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Vibrations of dam–plate of a hydro-technical structure under seismic load

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Abstract. In present paper, the problem of the vibration of a viscoelastic dam-plate of a hydro-technical structure is investigated, based on the Kirchhoff-Love hypothesis in the geometrically nonlinear statement. This problem is reduced to a system of nonlinear ordinary integro-differential equations by using the Bubnov-Galerkin method. The resulting system with a weakly-singular Koltunov-Rzhanitsyn kernel is solved using a numerical method based on quadrature formulas. The behavior of the viscoelastic dam-plate of hydro-technical structure is studied for the wide ranges of physical, mechanical, and geometrical material parameters.

1 Introduction

When solving energy and water management problems in Uzbekistan, one of the main tasks is creating economic and reliable structures of mountain hydro-technical structures, taking into account the fact that the construction site presents a zone of high seismicity. The design of hydro-technical structures subject to potential earthquakes significantly depends on their dynamic characteristics and the vibration processes over time. Therefore, the need arises to proceed to the dynamic theory of earthquake resistance.

The intensity of structures vibrations under dynamic influences substantially depends on the degree of energy dissipation in them. It can be expected that the higher the energy dissipation in the structure, the less intense the resonant vibrations at a given level of excitation.

Theoretical description of the processes of strain during vibrations of rigid bodies and structures, taking into account internal friction, is often limited to studying the general laws of the external manifestation of the dissipation mechanism. Hypotheses and linear models of frequency-independent internal friction [1, 2] are widely used to solve the dynamics of structures. These hypotheses, reflecting the manifestation of elastic imperfections in the materials, do not describe the creep of strains and relaxation of stresses, called "hereditary properties".

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The foundations of the modern hereditary theory of viscoelasticity, which reflects almost all the features of the quasistatic and dynamic behavior of the material, are found in classical works of Boltzmann and Volterra. The hereditary theory of viscoelasticity is more general, and it describes more accurately the mechanism of dissipation in the materials [3 - 7].

It is well known that the rheological properties of the medium significantly affect the strain process as a whole, i.e. all three of its stages: elastic, plastic and the stage of destruction. Therefore, the problems of the theory of viscoelasticity have attracted the special attention of researchers in recent years [8 - 32].

This paper is devoted to mathematical modeling and creating an algorithm for the numerical solution of dynamic problems of hereditary-deformable systems.

1.1 Statement of the problem and nonlinear equations of motion

Consider the problem of vibrations of a dam-plate made of a homogeneous viscoelastic isotropic material. The inertia forces acting on the dam-plate are generated from the motion and dam deformation, and the hydrodynamic pressure of water arises from the dam motion as a rigid body and the dam strain.

A mathematical model of the problem, relative to the transverse deflection $w_1 = w_1(x, y, t)$, under certain assumptions, taking into account viscoelastic properties of the dam-plate material, is reduced to solving equations of the form

$$D\left(1-R^*\right)\nabla^4\left[w_1(x,y,t)\right]+\rho_1 h\frac{\partial^2\left(w_1+w_0\right)}{\partial t^2}-\rho\left.\frac{\partial\varphi_1}{\partial t}\right|_{x=0}-\rho\left\{\frac{\partial\varphi_0}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\varphi_0}{\partial z}\right)^2+\left(\frac{\partial\varphi_0}{\partial y}\right)^2\right]\right\}_{x=w_0(t)}=0, \tag{1}$$

where $w_1(x, y, t)$ is the deflection of the dam-plate; h is the thickness of the dam-plate; ρ_1 is the density of the dam material; ρ is the density of water; $\varphi_1(x, y, z, t)$ is the function of the potential of fluid flow rate arising from dam-plate strain; $\varphi_0(x, y, t)$ is the function of the potential fluid flow rate, arising from the dam motion as a rigid body; $w_0(t)$ is the law of base motion during an earthquake:

$$w_0(t)=a_0 e^{-\varepsilon_0 t}\sin\omega_0 t\,; \tag{2}$$

here a_0 is the initial maximum amplitude; ε_0 is the soil attenuation coefficient; ω_0 is frequency of soil vibrations; t is time. All these values are determined from the analysis of the earthquake seismogram of corresponding intensity.

2 Methods

Solution of integro-differential equations (1), satisfying the boundary conditions of the problem, is given in the form

$$w_1(y,z,t)=\sum_{k=1,3,\dots}^{\infty} C_k(t) w_k(y,z), \qquad (3)$$

where $C_k = C_k(t)$ is the sought for time functions; coordinate functions $w_k(y,z)$ satisfy the boundary conditions for fixing the dam-plate edges.

Consider a plate with the following boundary conditions:

1. Edges $z = \pm\alpha$ are freely supported

$$w_1 = 0, \frac{\partial w_1}{\partial y^2} + \mu \frac{\partial w_1}{\partial z^2} = 0$$

2. Edge $y = 0$ is rigidly fixed,

$$w_1 = 0, \frac{\partial w_1}{\partial x} = 0 \qquad (4)$$

3. Edge $y = b$ is free.

$$\frac{\partial w_1}{\partial y^2} + \mu \frac{\partial w_1}{\partial z^2} = 0, \frac{\partial^3 w_1}{\partial y^3} + (2-\mu)\frac{\partial^3 w_1}{\partial z \partial y^2}$$

Functions $w_k(y,z)$ satisfying these boundary conditions of fixation are taken in the form

$$w_k(y,z) = V_k(y) H_k(z),$$

where

$$H_k(z) = \cos\frac{k\pi z}{2a};$$
$$V_k(y) = V_{1k}(y) + E_k V_{2k}(y).$$

Here

$$E_k = \frac{1}{V''_{2k}(b)}\left[V''_{1k}(b) - \left(\frac{k\pi}{2a}\right)^2 V_{1k}(b)\mu\right],$$

$$V_{1k}(y) = \frac{1}{k}\sin\frac{k\pi y}{2b} - \frac{1}{k+2}\sin\frac{(k+2)\pi y}{2b}, V_{2k}(y) = \cos\frac{k\pi y}{2b} - \cos\frac{(k+2)\pi y}{b}. k = 1,3,5,...$$

So, the expression for function $w_k(y,z)$, is determined and has the form

$$w_k(y,z) = \cos\frac{k\pi z}{2a}\left[\frac{1}{R}\sin\frac{k\pi y}{2b} - \frac{1}{k+2}\sin\frac{(k+2)\pi y}{2b} + E_k\left(\cos\frac{k\pi y}{b} - \cos\frac{(k+2)\pi y}{b}\right)\right]. \quad (5)$$

Substituting (5) and (3) into equation (1) and applying the Bubnov-Galerkin procedure to determine the unknowns $C_k = C_k(t), k = 1,3,5,\ldots$, the following system of integro-differential equations is obtained:

$$\sum_{k=1,3,\ldots}^{\infty}\left[L_{mk}\ddot{C}_k(t) + \omega^2\left(1 - R^*\right)M_{mk}C_k(t)\right] + a_0\omega^2 N_m(t) = 0 . \quad (6)$$

$$C_k(0) = C_{0k}, \quad \dot{C}_k(0) = \dot{C}_{0k}, \; k,m = 1,3,5,\ldots$$

Here

$$L_{mk} = \frac{1}{ab}\left\{\int_0^b\int_{-a}^a w_k(y,z)w_m(y,z)dydz + \frac{2}{k\pi}\frac{\rho}{\rho_1}\left(\frac{b}{h}\right)\times \right.$$

$$\left.\frac{\int_0^b\int_{-a}^a (z-a)w_k(y,z)\cos\gamma_k y dydz \int_0^b\int_{-a}^a (z-a)w_m(y,z)\cos\gamma_k y dydz}{\int_0^b\int_{-a}^a (z-a)^2\cos^2\gamma_k y dydz}\right\};$$

$$M_{mk} = \frac{b^3}{a\pi^4}\int_0^b\int_{-a}^a \nabla^4 w_k(y,z)w_m(y,z)dydz;$$

$$N_m(t) = \frac{1}{aba_0\omega^2}\int_0^b\int_{-a}^a\left\{\ddot{w}_0(t) - \frac{\rho}{\rho_1}\frac{1}{h}\left[\frac{\partial\varphi_0}{\partial t} + \frac{1}{2}\left(\frac{\partial\varphi_0}{\partial x}\right)^2 + \right.\right.$$

$$\left.\left. + \frac{1}{2}\left(\frac{\partial\varphi_0}{\partial y}\right)^2\right]_{x=w_0(t)}\right\}w_m(y,z)dydz.$$

where ω is the frequency of the basic mode of vibration: $\omega = \sqrt{\frac{D}{\rho_1 h}\left(\frac{\pi}{b}\right)^4}$

In calculations, the Koltunov–Rzhanitsyn kernel is used:

$$R(t) = At^{\alpha-1}\exp(-\beta t), \qquad A,\beta > 0, \qquad 0 < \alpha < 1.$$

Integration of the system of equations (6), obtained based on numerous approximations of deflections, was performed using a numerical method based on the use of quadrature formulas [4, 6, 7, 21].

3 Results and discussion

In figure 1 the graphs of curves $w\left(\frac{1}{2},\frac{1}{2},t\right)$ for various values of rheological parameter A are presented. An analysis of results shows that in the initial point in time, the solutions of elastic and viscoelastic problems differ little from each other. Over time, the vibrations at $A = 0$ occur closer to the harmonic law, and with increasing A, the amplitude and frequency of vibrations decrease significantly.

The effect of the hydrodynamic pressure of water on the dam behavior was investigated.

Figure 2 shows the graphs of curves $w\left(\frac{1}{2},\frac{1}{2},t\right)$ for various values of the parameter ρ/ρ_1. The results obtained here show that in the initial point in time, the curves almost coincide, and over time they differ significantly from each other. An analysis shows that with an increase in the parameter value ρ/ρ_1, the amplitude of dam-plate vibrations decreases. So, an account for hydrodynamic pressure of water leads to a decrease in the vibration amplitude, and the frequency of vibrations does not change significantly.

Figure 3 shows the graphs of curves $w\left(\frac{1}{2},\frac{1}{2},t\right)$ for various values of the parameter λ. With increasing values of λ, the amplitude of vibrations decreases and a phase shift to the right is observed.

Figure 4 shows the graphs of curves $w\left(\frac{1}{2},\frac{1}{2},t\right)$ for various values of the rheological parameter α. An analysis of results shows that an increase in the value of this parameter leads to an increase in the amplitude and frequency of vibrations.

Figure 5 shows the graphs of curves $w\left(\frac{1}{2},\frac{1}{2},t\right)$ for various values of the rheological parameter β. An analysis of results shows that taking parameter β into account does not significantly affect the amplitude and frequency of vibrations of a dam-plate.

When calculating the deflection value by formula (1), the first five harmonics were held (N=5). The calculations showed that a further increase in term number does not significantly affect the amplitude of dam-plate vibrations (figures 6 and 7).

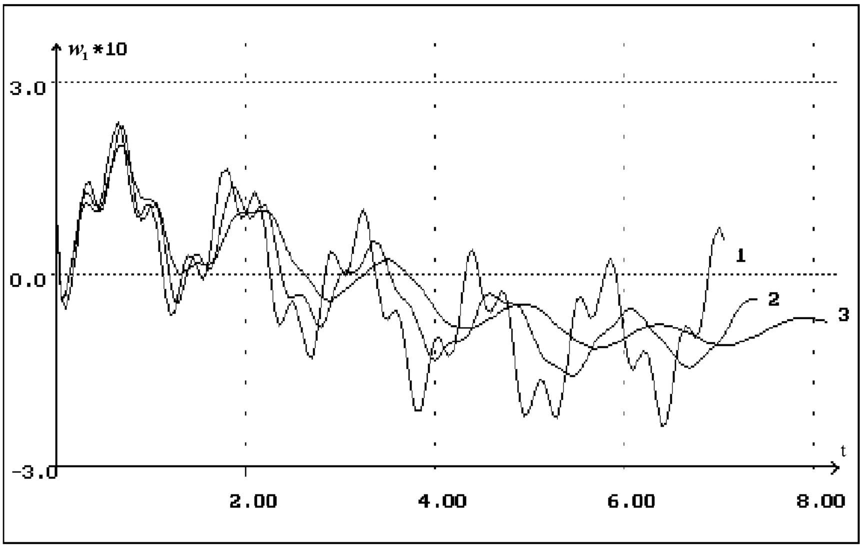

Fig 1. $\alpha = 0.25$; $\beta = 0.05$; $\lambda = 1$; $\mu = 0.3$; $\rho / \rho_1 = 1/2.4$; $A = 0(1)$; $0{,}05(2)$; $0{,}1(3)$.

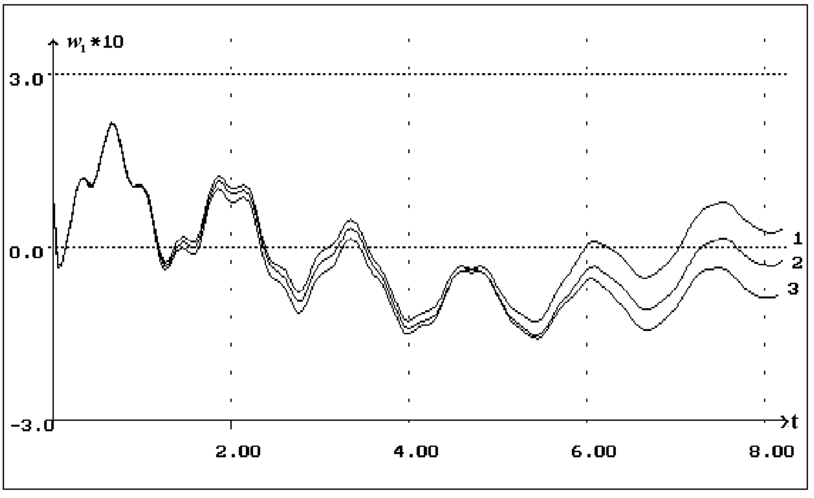

Fig 2. $A = 0.05$; $\alpha = 0.25$; $\beta = 0.05$; $\lambda = 1$; $\mu = 0.3$; $\rho / \rho_1 = 0(1)$; $1/5(2)$; $1/2.4(3)$.

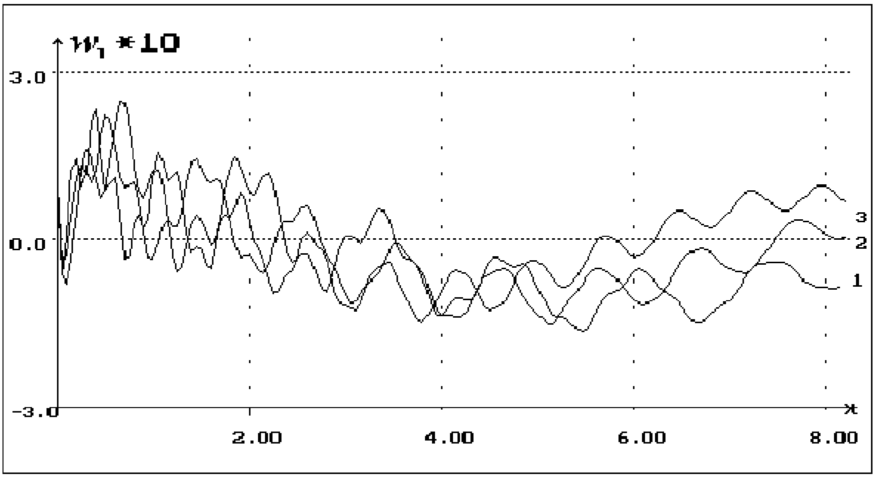

Fig 3. $A = 0.05;\ \alpha = 0.25;\ \beta = 0.05; \mu = 0.3;\ \rho / \rho_1 = 1/2.4;\ \lambda = 1(1);\ 1/5(2);\ 2(3)$.

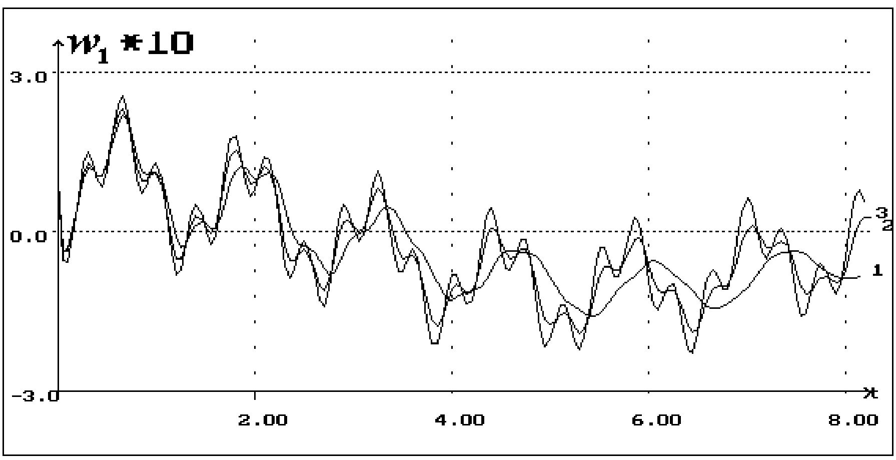

Fig 4. $A = 0.05;\ \beta = 0.05; \lambda = 1;\ \mu = 0.3;\ \rho / \rho = 1/2.4;\ \alpha = 0.25(1);\ 0.5(2);\ 0.75(3)$.

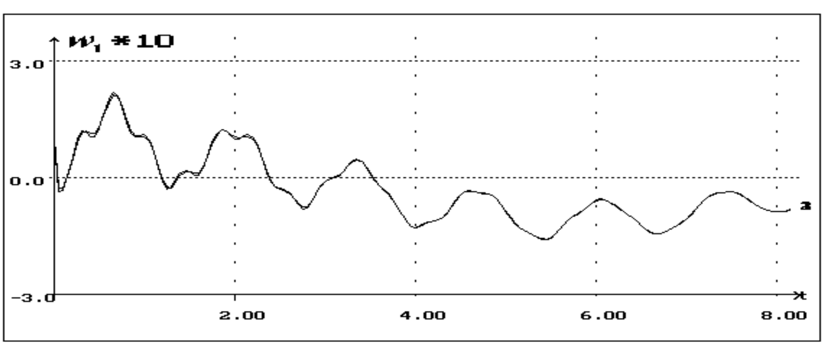

Fig 5. $A = 0.05;\ \alpha = 0.25;\ \lambda = 1;\ \mu = 0.3;\ \rho / \rho = 1/2.4;\ \beta = 0.05(1);\ 0.075(2);\ 0.1(3)$.

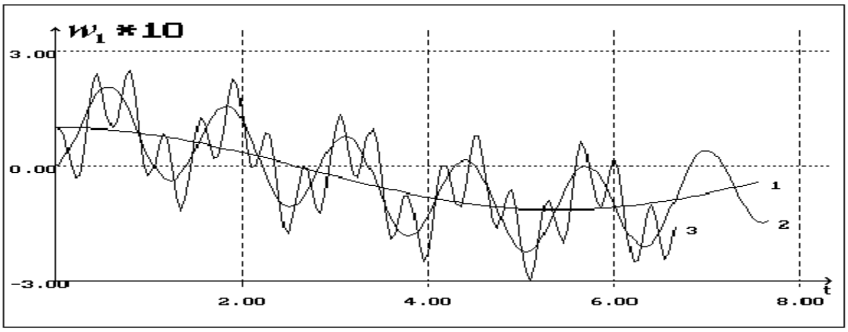

Fig 6. $A = 0.05;\ \alpha = 0.25;\ \lambda = 1;\ \mu = 0.3;\ \rho / \rho_1 = 1/2.4;\ \beta = 0.05;\ \mathrm{N} = 1(1);\ 3(2);\ 5(3)$.

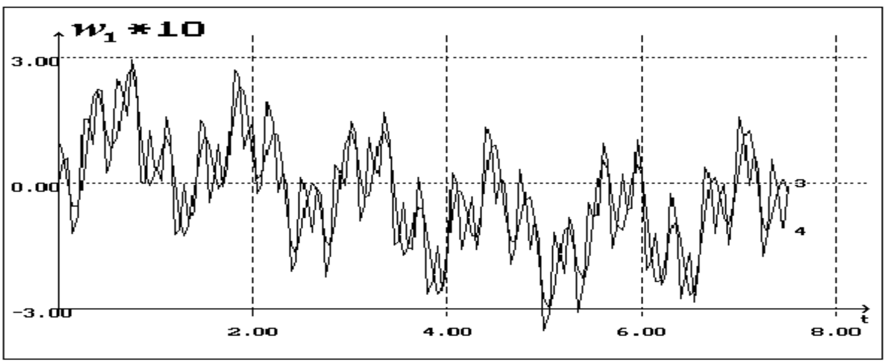

Fig 7. $A = 0.05;\ \alpha = 0.25;\ \lambda = 1;\ \mu = 0.3;\ \rho / \rho_1 = 1/2.4;\ \beta = 0.05;\ \mathrm{N} = 5(1);\ 7(2)$.

4 Conclusions

Mathematical models of the dynamics problems of a dam-plate of constant and variable thickness are constructed taking into account:

- viscoelastic properties of the material;
- inertia forces arising from the dam-plate motion as a rigid body and from its deformation.

Based on the Bubnov-Galerkin method in combination with a numerical method based on the use of quadrature formulas:

- the methods have been developed for solving interconnected integro-differential equations of Volterra type;
- a computational algorithm has been developed that allows studying the vibration problems of a dam-plate of constant thickness, taking into account the viscoelastic properties of the material.

It is established that an account for viscoelastic properties of the dam-plate material leads to a decrease in the amplitudes and frequencies of vibrations.

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