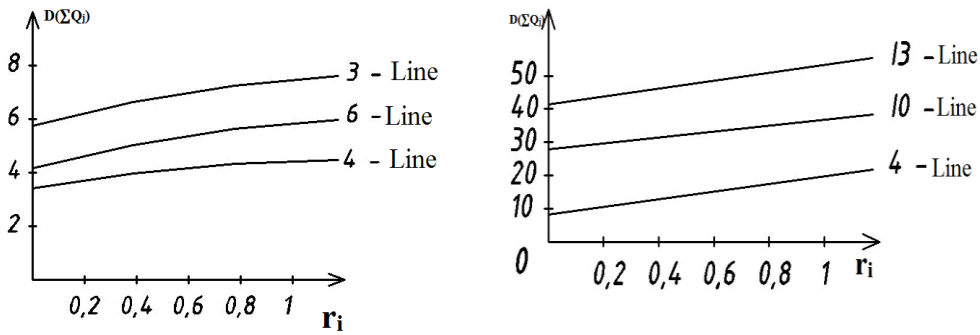

Fig. 6. Change in variance of total load network depending on the value of the coefficient correlations between the target consumption process product at network nodes. a is the network in Figure 1; b is the network in Figure 2. $D(\sum Q_j)$ is variance value total loads obtained by imitation modelling.

From (2), the probability of the appearance of different values H_Δ at $\sum Q_j = E$. In (2), all the necessary quantities are known from the results of the above-described calculation of the stochastic flow distribution. The correlation coefficient between и can be refined based on the simulation results. For the network figure 2, the graph of values H_Δ for various $\sum Q_j$ shown in Figure 5 - the correlation coefficient here is 0.3, which is quite close to the one used earlier. Results of calculating conditional probability distribution functions H_Δ at 6 values $\sum Q_j$ shown in figure 3.8, and the parameters of these functions are given in table 3.5, the data of which show that the calculation for 2 quite well converges with simulation modelling and is quite consistent with the data of field experiments in engineering networks shown in figure 7.

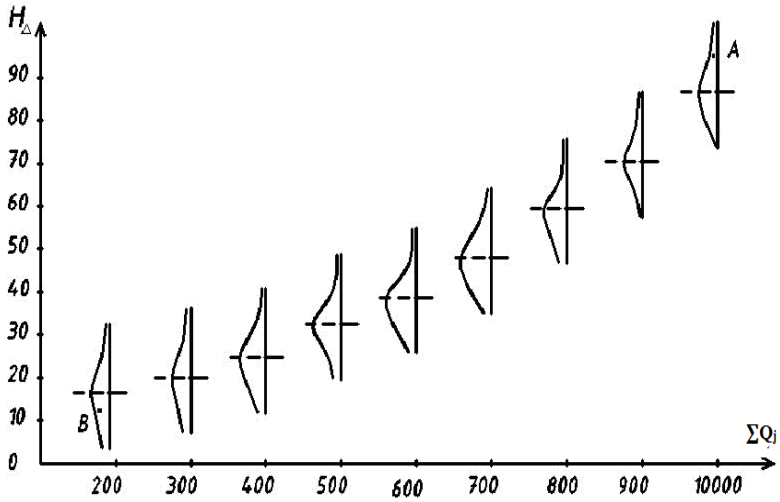

Fig. 7. Distribution functions of possible changes in head losses in the network: A - the highest value of head loss; B - the smallest value of the head loss

Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 1

Rooms plot networks	Simulation modelling				Math modelling			
	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\overline{q_i}$	υ_{q_i}
1	27.05	0.217	766	0.411	27.27	0.259	791.18	0.416
2	36.86	0.215	755	0.403	26.64	0.265	759.15	0.499
3	3.08	0.782	15.3	1.37	3.19	0.980	19.9	1.22
4	15.29	0.258	249	0.484	15.12	0.262	244.4	0.498
5	15.29	0.264	260	0.508	15/57	0.256	258.3	0.487
A source (Node 0)	$\overline{\sum Q_j}$ = 513,56	$\upsilon_{\sum Q_i}$ = 0,205	$\overline{H_\Delta}$ = 1,4	υ_{H_Δ} = 0,410	$\overline{\sum Q_j}$ = 513,1	$\upsilon_{\sum Q_i}$ = 0,189	$\overline{H_\Delta}$ = 1,35	υ_{H_Δ} = 0,408

Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 2

Rooms plot networks	Simulation modelling				Math modelling			
	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\overline{q_i}$	υ_{q_i}
1	27.05	0.217	766	0.411	27.27	0.259	791.18	0.416
2	36.86	0.215	755	0.403	26.64	0.265	759.15	0.499
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Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 3

Rooms plot networks	Simulation modelling				Math modelling			
	$\overline{q}_i$	υ_{q_i}	$\overline{h}_i$	$\upsilon_{h_{ш}}$	$\overline{q}_i$	υ_{q_i}	$\overline{h}_i$	$\upsilon_{h_{ш}}$
1	2	3	4	5	6	7	8	9
1	136.89	0.388	1.28	0.684	137.0	0.391	1.33	0.691
2	376.67	0.396	2.13	0.703	376.1	0.391	2.01	0.701
3	80.22	0.396	3.45	0.705	80	0.39	3.25	0.695
4	50.95	0.381	1.72	0.679	51	0.389	1.8	0.683
5	14.51	0.389	4.16	0.711	14.5	0.394	4.12	0.699
6	30.74	0.385	2.34	0.707	30.8	0.388	2.41	0.715
7	14.99	0.399	4.44	0.71	15.1	0.403	4.51	0.78
8	169.23	0.397	2.9	0.689	161.3	0.399	2.95	0.702
9	194.6	0.391	2.59	0.694	195.1	0.402	2.71	0.71
10	159.8	0.393	2.79	0.695	159.1	0.389	2.67	0.691
11	95.5	0.384	2.43	0.721	95.4	0.381	2.41	0.72
12	15.07	0.401	2.63	0.711	15.3	0.411	2.72	0.719
13	75.15	0.409	2.7	0.771	74.8	0.4	2.63	0.769
14	14.39	0.448	3.64	0.745	14.5	0.451	3.72	0.75
15	52.1	0.444	4.68	0.691	52.9	0.449	4.76	0.688
16	90.41	0.388	2.49	0.699	90.1	0.381	2.41	0.698
17	139.7	0.399	2.33	0.72	138.3	0.388	2.21	0.69
18	75.67	0.428	3.14	0.714	75.2	0.417	3.1	0.702
19	107.3	0.409	3.56	0.722	108.3	0.415	3.72	0.735
20	86.41	0.419	4.04	0.807	86.2	0.403	3.91	0.798
21	37.99	0.446	2.49	0.701	37.7	0.425	2.33	0.692
22	54.15	0.399	1.54	0.734	54.01	0.391	1.52	0.733
23	16.54	0.428	5.39	0.655	16.1	0.421	5.23	0.651
24	3.31	0.345	0.94	0.755	3.44	0.355	0.99	0.761
25	37.92	0.448	4.62	0.709	37.7	0.432	4.24	0.697
26	18.1	0.399	3.24	0.727	18.2	0.405	3.41	0.731
27	52.61	0.424	4.99	0.686	52.2	0.421	4.91	0.683
28	21.33	0.37	1.07	0.726	20.6	0.362	1.08	0.701
29	70.52	0.426	0.89	0.759	71	0.431	0.95	0.772
30	10.01	0.441	6.34	0.726	9.97	0.417	0.14	0.711
31	46.3	0.423	6.49	0.709	46.2	0.421	6.21	0.695
32	16.02	0.405	3.63	0.751	16.41	0.396	3.47	0.742
33	3.86	0.386	1.32	0.696	4.22	0.392	1.41	0.707
34	14.87	0.371	3.06	0.898	14.33	0.361	3	0.876
35	8.85	0.58	0.89	0.778	8.49	0.471	0.83	0.77
36	48.79	0.449	0.95	0.717	49.2	0.457	0.99	0.731
37	8.6	0.411	6.66	0.777	8.56	0.4	6.51	0.769
38	32.67	0.471	3.29	3.29	0.775	32.4	0.469	0.768
39	21.13	0.432	3.48	3.48	0.931	21	0.43	0.927
40	4.9	0.632	0.402	0.807	4.88	0.622	0.389	0.8
41	21.18	0.474	0.44	1.39	20.3	0.465	0.37	1.2

Continuation

Rooms plot networks	Simulation modelling				Math modelling			
	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\upsilon_{h_{ш}}$	$\overline{q_i}$	υ_{q_i}	$\overline{h_i}$	$\upsilon_{h_{ш}}$
42	2.75	0.791	0.83	0.792	2.92	0.81	0.98	0.82
43	26.12	0.449	1.47	0.678	26.3	0.44	1.41	0.67
44	18.39	0.382	3.77	0.754	18.61	0.389	0.98	0.781
45	18.67	0.444	0.86	0.784	18.5	0.437	0.78	0.765
46	7.56	0.429	3.52	0.769	7.69	0.435	8.68	0.782
47	6.37	0.418	3.68	0.769	6.32	0.401	3.52	0.761
48	7.28	0.426	0.13	0.836	7.15	0.397	0.12	0.811
49	8.62	0.391	6.59	0.707	8.53	0.376	6.31	0.696
50	2.16	0.822	5.48	1.71	2.03	0.802	5.31	1.63
A source (Node 0)	$\overline{\sum Q_j} =$ 513.56	$\upsilon_{\sum Q_i} =$ 0.205	$\overline{H_\Delta} =$ 1.4	$\upsilon_{H_\Delta} =$ 0.410	$\overline{\sum Q_j} =$ 513.1	$\upsilon_{\sum Q_i} =$ 0.189	$\overline{H_\Delta} =$ 1.35	$\upsilon_{H_\Delta} =$ 0.408

4 Conclusions

In this article, we propose ways to implement a quasilinear mathematical model of flow distribution in pipeline engineering networks, which determines the matrices of generalized network parameters – load distribution coefficients along the branches of the scheme, calculated at a point corresponding to the mathematical expectation of node loads. Based on the model obtained in the work, the convergence of the obtained results with the results of simulation modelling of engineering networks is proved by numerical experiment on an electronic computer.

The effectiveness of the developed model, the corresponding algorithms and the software package is proved. The values of the criterion of reduced costs for parametric optimization of engineering networks are given by comparing the results obtained, it is shown that they can be reduced in comparison with the methods currently used in practice. The article indicates the possibility of obtaining equivalent hydraulic characteristics of engineering networks at the design stage in the following cases.

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