Joint selection of the MCS's and power allocation coefficients in the two-user downlink PD-NOMA system

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Abstract. Power Domain Non-Orthogonal Multiple Access (PD-NOMA) is a perspective multiplexing technique for future cellular networks. Nevertheless, it is poorly studied and not applied in the existing systems due to the complexity of PD-NOMA signal processing, resource scheduling, and power allocation. The issue is that a modulation and coding scheme (MCS) selection, including power allocation, is a cooperative procedure considering the channel state information of each multiplexed user. It can be solved by enumerating all possible multiplexing combinations but at the expense of the high computational complexity. In our work, we propose a composed table with the joint MCS's, which can be selected by the base station (BS) for the user multiplexing in a downlink PD-NOMA system based on their signal-to-noise (SNR) ratios. It allows selecting two MCS's with two power allocation coefficients for both users and guarantees the 10% block error rate (BLER) performance in the additive white Gaussian noise (AWGN) channel. The joint MCS selection method is based on a max-rate scheduling strategy and provides system capacity maximization ignoring fairness between users. The proposed table is given in the Appendix.

1 Introduction

PD-NOMA is one of the perspective techniques for user multiplexing in future networks. It has been shown [1–5] that PD-NOMA allows increasing the system capacity in the multiuser communication systems such as cellular (3GPP long term evolution, LTE) and wireless local area networks (IEEE 802.11). The basic PD-NOMA idea is the non-orthogonal multiplexing of several users in a power domain of the same time-frequency resource segment by the superposition of their signals with different power weights. Thus, there is a controlled co-channel interference that is cancelled at the receiver side by the successive interference cancellation (SIC) method [6]. The issue is that user signals are dependent on each other, and it should be considered during resource scheduling, cooperative MCS selection, and multiplexing.

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To achieve the maximum system capacity, it is necessary to use the MCS that has the highest spectral efficiency (SE) and guarantees the required error rate performance within the given channel conditions. Therefore, adaptive modulation and coding (AMC) has been proposed [7,8]. It is a simple approach that selects the most efficient MCS according to the propagation channel quality. The AMC is widely used in actual wireless networks with orthogonal multiple access (OMA). Thus, in conventional OMA systems, MCS's for different users are independently selected because they are orthogonal to each other. Unlike OMA, in PD-NOMA systems, the users are non-orthogonal in the power domain, and it is a reason for the joint MCS selection. Besides, the power allocation coefficients should be calculated by taking into account the channel quality of each user. It can be carried out by enumerating all possible multiplexing configurations and selecting the most effective of them (joint MCS's and power allocation coefficients) at the expense of the high computational complexity.

Many researchers have developed power allocation schemes for PD-NOMA within the conditions of the Shannon system model, considering the infinite number of MCS levels. However, during the literature searching, we have found the only paper devoted to the joint MCS selection in PD-NOMA systems using finite number of MCS levels from the practical system and considering the efficiency of used error-correcting methods. In [9], the authors propose their method of a joint power allocation and MCS selection considering max-min user fairness and determining the power allocation coefficient according to the MCS level of the user with a poor channel. It allocates the maximum power shared to the user with better channel conditions while guaranteeing the decoding of the second user. However, their algorithm is iterative, so it has high computational complexity.

In our work, we propose a table that makes it possible to select the pair of MCS levels depending on the user SNR values considering the max C/I scheduling strategy. It maximizes the system throughput instantly by selecting two MCS's that have the highest sum SE. This approach has a very low computing complexity and may be a temporary solution for the AMC in a two-user PD-NOMA system until an optimal approach is found.

2 Downlink PD-NOMA system

We are using the PD-NOMA system model with multiplexing of two users in the power domain. According to its principles, the group signal is a superposition of two user signals with different power weights. The user with a weak propagation channel gets the largest power share compared to the user with a strong propagation channel. We sorted users in the ascending order of their channel quality and identified the weak user (UE₁) with power allocation coefficient p_1 and the strong user (UE₂) with p_2 ; therefore, $p_1 > p_2$ and $p_1 + p_2 = 1$. The downlink group signal $\mathbf{s}$ at the BS output is given by

$$\mathbf{s} = \sqrt{p_1} \mathbf{x} + \sqrt{p_2} \mathbf{y}, \quad (1)$$

where $\mathbf{x} \in \mathbf{A}_1$ and $\mathbf{y} \in \mathbf{A}_2$ are the symbols of the joint MCS's with the $\mathbf{A}_1$, $\mathbf{A}_2$ alphabets.

The signal $\mathbf{s}$ is transmitted via two independent propagation channels to UE₁ and UE₂ and the received signal for $k \in 1, 2$ user is expressed as

$$\mathbf{z}_k = \alpha_k \cdot \mathbf{s} + \mathbf{n}_k, \quad (2)$$

where $\mathbf{n}_k \sim CN(0, N_0)$ is a component of the complex white Gaussian noise; and α_k is a channel propagation coefficient of the k^{th} user. We denoted above that UE₁ possesses a weak propagation

channel; therefore, $|\alpha_1| \leq |\alpha_2|$. Let us denote W_k as SNR of the k^{th} user. Considering $E\{|\mathbf{s}|^2\} = 1$ (where $E\{\cdot\}$ – expected value), SNR values relatively received group signal power for two users can be obtained by $W_k = |\alpha_k|^2 / N_0$.

The decoding of a PD-NOMA signal is employed by the SIC. Thus, the symbols $\mathbf{x}$ owned by UE_1 are decoded firstly using the minimum Euclidean distance criterion and treating $\mathbf{y}$ as an interference. It can be expressed as

$$\mathbf{x} = \arg \min_{\mathbf{x} \in \mathbf{A}_1} \left| \mathbf{z}_1 - \sqrt{p_1} \mathbf{x} \right|^2 \quad (3).$$

The symbols $\mathbf{y}$ owned by UE_2 are decoded using the SIC by these three steps: first, obtain $\mathbf{x}$ same as UE_1 by (3); secondly, cancel $\mathbf{x}$ from the received PD-NOMA signal by $\mathbf{z}_2^* = \mathbf{z}_2 - \sqrt{p_1} \mathbf{x}$; finally, decode $\mathbf{y}$ from the updated signal $\mathbf{z}_2^*$.

$$\mathbf{y} = \arg \min_{\mathbf{y} \in \mathbf{A}_2} \left| \mathbf{z}_2^* - \sqrt{p_2} \mathbf{y} \right|^2 \quad (4).$$

3 Proposed joint MCS Table

In this section, we have described the joint MCS table design process. Our goal is to search for the most efficient joint MCS combination, which guarantees 10% BLER performance and provides the maximization of the system capacity, considering the user's SNR values. Our approach to the search includes the enumeration of all possible combinations for each joint user's SNR values and their verification with the help of a simulation. The combinations that do not guarantee 10% BLER are eliminated from a valid range of searches. Then, one with the most SE is selected from the remaining combinations.

3.1 Used MCS levels

A downlink system capacity significantly depends on a user non-orthogonal multiplexing technique at the BS side, which includes the joint MCS selection and power allocation between users. Commonly, all possible MCS's are given in a special table in a technical specification. In our system model, we use a bank of the finite number of MCS levels given by [10]. Their main characteristics are presented in Table 1. **MCS Table in LTE Downlink..** Each MCS level is indexed by m and based on a square Q -QAM and Turbo encoding with the code rate R . The SNR threshold with 10% BLER is used for switching m together with changing the channel state. It allows increasing the SE of transmission due to improving the channel quality.

3.2 Simulation model

The structure of the simulation model is shown in Fig. 1. **The structure of the simulation model** Random bits $\mathbf{b}_1, \mathbf{b}_2$ are encoded and modulated according to the MCS indices m_1, m_2 . Next, modulation symbols $\mathbf{x}, \mathbf{y}$ are weighed by the power allocation coefficients $\sqrt{p_1}, \sqrt{1-p_1}$, and the group PD-NOMA signal $\mathbf{s}$ is obtained by (1). Thereafter, it

transmits via two AWGN channels by (2) with SNR values W_1, W_2 . UE_1 receives z_1 and decodes their signal directly by (3) under the channel conditions with the SNR value equal to W_1 ; the onward obtains the received bits b_1 . In their turn, UE_2 receives z_2 with the SNR value equal to W_2 , then cancels x by the SIC, and decodes their signal receiving b_2 by (4). Next, BLER values for both users are calculated.

Table 1. MCS Table in LTE Downlink.

m	Q	R^{x1024}	SE (bits/s/Hz)	SNR threshold, dB
1	4	78	0.1523	-9.478
2	4	120	0.2344	-6.658
3	4	193	0.3770	-4.098
4	4	308	0.6010	-1.798
5	4	449	0.8770	0.399
6	4	602	1.1758	2.424
7	16	378	1.4766	4.489
8	16	490	1.9141	6.367
9	16	616	2.4063	8.456
10	64	466	2.7305	10.266
11	64	567	3.3223	12.218
12	64	666	3.9023	14.122
13	64	772	4.5234	15.849
14	64	873	5.1152	17.786
15	64	948	5.5547	19.809

In the simulation, we change W_1, W_2 from -10 to 40 dB with 1 dB step always keeping $W_1 \leq W_2$. For each combination W_1, W_2 , we enumerate all possible combinations of MCS levels with the indices $m_1, m_2 \in 1...15$ (taken from Table 1. **MCS Table in LTE Downlink.**) and all possible power allocation coefficients controlled by p_1 in the range from 0.5 to 1 with the 0.05 step. Next, we initiate the simulation of a data transmission process and estimate a BLER performance. Further, the m_1, m_2, p_1 combination that guarantees 10% BLER performance and maximizes the system capacity is selected and added in the table for the SNR thresholds W_1, W_2 . In case the identical sum SE is given by several combinations, the priority is given to the weak user, and the combination that simultaneously maximizes throughput and the SE of UE_1 is added in the table.

Fig. 1. The structure of the simulation model.

3.3 Simulation result

The designed table is given in the Appendix. It consists of 628 various combinations written as $p_1[m_1, m_2]$ with SNR thresholds W_1, W_2 given in dB. It means that UE₁ gets the MCS index m_1 with the power allocation coefficient p_1 , and UE₂ gets m_2 with $p_2 = 1 - p_1$. Therefore, the group PD-NOMA signal is obtained by the superposition of user signals by (1) as $\mathbf{s} = \sqrt{p_1}\mathbf{x} + \sqrt{1 - p_1}\mathbf{y}$, where $\mathbf{x} \in \mathbf{A}_{m_1}$ and $\mathbf{y} \in \mathbf{A}_{m_2}$. For example, 0.950[3,8] means that UE₁ and UE₂ get the 3rd and 9th MCS's from Table 1. **MCS Table in LTE Downlink**, and the group PD-NOMA signal is a superposition of the signals using power allocation coefficients $p_1 = 0.95$ and $p_2 = 0.05$, respectively.

There are 32 possible values for p_1 , so only five additional bits are required to transfer the control information about the power shared between the users.

3.4 Joint MCS's selection

Exactly as for the AMC in the OMA, in the PD-NOMA, the BS should select the MCS combination with the highest sum SE for the given users' SNR values, considering the thresholds W_1, W_2 . These methods are well described in [10]. For example, if UE₁ and UE₂ possess $SNR_1 = 10$ dB and $SNR_2 = 25$ dB, the BS selects from table the combination 0.810[6,14] with the SE equal to 6.291 bits/s/Hz.

There are no joint MCS's that guarantee the required BLER performance within the channel conditions $SNR_1 < -9$ dB and $SNR_2 < 5$ dB. It can be solved with the help of the MCS table (Table 1. **MCS Table in LTE Downlink**.) extension by adding MCS with a higher BLER performance. Within the conditions $SNR_1 > 38$ dB and $SNR_2 > 39$ dB, the BS selects the most efficient MCS's for both users.

4 Conclusion

The paper proposes the table of joint MCS's and power allocation coefficients for the downlink two-user PD-NOMA system. The finite number of MCS levels used is taken from the LTE specification and presented in Table 1. **MCS Table in LTE Downlink**. The table allows a scheduler to merely pick the MCS for UE1 and UE2 and multiplex them by a power domain based on their SNR values. Hence, the complexity of user PD-NOMA multiplexing by the proposed scheme is well reduced. All multiplexing combinations guarantee 10% BLER performance at the SNR and satisfy the necessary thresholds. Also, the number of power share states is equal to 32, and only five redundancy bits are required to transfer it.

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Appendix

Table 2. Proposed table of the joint MCS’s and power allocation coefficients for PD-NOMA system.

$W_1 = -9$	W_2	$W_1 = -8$	W_2	$W_1 = -7$	W_2	$W_1 = -6$	W_2
0.925[1,1]	5	0.865[1,1]	4	0.680[1,1]	3	0.530[1,1]	2
0.940[1,2]	7	0.805[1,2]	5	0.805[1,2]	5	0.925[2,1]	5
0.920[1,3]	8	0.875[1,3]	7	0.875[1,3]	7	0.900[2,2]	6
0.925[1,4]	10	0.835[1,4]	8	0.835[1,4]	8	0.875[1,3]	7
0.925[1,5]	12	0.870[1,5]	10	0.870[1,5]	10	0.835[1,4]	8
0.910[1,6]	13	0.830[1,6]	11	0.830[1,6]	11	0.895[2,4]	9
0.910[1,7]	15	0.785[1,7]	12	0.785[1,7]	12	0.865[1,5]	10
0.915[1,8]	17	0.815[1,8]	14	0.815[1,8]	14	0.830[1,6]	11
0.910[1,9]	19	0.820[1,9]	16	0.820[1,9]	16	0.785[1,7]	12
0.915[1,10]	21	0.770[1,10]	17	0.770[1,10]	17	0.815[1,8]	14
0.915[1,11]	23	0.780[1,11]	19	0.780[1,11]	19	0.820[1,9]	16
0.915[1,12]	25	0.790[1,12]	21	0.790[1,12]	21	0.770[1,10]	17
0.920[1,13]	27	0.805[1,13]	23	0.805[1,13]	23	0.780[1,11]	19
0.920[1,14]	29	0.810[1,14]	25	0.810[1,14]	25	0.790[1,12]	21
0.920[1,15]	31	0.750[1,15]	26	0.750[1,15]	26	0.805[1,13]	23
$W_1 = -5$	W_2	$W_1 = -4$	W_2	$W_1 = -3$	W_2	0.805[1,14]	25
0.530[1,1]	2	0.530[1,1]	2	0.530[1,1]	2	0.750[1,15]	26
0.860[2,1]	4	0.675[2,1]	3	0.680[2,1]	3	0.900[2,15]	30
0.805[2,2]	5	0.805[2,2]	5	0.860[3,1]	4	$W_1 = -2$	W_2
0.875[2,3]	7	0.875[2,3]	7	0.900[3,2]	6	0.530[2,1]	2
0.835[2,4]	8	0.835[2,4]	8	0.875[3,3]	7	0.865[3,1]	4
0.870[2,5]	10	0.870[2,5]	10	0.835[2,4]	8	0.805[3,2]	5
0.830[2,6]	11	0.830[2,6]	11	0.895[3,4]	9	0.870[3,3]	7
0.785[2,7]	12	0.785[2,7]	12	0.870[3,5]	10	0.830[3,4]	8
0.815[2,8]	14	0.815[2,8]	14	0.830[2,6]	11	0.870[3,5]	10
0.820[2,9]	16	0.820[2,9]	16	0.785[2,7]	12	0.830[3,6]	11
0.770[2,10]	17	0.770[2,10]	17	0.850[3,7]	13	0.790[3,7]	12
0.785[2,11]	19	0.780[2,11]	19	0.815[2,8]	14	0.815[3,8]	14
0.790[2,12]	21	0.790[2,12]	21	0.860[3,8]	15	0.820[3,9]	16
0.805[2,13]	23	0.805[2,13]	23	0.820[2,9]	16	0.770[3,10]	17
0.805[2,14]	25	0.805[2,14]	25	0.770[2,10]	17	0.780[3,11]	19
0.750[1,15]	26	0.750[2,15]	26	0.780[2,11]	19	0.790[3,12]	21
0.805[2,15]	27	0.985[3,15]	39	0.790[2,12]	21	0.805[3,13]	23

$W_1 = -1$	W_2	0.765[2,15]	26	0.805[2,13]	23	0.805[3,14]	25
0.530[2,1]	2	0.985[3,15]	40	0.845[3,13]	24	0.750[3,15]	26
0.680[3,1]	3	$W_1 = 0$	W_2	0.805[2,14]	25	$W_1 = 2$	W_2
0.925[4,1]	5	0.530[2,1]	2	0.750[2,15]	26	0.535[3,1]	2
0.940[4,2]	7	0.675[3,1]	3	0.845[3,15]	28	0.680[4,1]	3
0.920[4,3]	8	0.860[4,1]	4	$W_1 = 1$	W_2	0.865[5,1]	4
0.870[3,5]	10	0.805[4,2]	5	0.530[2,1]	2	0.900[5,2]	6
0.830[3,6]	11	0.875[4,3]	7	0.685[3,1]	3	0.875[5,3]	7
0.785[3,7]	12	0.835[4,4]	8	0.860[4,1]	4	0.895[5,4]	9
0.815[3,8]	14	0.870[4,5]	10	0.805[4,2]	5	0.870[5,5]	10
0.820[3,9]	16	0.830[4,6]	11	0.950[5,1]	6	0.830[4,6]	11
0.770[3,10]	17	0.785[3,7]	12	0.940[5,2]	7	0.785[4,7]	12
0.785[3,11]	19	0.850[4,7]	13	0.835[4,4]	8	0.815[4,8]	14
0.790[3,12]	21	0.815[4,8]	14	0.940[5,3]	9	0.860[5,8]	15
0.805[3,13]	23	0.820[4,9]	16	0.870[4,5]	10	0.820[4,9]	16
0.805[3,14]	25	0.770[3,10]	17	0.830[4,6]	11	0.770[4,10]	17
0.750[2,15]	26	0.830[4,10]	18	0.785[4,7]	12	0.780[4,11]	19
0.805[3,15]	27	0.785[3,11]	19	0.815[4,8]	14	0.790[4,12]	21
0.920[4,15]	31	0.830[4,11]	20	0.820[4,9]	16	0.805[4,13]	23
$W_1 = 3$	W_2	0.790[3,12]	21	0.770[4,10]	17	0.805[4,14]	25
0.675[4,1]	3	0.835[4,12]	22	0.785[4,11]	19	0.750[4,15]	26
0.865[5,1]	4	0.805[4,13]	23	0.790[4,12]	21	0.875[5,15]	29
0.805[5,2]	5	0.810[4,14]	25	0.805[4,13]	23	$W_1 = 6$	W_2
0.965[6,1]	7	0.750[3,15]	26	0.805[4,14]	25	0.900[6,2]	6
0.835[5,4]	8	0.805[4,15]	27	0.750[3,15]	26	0.875[6,3]	7
0.870[5,5]	10	$W_1 = 4$	W_2	0.805[4,15]	27	0.830[6,4]	8
0.830[5,6]	11	0.865[5,1]	4	0.950[5,15]	33	0.870[6,5]	10
0.785[4,7]	12	0.925[6,1]	5	$W_1 = 5$	W_2	0.830[6,6]	11
0.850[5,7]	13	0.900[6,2]	6	0.925[6,1]	5	0.785[5,7]	12
0.815[5,8]	14	0.920[6,3]	8	0.900[6,2]	6	0.850[6,7]	13
0.820[5,9]	16	0.895[6,4]	9	0.875[6,3]	7	0.815[6,8]	14
0.770[4,10]	17	0.900[6,5]	11	0.895[6,4]	9	0.820[6,9]	16
0.830[5,10]	18	0.785[5,7]	12	0.870[6,5]	10	0.770[5,10]	17
0.780[4,11]	19	0.815[5,8]	14	0.785[5,7]	12	0.830[6,10]	18
0.830[5,11]	20	0.820[5,9]	16	0.850[6,7]	13	0.780[5,11]	19
0.790[4,12]	21	0.770[5,10]	17	0.815[5,8]	14	0.830[6,11]	20
0.835[5,12]	22	0.780[5,11]	19	0.860[6,8]	15	0.790[5,12]	21
0.805[5,13]	23	0.790[5,12]	21	0.820[5,9]	16	0.835[6,12]	22
0.810[5,14]	25	0.805[5,13]	23	0.770[5,10]	17	0.805[6,13]	23
0.750[4,15]	26	0.805[5,14]	25	0.780[5,11]	19	0.805[6,14]	25
0.805[5,15]	27	0.750[5,15]	26	0.790[5,12]	21	0.805[6,15]	27
0.950[6,15]	34	0.900[6,15]	30	0.805[5,13]	23	0.950[7,15]	33
$W_1 = 7$	W_2	$W_1 = 8$	W_2	0.845[6,13]	24	$W_1 = 10$	W_2
0.875[6,6]	7	0.835[6,4]	8	0.810[5,14]	25	0.870[6,5]	10
0.835[6,4]	8	0.870[6,5]	10	0.750[5,15]	26	0.830[6,6]	11
0.870[6,5]	10	0.830[6,6]	11	0.845 [6,15]	28	0.785[6,7]	12
0.830[6,6]	11	0.785[6,7]	12	0.985 [7,15]	39	0.815[6,8]	14
0.785[6,7]	12	0.815[6,8]	14	$W_1 = 9$	W_2	0.820[6,9]	16
0.815[6,8]	14	0.820[6,9]	16	0.895 [6,4]	9	0.770[6,10]	17
0.820[6,9]	16	0.770[6,10]	17	0.870 [6,5]	10	0.785[6,11]	19
0.770[5,10]	17	0.780[6,11]	19	0.830 [6,6]	11	0.790[6,12]	21
0.830[6,10]	18	0.790[6,12]	21	0.785 [6,7]	12	0.805[6,13]	23

0.785 [6,11]	19	0.805[6,13]	23	0.815 [6,8]	14	0.810[6,14]	25
0.790[6,12]	21	0.805[6,14]	25	0.820 [6,9]	16	0.750[6,15]	26
0.805[6,13]	23	0.750[5,15]	26	0.770 [6,10]	17	0.950[8,14]	31
0.805[6,14]	25	0.805[6,15]	27	0.780 [6,11]	19	0.965[9,14]	33
0.750[5,15]	26	0.950[8,14]	31	0.790 [6,12]	21	0.965[9,15]	35
0.805[6,15]	27	0.950[8,15]	33	0.805 [6,13]	23	$W_1 = 14$	W_2
0.950[7,15]	33	$W_1 = 12$	W_2	0.805 [6,14]	25	0.815[6,8]	14
0.965[8,15]	36	0.785[6,7]	12	0.750 [6,15]	26	0.820[6,9]	16
$W_1 = 11$	W_2	0.815[6,8]	14	0.950 [8,14]	31	0.770[6,10]	17
0.830[6,6]	11	0.820[6,9]	16	0.950 [8,15]	33	0.780[6,11]	19
0.785[6,7]	12	0.770[6,10]	17	0.990[9,14]	38	0.790[6,12]	21
0.815[6,8]	14	0.780[6,11]	19	0.990[9,15]	40	0.805[6,13]	23
0.820[6,9]	16	0.790[6,12]	21	$W_1 = 13$	W_2	0.805[6,14]	25
0.770[6,10]	17	0.805[6,13]	23	0.850[6,7]	13	0.750[6,15]	26
0.780[6,11]	19	0.805[6,14]	25	0.815[6,8]	14	0.950[9,13]	29
0.790[6,12]	21	0.750[6,15]	26	0.820[6,9]	16	0.950[9,14]	31
0.805[6,13]	23	0.950[9,13]	29	0.770[6,10]	17	0.950[9,15]	33
0.805[6,14]	25	0.950[9,14]	31	0.780[6,11]	19	0.985[11,14]	37
0.750[6,15]	26	0.950[9,15]	33	0.790[6,12]	21	0.985[11,15]	39
0.950[9,13]	29	0.985[10,15]	39	0.805[6,13]	23	$W_1 = 18$	W_2
0.950[9,14]	31	$W_1 = 16$	W_2	0.805[6,14]	25	0.990[12,1]	18
0.950[9,15]	33	0.820[6,9]	16	0.750[6,15]	26	0.995[13,1]	19
0.990[10,15]	40	0.770[6,10]	17	0.950[9,13]	29	0.995[13,2]	20
$W_1 = 15$	W_2	0.990[12,1]	18	0.950[9,14]	31	0.790[6,12]	21
0.860[6,8]	15	0.780[6,11]	19	0.950[9,15]	33	0.805[6,13]	23
0.820[6,9]	16	0.790[6,12]	21	0.985[10,15]	39	0.805[6,14]	25
0.770[6,10]	17	0.805[6,13]	23	$W_1 = 17$	W_2	0.750[6,15]	26
0.780[6,11]	19	0.805[6,14]	25	0.770[6,10]	17	0.950[9,13]	29
0.790[6,12]	21	0.750[6,15]	26	0.990[12,1]	18	0.950[9,14]	31
0.805[6,13]	23	0.950[9,13]	29	0.995[13,1]	19	0.950[9,15]	33
0.805[6,14]	25	0.950[9,14]	31	0.995[13,2]	20	0.985[12,13]	35
0.750[6,15]	26	0.950[9,15]	33	0.790[6,12]	21	0.985[12,14]	37
0.950[9,13]	29	0.990[12,13]	36	0.805[6,13]	23	0.985[12,15]	39
0.950[9,14]	31	0.985[11,14]	37	0.805[6,14]	25	$W_1 = 22$	W_2
0.950[9,15]	33	0.990[12,14]	38	0.750[5,15]	26	0.995[15,3]	22
0.985[11,14]	37	0.990[12,15]	40	0.805[6,15]	27	0.995[15,4]	23
0.985[11,15]	39	$W_1 = 20$	W_2	0.950[9,13]	29	0.995[15,5]	24
$W_1 = 19$	W_2	0.995[14,2]	20	0.950[9,14]	31	0.995[15,6]	26
0.990[13,2]	19	0.995[14,3]	21	0.950[9,15]	33	0.990[14,8]	27
0.990[13,3]	20	0.995[14,4]	23	0.985[12,13]	35	0.990[14,9]	29
0.790[6,12]	21	0.995[14,5]	24	0.985[12,14]	37	0.990[14,10]	31
0.990[13,4]	22	0.810[6,14]	25	0.985[12,15]	39	0.990[14,11]	33
0.805[6,13]	23	0.750[6,15]	26	$W_1 = 21$	W_2	0.985[13,13]	35
0.805[6,14]	25	0.990[13,9]	29	0.995[14,3]	21	0.990[14,13]	36
0.750[6,15]	26	0.995[14,8]	30	0.995[14,4]	23	0.990[14,14]	38
0.990[13,9]	29	0.950[9,14]	31	0.995[14,5]	24	0.990[14,15]	40
0.950[9,14]	31	0.950[9,15]	33	0.805[6,14]	25	$W_1 = 26$	W_2
0.950[9,15]	33	0.990[13,12]	35	0.750[6,15]	26	0.990[15,7]	26
0.990[13,12]	35	0.990[13,13]	36	0.985[13,9]	28	0.990[15,8]	27
0.990[13,13]	36	0.990[13,14]	38	0.985[13,10]	30	0.985[14,9]	28
0.990[13,14]	38	0.990[13,15]	40	0.950[9,14]	31	0.990[15,9]	29
$W_1 = 23$	W_2	$W_1 = 24$	W_2	0.985[13,11]	32	0.990[15,10]	31

0.995[15,4]	23	0.990[15,6]	24	0.985[13,12]	33	0.985[14,11]	32
0.995[15,5]	24	0.990[14,8]	27	0.985[13,13]	35	0.985[14,12]	33
0.995[15,6]	26	0.995[15,7]	28	0.985[13,14]	37	0.985[14,13]	35
0.990[14,8]	27	0.990[14,9]	29	0.985[13,15]	39	0.990[15,13]	36
0.995[15,7]	28	0.990[14,10]	31	$W_1 = 25$	W_2	0.985[14,14]	37
0.990[14,9]	29	0.995[15,9]	32	0.990[15,6]	25	0.990[15,14]	38
0.990[14,10]	31	0.990[14,11]	33	0.990[14,8]	27	0.990[15,15]	40
0.995[15,9]	32	0.985[13,13]	35	0.985[14,9]	28	$W_1 = 30$	W_2
0.990[14,11]	33	0.990[14,13]	36	0.990[14,10]	31	0.990[15,9]	30
0.985[13,13]	35	0.990[14,14]	38	0.995[15,9]	32	0.990[15,10]	31
0.990[14,13]	36	0.990[14,15]	40	0.990[14,11]	33	0.985[14,11]	32
0.990[14,14]	38	$W_1 = 28$	W_2	0.985[13,13]	35	0.985[14,12]	33
0.990[14,15]	40	0.985[14,9]	28	0.990[14,13]	36	0.985[14,13]	35
$W_1 = 27$	W_2	0.990[15,9]	29	0.990[14,14]	38	0.990[15,13]	36
0.990[15,8]	27	0.990[15,10]	31	0.990[14,15]	40	0.985[14,14]	37
0.985[14,9]	28	0.985[14,11]	32	$W_1 = 29$	W_2	0.990[15,14]	38
0.990[15,9]	29	0.985[14,12]	33	0.990[15,9]	29	0.990[15,15]	40
0.990[15,10]	31	0.985[14,13]	35	0.990[15,10]	31	$W_1 = 34$	W_2
0.985[14,11]	32	0.990[15,13]	36	0.985[14,11]	32	0.985[15,12]	34
0.985[14,12]	33	0.985[14,14]	37	0.985[14,12]	33	0.985[15,13]	35
0.985[14,13]	35	0.990[15,14]	38	0.985[14,13]	35	0.985[15,14]	37
0.985[14,14]	37	0.990[15,15]	40	0.990[15,13]	36	0.985[15,15]	39
0.990[15,14]	38	$W_1 = 32$	W_2	0.985[14,14]	37	$W_1 = 36$	W_2
0.990[15,15]	40	0.985[15,11]	32	0.990[15,14]	38	0.990[15,13]	36
$W_1 = 31$	W_2	0.985[15,12]	33	0.990[15,15]	40	0.985[15,14]	37
0.990[15,10]	31	0.985[15,13]	35	$W_1 = 33$	W_2	0.985[15,15]	39
0.985[14,11]	32	0.985[15,14]	37	0.985[15,12]	33	$W_1 = 37$	W_2
0.985[14,12]	33	0.985[15,15]	39	0.985[15,13]	35	0.985[15,14]	37
0.985[14,13]	35	$W_1 = 35$	W_2	0.985[15,14]	37	0.985[15,15]	39
0.990[15,13]	36	0.985[15,13]	35	0.985[15,15]	39	$W_1 = 38$	W_2
0.985[14,14]	37	0.985[15,14]	37			0.990[15,14]	38
0.990[15,14]	38	0.985[15,15]	39			0.985[15,15]	39
0.990[15,15]	40						