Flow spreading behind a combined dam with a through part of tetrahedrons on foothill rivers

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Abstract. The areas of the foothill rivers are distinguished by large bottom slopes reaching 0.004, with increased kinetics of the flow of more than 0.15, and by gravel-pebble sediments of the river bed and flow. An analysis of the studies performed on the regulatory structures showed that the bulk of them was performed for the conditions of lowland rivers. In the article, a design of a combined dam with a through part of tetrahedrons is proposed. The experiments were conducted in a flume with a rigid bottom and with a variable slope of. The modeling was performed according to Froude in a self-similar area. Experimental studies revealed the presence of two flow spreading modes depending on the bottom slope: a "calm" mode at \( i_s < i_w \) and a "critical" mode at \( i_s > i_w \). These modes are mainly influenced by the degree of constraint and the Froude number. In the previous articles, a solution to the problem was provided for the case \( i_s < i_w \), which covers the foothill sections of rivers at a "calm" mode. At a further increase in the degree of flow constraint \( n > 0.3 \) and the Froude number \( F_r > 0.15 \), a "critical mode" is observed. Here, a solution to the problem for this case is given. The problem, in this case, differs from the previously considered one by non-uniform distribution of velocities in the weakly disturbed core, a significant reduction in the length of the vortex zone; the vertical compression of the flow continues to the end of the vortex zone. The versatility of the velocity distribution in the zones of weakly disturbed core and intense turbulent mixing is experimentally substantiated. With theoretical studies, using the basic equations of applied mechanics, a method for calculating the velocity field was developed, and the planned dimensions of the vortex zones were established. The comparison with experimental data showed satisfactory agreement.

1 Introduction

Stream-bank erosion brings huge losses worldwide to the economies of countries located in coastal zones. Agricultural lands, settlements and cities are being washed away [1-3]. Therefore, it is not surprising that there are many theoretical and experimental studies devoted to improving structures and aimed to develop the methods of the design

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justification of regulatory structures [4-10]. The researchers paid main attention to determining the depth of local erosion near blind dams [11-15] and the conditions of lowland rivers.

The foothill sections of the rivers have their own features, which consist of the morphology and the hydraulics of the flows. [16, 17]. On the foothill rivers of Uzbekistan (Zarafshan, Chirchik, Kashkadarya, Akhangaran and others), the slopes of the channel vary within \( i = 0.001 + 0.004 \), the kinetics of the flow is \( F_e = 0.15 + 0.5 \). Research for these conditions was conducted for blind and through transverse structures [17-19]. In the laboratory of Hydrotechnical Engineering of the Institute, new structures were developed, and large-scale studies of the operation of combined dams with a through part of piles driven into the river bottom were carried out [20-24]. They are the most capital, and the disadvantage is their high cost.

A combined dam is proposed, consisting of a blind part built of local soil and a through part built of reinforced concrete tetrahedrons laid in the dam's head.

Theoretical and experimental studies were carried out in a flume of a variable slope, the physical picture of the flow constrained by a combined dam with a through part of tetrahedrons was revealed, and a method for their calculation was developed.

2 Materials and Methods

The experimental research methods were described in detail in our previous articles [1, 2]. Here we give the main characteristics of the flow and the channel: the dimensions of the flume, the building factor of the through part \( P = 0.1 \div 0.4 \), \( P = \frac{W_r}{W} \) (building area of the through part, the total area), the angle of the dam installation \( \alpha = 75 \div 90^\circ \), the bottom slope \( \varepsilon_i = 0.0001 \) before \( 0.004 \).

Modeling was performed according to Froude. In all experiments, the turbulent mode was maintained. The condition of the planned task \( B/h > 6 \) was met. Water flow rates were measured by the Thompson triangular weir. The free surface was fixed using a measuring needle with leveling. Water velocities were measured with a SANIIIRI micro-spinner with a TsISNV-5 electronic sensor. The main provisions of the theory of turbulent jets propagating in a confined space were used in theoretical studies: the scheme of dividing the flow into hydraulic homogeneous zones: a weakly disturbed core, intense turbulent mixing, and reverse currents.

To solve the problem, the basic equations of applied mechanics, law of conservation of momentum in the flow, conservation of the flow rate, and differential equation of non-uniform motion recorded for the transit flow were used, considering the tangential turbulent stresses on the side surfaces, according to Prandtl.

3 Results and Discussion

The physical nature of the flow around the combined dam, the through part of which is made of tetrahedrons, has much in common with the flow around the combined dam with the through part from the pile rows [23-28] for flat rivers; though it differs both qualitatively and quantitatively. The flow occurs with the formation of a backwater section between sections F-F and 0-0, planned compression between sections 0-0 and ПС, vertical compression between sections ПС-ВС, a recovery area between sections BC and Б-Б (Fig. 1).
In the design scheme: beam 0'-1 is the boundary between the core and the zone of intense turbulent mixing; 0'-2 is the outer boundary of the zone of intense turbulent mixing; 0'-3 is the boundary of the zone of zero velocities; 0'-4 is the transit flow boundary; МП, ПС, О-О, К-К; ВС, Б-Б – are the sections of the maximum backwater, constraint, planned compression, the end of the vortex zone, vertical compression, common flow state.

The boundaries of zones of intense turbulent mixing $C_1 = 0,0$, $C_2 = 0,37$ were experimentally established (Figure 1).

It was also found experimentally that the distribution of velocities in the zone of a weakly disturbed core obeys the theoretical Schlichting – Abramovich dependence (Fig. 2).

$$\frac{u-u_s}{u_{\text{max}}-u_s} = (1-\eta^{1.5})^2,$$

where $\eta = \frac{y_1 - y}{\theta_s}$

and the distribution of velocities in the zone of intense turbulent mixing (Fig. 3) is

$$\frac{u_{\text{max}} - u}{u_{\text{max}} - u_{\text{t}}} = (1-\eta^{1.5})^2;$$

where

$$\eta = \frac{y_2 - y}{b} = \frac{y_2 - y_1}{y_2 - y_1}$$
Theoretically, it was necessary to evaluate the influence of the longitudinal slope and other characteristics of the flow and structure on the patterns of change in the maximum velocities in the core $u_\text{max}$, along the opposite bank $u_*$ and the determination of the vortex zone length $L_v$. 

**Fig.2.** Dimensionless velocity profiles in the core

**Fig.3.** Dimensionless velocity profiles in the region of intensive turbulent mixing
To determine $u_{\text{max}}$, we use the equation of conservation of momentum in the flow, written for the cross-sections of ПС and X-X

$$
\rho h_{nc} \int_{0}^{\xi_{nc}} u^{2} \, dy + \rho h_{nc} \int_{y_{1}}^{y_{2}} u^{2} \, dy + \rho u_{wnc}^{2} h_{nc}(B - \beta e_{nc} - \epsilon_{c}) = \rho h_{x} \int_{0}^{\xi_{nc}} u^{2} \, dy + \rho h_{x}
$$

$$
\int_{y_{1}}^{y_{2}} u^{2} \, dy + \rho u_{wnc}^{2} h_{x}(B - \beta e_{x} - \epsilon) + \rho \int_{0}^{x} \left( u_{wnc}^{2} + \beta u_{nc}^{2} \right) \, dy \cdot dx + \frac{gB}{2} (h_{x}^{2} - h_{nc}^{2}) - \frac{gB}{2} (h_{nc} + h_{x}) Bi_{\beta} \, x
$$

where the depth of the flow in the X-X section is determined as

$$
h_{x} = h_{nc} + i_{x} x - ix = h_{nc} + x(i_{g} - i) = h_{nc} + I x; \quad I = i_{x} - i
$$

where $I$ is the average value of the depth increment in the spreading section

$$
\frac{\partial h}{\partial x} = I = \frac{h_{nc} - h}{\ell_{a}}
$$

Performing the integration considering the given dependencies, we obtain

$$
\begin{align*}
    u_{\text{max},nc}^{2} \beta_{nc} h_{nc} (0.316 + 0.268 m_{nc} + 0.416 m_{nc}^{2} + 0.416 u_{\text{max},nc}^{2} e_{c} h_{nc}) &= \\
    = h_{x} \beta + \frac{\lambda}{2} \int_{0}^{x} \beta dx + \frac{gB}{2} (h_{x}^{2} - h_{nc}^{2}) - \frac{gB}{2} (h_{nc} + h_{x}) i_{g} x
\end{align*}
$$

(4)
Where

\[ \beta = \int_{c}^{6} \int \theta_{xx} + \theta u^2 dy \quad F_1 = 0.316 + 0.268m_{nc} + 0.416m_{nc}^2 \]

\[ m_{nc} = \frac{u_{*nc}}{u_{\text{max},nc}} \]

The general solution of the integral equation has the form

\[
\beta = -2gBI \frac{(h_{nc} + Lx)}{4I + \lambda} - gBi\frac{x^2}{2(h_{nc} + Lx)} + c(h_{nc} + Lx)^{-\left(\frac{\lambda}{2I}\right)}
\]

(5)

The constant \(c\) is determined from the boundary conditions at \(x = 0\)

\[ \beta = u_{\text{max},nc}^2 \left( e_{nc} F_1 + 0.416 e_c \right) \]

Then from (5)

\[ \beta = -2gBI \frac{h_{nc}}{4I + \lambda} + ch_{nc}^{-\left(\frac{\lambda}{2I}\right)} \]

Hence,

\[
c = \frac{\beta + 2gBI \frac{h_{nc}}{4I + \lambda}}{h_{nc}^{-\left(\frac{\lambda}{2I}\right)}} = \frac{u_{\text{max},nc}^2 \left( e_{nc} F_1 + 0.416 e_c \right) + 2gBI \frac{h_{nc}}{4I + \lambda}}{h_{nc}^{-\left(\frac{\lambda}{2I}\right)}}
\]

Substituting it into (5), we obtain a partial solution

\[
\beta = -2gBI \frac{(h_{nc} + Lx)}{4I + \lambda} - gBi\frac{x^2}{2(h_{nc} + Lx)} + u_{\text{max},nc}^2 \left( e_{nc} F_1 + 0.416 e_c \right) + 2gBI \frac{h_{nc}}{4I + \lambda} \left( h_{nc} + Lx \right)^{-\left(\frac{\lambda}{2I}\right)}
\]

On the other hand,

\[ \beta = \int_{c}^{6} \int \theta_{xx} + \theta u^2 dy = u_{\text{max}}^2 \left( e_{g} F_2 + 0.416 e_c \right) \]

where
\[ F_2 = 0.316 + 0.268 m_1 + 0.416 m_1^2 \]

\[ m_1 = \frac{u_\star}{u_{\text{max}}} \]

\[ u_{\text{max}}^2 (\bar{\theta}_c F_2 + 0.416 \bar{\theta}_c) = u_{\text{max,nc}}^2 (\bar{\theta}_{\text{nc}} F_1 + 0.416 \bar{\theta}_c) \left( \frac{h_{\text{nc}}^{1 + \frac{\lambda}{2\ell}}}{(h_{\text{nc}} + Ix)^{1 + \frac{\lambda}{2\ell}}} \right) + \]

\[ + 2gBl \frac{h_{\text{nc}}}{(4I + \lambda)} \cdot \frac{h_{\text{nc}}^{(1 + \frac{\lambda}{2\ell})}}{(h_{\text{nc}} + Ix)^{(1 + \frac{\lambda}{2\ell})}} - 2gBl \frac{(h_{\text{nc}} + Ix - gBi}{2(4I + \lambda)} - \frac{\bar{\theta}_c}{2Fr_{\text{nc}} (1 - n)} \kappa^2 \zeta^2 \]

\[
\left( \frac{u_{\text{max}}}{u_{\text{max,nc}}} \right)^2 = \frac{1}{(\bar{\theta}_c F_2 + 0.416 \bar{\theta}_c)} \left[ \frac{M + P}{(1 + I\kappa \zeta)} - P(1 + I\kappa \zeta) - \frac{i\lambda}{2Fr_{\text{nc}} (1 - n)} \kappa^2 \zeta^2 \right]
\]

Where

\[ M = \bar{\theta}_{\text{nc}} F_1 + 0.416 \bar{\theta}_c \]

We check at \( X=0 \)

\[
\left( \frac{u_{\text{max}}}{u_{\text{max,nc}}} \right)^2 = \frac{\bar{\theta}_{\text{nc}} F_1 + 0.416 \bar{\theta}_c}{\bar{\theta}_{\text{nc}} F_1 + 0.416 \bar{\theta}_c} \quad u_{\text{max}} = u_{\text{max,nc}}
\]

Suppose we neglect the last term that considers the value of the component of the fluid weight and assume a uniform distribution of velocities in the weakly disturbed core \( F_1 = 1 \).

In that case, we arrive at the dependence obtained earlier [7].

Equation of conservation of flow rate for sections \( \text{PS and } X-X \) is written as \( \Pi \text{ and } X-X \)

\[ h_{nc} \int_{0}^{\theta_{\text{nc}}} u dy + h_{nc} \int_{\theta_{\text{nc}}}^{\theta_{\text{nc}}+\theta_{c}} u dy = h_x \int_{0}^{\theta_{x}} u dy + h_x \int_{\theta_{x}}^{\theta_{x}+\theta_{c}} u dy \]

(7)

Integrating it with (1) and (2), we obtain

\[ u_{\text{max,nc}} \bar{\theta}_c h_{nc} (0.45 + 0.55m_1) + 0.55u_{\text{max,nc}} \bar{\theta}_c h_{nc} = \]

\[ = u_{\text{max}} \bar{\theta}_c h_x (0.45 + 0.55m_1) + 0.55u_{\text{max}} \bar{\theta}_c h_x \]

(8)

where

\[ m_1 = \frac{u_\star}{u_{\text{max}}} \]
from (9) we obtain

\[ u_{\text{max}} = \frac{[\bar{a}_m (0.45 + 0.55m_{nc}) + 0.55\bar{a}_m]}{[\bar{a}_m (0.45 + 0.55m_{nc}) + 0.55\bar{a}_m](1 + \text{ik}\zeta)} \]  (9)

Solving (9) and (6) together, we arrive at a quadratic equation for \( m_1 \)

\[ A_1 m_1 + A_2 m_1 + A_3 = 0 \]  (10)

Where

\[ A_1 = 0.416\bar{a}_m \cdot N^2 - 0.302\bar{a}_m^2 (1 + \text{ik}\zeta)^2 R \]
\[ A_2 = 0.268\bar{a}_m \cdot N^2 - (1 + \text{ik}\zeta)^2 R\bar{a}_m (0.49\bar{a}_m + 0.605\bar{a}_m) \]
\[ A_3 = \bar{a}_m \cdot N^2 (0.316 + 0.416\bar{a}_m) - (0.45\bar{a}_m + 0.55\bar{a}_m)^2 (1 + \text{ik}\zeta)^2 R \]
\[ N = \bar{a}_m (0.45 + 0.55m_{nc}) + 0.55\bar{a}_m \]
\[ R = \frac{M + \frac{P}{\sqrt{1 + \text{ik}\zeta}^{1 + \frac{2}{27}}} - P (1 + \text{ik}\zeta) - \frac{i_d \kappa^2 r^2}{2Fr_{nc}(1-n)}}{2Fr_{nc}(1-n)} \]
\[ M = \bar{a}_m F_1 + 0.416\bar{a}_m \]
\[ P = \frac{2I}{Fr_{nc}(4I + \lambda)(1-n)} \]
\[ m_1 = \frac{u_\text{max}}{u_\text{max}}, \quad m_{nc} = \frac{u_{\text{max.nc}}}{u_{\text{max.nc}}} \]

In equation (10), a root less than one is taken as the main root, and a root greater than one is discarded as it contradicts the physics of the phenomenon.

The length of the vortex zone is determined from the equation of non-uniform motion recorded with account for the tangential turbulent stresses on the side surfaces [1]. The order of the solution remains the same, so we write it down finally as

\[ L_\text{v} = \frac{A}{E} \frac{\ell n \theta_{\text{nc}}}{\theta_{\text{nc}}} \sqrt{\frac{D\theta_{\text{nc}} + E}{D\theta_{\text{nc}}^2 + E}} \]  (11)

Where

\[ A = 2\alpha Q^2 h_{cp}; \quad D = -2g_i h_{cp}; \quad E = Q^2 \left( \frac{\lambda_{\text{cp}} h_{cp} + \varphi_{\text{cp}}}{B_{\text{cp}}} + 2.88\chi^2 \frac{h_{cp}}{\theta_{\text{cp}}} - 4\alpha d \right) \]
\[ h_{cp} = \frac{h_{nc} + h_{c}}{2}; \quad \theta_{\text{cp}} = \frac{\theta_{\text{nc}} + \theta_{\text{c}}}{2}; \quad B_{\text{cp}} = \frac{\theta_{\text{nc}} + \theta_{\text{c}}}{2}; \quad \chi = 0.21 \]

\( \lambda_{\text{cp}}, \lambda_{\text{cp}} \) are the resistance factors of the bank and bottom.

The design of a combined dam is considered, the blind part of which is made of local soil and the through part is made of tetrahedrons. Such a dam combines the positive features of the blind and through structures. Because the through part does not require driving into the bottom of the reservoir, construction costs are significantly reduced.
The features of such structures operation were experimentally revealed. The formation of two flow modes was established: a "calm" mode at \( n_d < 0.3, \) \( F_r < 0.15 \) and a "critical" mode at \( n_d > 0.3, \) \( F_r > 0.15. \) The second flow scheme is considered here. A jet character of the flow around a combined dam with a through part of tetrahedrons was stated. It was found that in the zone of a weakly disturbed core, the distribution of velocities in the plan has a non-uniform character and obeys the theoretical Schlichting – Abramovich dependence (Fig. 2).

The distribution of velocities in the zone of intense turbulent mixing is universal and obeys the theoretical Schlichting-Abramovich dependence (Figure 3).

The boundaries of zones of intense turbulent mixing \( 0'-1 \) \( C_1 =0.0, \) \( 0'-2 \) \( C_2 =0.37 \) were experimentally established (Figure 1)

A method for calculating the field of flow velocities constrained by a combined dam with a through part of tetrahedrons for the conditions of the foothill sections of rivers was developed. The task is implemented for the second "calm" mode using the integral relation characterizing the law of conservation of momentum, the equations of conservation of flow rate and non-uniform motion recorded, taking into account the tangential turbulent stresses on the side surfaces of the vortex according to Prandtl. Calculated dependencies are obtained to determine the change in the maximum velocities in the weakly disturbed core \( U_a \) at the opposite bank \( U \) and the length of the vortex zones in the spreading area \( l_{a1}, l_{a2}. \) Direct calculations and their comparison with the experimental data (Fig. 4) showed the correctness of the theoretical solutions obtained.

4 Conclusions

1. A change in the longitudinal slope of the bottom, typical for foothill rivers areas, from \( i_D = 0.002 \) to \( i_D = 0.004 \) leads to a change in the hydraulic and kinematic characteristics of the flow. At \( n > 0.3 \) and the Froude number \( F_r > 0.15, \) a "critical" mode is observed.
2. In the presence of a "critical" mode, the distribution of velocities in the zone of a weakly disturbed core is non-uniform and obeys the theoretical Schlichting-Abramovich dependence.
3. The expansion coefficient of the zone of intense turbulent mixing is 0.37 versus 0.27 in the theory of turbulent jets. The distribution of velocities in this zone is also universal.
4. The section of the maximum vertical compression coincides with the end of the vortex zone.
5. Using the main provisions of the theory of turbulent jets, a method for calculating the velocity field and dependence for determining the length of the vortex zone are proposed.

References

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