# On a Method in Dynamic Elasticity Problems for Heterogeneous Wedge-Shaped Medium

*Vyacheslav* Berkovich[1,*] and *Victor* Poltinnikov[2]

[1]Don Cossacks' State Institute of Food Industry and Business (brunch) «MGUTU» named after K.G.Razumovsky (First Cossacks' University), 55, Semashko, 344007 Rostov-on-Don, Russia
[2] Don State Technical University, 1 Gagarin sq., 344010 Rostov-on-Don, Russia

**Abstract.** The method of analysis of steady oscillations arising in the piecewise homogeneous wedge-shaped medium composed by two homogeneous elastic wedges with different mechanical and geometric characteristics is presented. Method is based on the distributions' integral transform technique and allows reconstructing the wave field in the whole medium by displacement oscillations given in the domain on the boundary of the medium. The problem in question is reduced to a boundary integral equation (BIA). Solvability problems of the BIA are examined and the structure of its solution is established.

## 1 Introduction

The aim of the present paper is mathematical modeling the dynamics of a massive body of composite material under harmonic oscillations. The investigation of the stressed and deformable state for such a body is of great interest for theoretical and practical analysis of strength of materials and reliability problems of technical constructions under long exploitation both in hard industry enterprises and agricultural machinery ones. In part, the problem in question appears when analyzing construction elements by nondestructive testing as well. Problems enumerated have been investigated by number of authors [1-8] at al. Analogous problems arise in seismic prospects when analyzing the wave propagation in the skew-layered medium near the earth crust surface. The problems mentioned above are reduced to mixed dynamic boundary value problems for the elastic wedge-shaped composite body. Investigation of such problems for the homogeneous medium has been usually based on Kontorovich-Lebedev integral transform techniques:

$$u(r,\phi) = \frac{1}{\pi}\int_{\Gamma} \bar{u}(\tau,\phi) K_{-i\tau}(\kappa r) d\tau,\ \kappa = \text{-ik}$$

where $k$ is the wave number, $K_{-i\tau}$(kr) is McDonald function [9], infinite contour $\Gamma$ belongs to a neighborhood of the real axis $R^1$ and satisfied Summerfield radiation principle. However, the use of such a techniques by classic way for piecewise homogeneous medium dynamics deduces to the additional auxiliary integral equations complicating the problems in question essentially when satisfying media interface conditions on the dividing media

* Corresponding author: bvn06@yandex.ru

boundary . Below it is offered a new method based on distributions' integral transforms technique permitting to exclude the mentioned additional integral equation from the consideration.

Let us consider steady oscillations arising in the wedge-shaped medium $\Omega = \Omega_1 \cup \Omega_2$ under antiplane deformation, one being composed by two wedge-shaped elastic composants $\Omega_{1,2}$ of span angles $\alpha_{1,2}$ with the common vertex, mechanical densities $D_{1,2}$ and shear modules $\mu_{1,2}$ (Fig.1). Generators of harmonic oscillations $f(r)e^{-i\omega t}$ with circular frequency $\omega$ are located in the domain $(a, b)$ on the upper boundary of the medium $\Omega$, the rest of the boundary being assumed to be unloaded The lower boundary of $\Omega$ is stiffly connected. We state the problem of working out the method of reconstructing the wave field in the heterogeneous medium $\Omega$ described above.

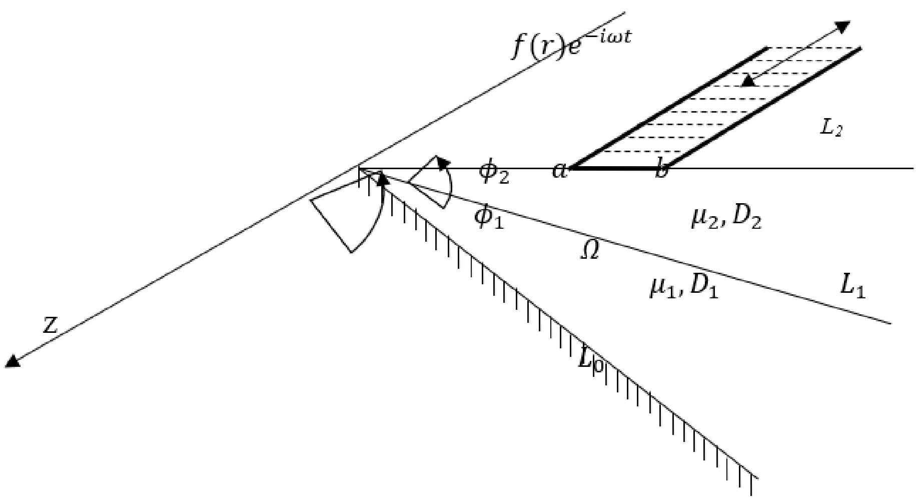


**Fig.1.** Composed wedge-shaped medium with generators

Let us search points' displacement $U(r, \phi, t)$ of the composants $\Omega_{1,2}$ in the form $U(r, \phi, t) = u(r, \phi)e^{-i\omega t}$. Then basic correlations of the dynamic elasticity deduce to the mixed boundary value problem about amplitude $u(r, \phi)$ of oscillations of the piecewise homogeneous wedge as follows:

$$\Delta u + k_i^2 u = 0\,, u = u(r, \phi)\,, k_i^2 = D_i\,\omega^2 \mu_i^{-1}\ (i = 1,2)$$

$$\sigma_{z\phi}^{(2)}|_{\phi=\phi_1+\phi_2} = \mu_2 \frac{1}{r}\frac{\partial u_2}{\partial \phi}|_{\ \phi=\phi_1+\phi_2} = 0\,, r \notin (\mathrm{a,b})$$

$$u|_{\phi=\phi_1+\phi_2} = f(r)\,, r \in (a, b)$$

$$[\sigma_{z\phi}]_{L_1} = [u]_{L_1} = 0 \qquad (1)$$

$$u|_{\phi=0} = 0$$

Conditions (1) are interface ones for the amplitude of displacements $u = u(r, \phi)$ and stresses $\sigma_{z\phi}$ on the line $L_1$, symbol $[\sigma_{z\phi}]_L = \sigma_{z\phi}^{(2)} - \sigma_{z\phi}^{(1)}$ means the function saltus via the line $L$ as well as $[u]_L$. Somerfield radiation principle is assumed to be satisfied at infinity in the whole medium $\Omega$

$$\frac{\partial u}{\partial r} - \mathrm{ik}_{1,2} u = o(r^{-12}), r = \sqrt{x^2 + y^2} \to \infty$$

We will use Kontorovich-Lebedev transform by the form:

$$\bar{f}(\tau) = \tau \mathrm{sh} \pi \tau \int_0^\infty f(r)\, K_{-i\tau}(\kappa r) \frac{\mathrm{dr}}{r}$$
$$f(r) = \frac{1}{\pi^2} \int_{-\infty}^\infty \bar{f}(\tau)\, K_{-i\tau}(\kappa r) d\tau$$

Let us search the solution of the mixed boundary value problem by the integral representation form for wedge – shaped mediums 1, 2 as follows ( $C_{1,2}(\tau), D_{1,2}(\tau)$ are sought-for-functions ) :

$$u^{(1)}(r, \phi) = \frac{1}{\pi} \int_{-\infty}^\infty [C_1(\tau) \mathrm{ch} \phi \tau + D_1(\tau) \mathrm{sh} \phi \tau]\, K_{-i\tau}(\kappa_1 r) d\tau \qquad (2)$$

$$u^{(2)}(r, \phi) = \frac{1}{\pi} \int_{-\infty}^\infty [C_2(\tau) \mathrm{ch} \phi \tau + D_2(\tau) \mathrm{sh} \phi \tau]\, K_{-i\tau}(\kappa_2 r) d\tau$$

We will construct the matrix (named in the sequel as “matrix-propagator”) connecting Kontorovich-Lebedev transforms of displacements and stresses on boundaries of the wedge-shaped medium of angle $\phi$.

By means of direct transformations using (2) it’s easy to establish the correlation:

$$u^{(1)}(r, \phi) = \frac{1}{\pi} \int_{-\infty}^\infty \left[\bar{u}_{10}(\tau) \mathrm{ch} \phi \tau + \frac{\bar{\sigma}_{10}(\tau)}{\mu_1 \tau} \mathrm{sh} \phi \tau\right] K_{-i\tau}(\kappa_1 r)\, d\tau \qquad (3)$$
$$\frac{1}{\mu_1} r \sigma^{(1)}(r, \phi) = \frac{1}{\pi} \int_{-\infty}^\infty \left[\tau\, \bar{u}_{10}(\tau) \mathrm{sh} \phi \tau + \frac{\bar{\sigma}_{10}(\tau)}{\mu_1} \mathrm{ch} \phi \tau\right] K_{-i\tau}(\kappa_1 r)\, d\tau$$

where $\bar{u}(\tau)$, $\bar{\sigma}(\tau)$ are Kontorovich-Lebedev transforms of displacements and stresses respectively .

The lower index 10 means that function value in (3) is considered as the limit while point of observation $(r, \phi)$ tends in the direction to the line $L_0$ within domain *1*. Going over to the Kontorovich – Lebedev transforms in the correlations (3) we can write down its for the wedge-shaped domain of angle $\phi_1$ by the matrix form as follows:

$$\begin{pmatrix} \bar{u}_{11}(\tau) \\ \bar{\sigma}_{11}(\tau) \end{pmatrix} = \begin{pmatrix} \mathrm{ch}\phi_1 \tau & \frac{\mathrm{sh}\phi_1 \tau}{\mu_1 \tau} \\ \mu_1 \tau \mathrm{sh}\phi_1 \tau & \mathrm{ch}\phi_1 \tau \end{pmatrix} \begin{pmatrix} \bar{u}_{10}(\tau) \\ \bar{\sigma}_{10}(\tau) \end{pmatrix} \qquad (4)$$

The lower index 11 means that function value in (4) is considered as the limit while point of observation tends to the boundary $L_1$ within domain *1.* In the correlations (4) the matrix $P(\tau|\phi_1, \mu_1)$ (“matrix- propagator”):

$$P(\tau|\phi_1, \mu_1) = \begin{pmatrix} \mathrm{ch}\phi_1 \tau & \frac{\mathrm{sh}\phi_1 \tau}{\mu_1 \tau} \\ \mu_1 \tau \mathrm{sh}\phi_1 \tau & \mathrm{ch}\phi_1 \tau \end{pmatrix}$$

connects boundary value transforms for displacement and stress transformations

$\begin{pmatrix}\bar{u}_{11}\\ \bar{\sigma}_{11}\end{pmatrix}, \begin{pmatrix}\bar{u}_{10}\\ \bar{\sigma}_{10}\end{pmatrix}$ in the medium 1. Going over to the medium 2 we get the analogous equality for the wedge-shaped domain of the angle $\varphi_2$:

$$\begin{pmatrix}\bar{u}_{22}(\tau)\\ \bar{\sigma}_{22}(\tau)\end{pmatrix} = P(\tau|\phi_2, \mu_2)\begin{pmatrix}\bar{u}_{21}(\tau)\\ \bar{\sigma}_{21}(\tau)\end{pmatrix} \tag{5}$$

The lower index 21 means that function value (5) is considered as the limit while point of observation tends to the boundary $L_1$ within domain 2 in the direction to the domain *1*.

## 2 Method

To satisfy interface conditions (1) of the boundary value problem for composite wedge by the classic way in connection with the use of the Kontorovich-Lebedev transform, to investigate the re-expansion integral $J(t,\tau)$ between systems of McDonald functions described in the monograph by Watson, G. H. [9] with different value of the wave number $(\kappa_1, \kappa_2)$ as follows:

$$J(\tau,\tau') = \int_0^\infty K_{-i\tau}(\kappa_1 t)\, K_{-i\tau'}(\kappa_2 t) t^{-1} \mathrm{dt} \quad \left(\kappa_1 \neq \kappa_2, \kappa_{1,2} > 0\right)$$

Asymptotic properties of this integral are established by Lemma below.

Lemma 2.1. There takes place asymptotic estimations for the integral $J(\tau,\tau')$ $(\tau = x + \mathrm{iy}, C, D = \mathrm{const})$:

$$J(\tau,\tau') = \begin{cases} O\left(|x|^{-32} e^{-C|x|}\right), x = |\mathrm{Re}\tau| \to \infty \\ O\left(|y|^{-32} e^{D|y|}\right), y = |\mathrm{Im}\tau| \to \infty \end{cases}$$

The proof is based on the representation of the integral $J(\tau,\tau')$ by virtue of the hyper geometric series and its asymptotic behavior described by [9].

Let us introduce Smirnov's classes $E_p(\Pi), p > 0$[10] of functions $Q(z)$ to be regular within the strip$\Pi$ , containing the real axis $R^1$ and obeying the condition:

$$\int_\Gamma |Q(z)|^p\, |\mathrm{dz}| < M_p(Q) = \mathrm{const}, p > 0$$
$$\Gamma \subset \Pi$$

Theorem 2.1. If the equality takes place for the class $E_p(\Pi), p > 0$ (the integral is singular)

$$\int_{-\infty}^{\infty} J(t,\tau)\; Q_1(t,\tau) d\tau = 0, t \in R^1 \subset \Pi \tag{6}$$

then there takes place the equality almost everywhere: $Q_1(t,t) = 0$ .

The proof points out of Lemma 2.1, theorems on vanishing the entire functions and well-known classic Sohotzki formulae for the singular integrals treated in classes $E_p(\Pi), p > 0$ described above.

To realize the distributions' integral transforms method offered, to introduced the functional spaces listed below.

2.1. Space $E_+$ of trial functions $\phi(t)$ with compact support in $R_+^1$

$$E_+ = \left\{\phi(t) : \gamma_k(\phi) = \sup_{t>0} |D^k\phi(t)| < \infty, k = 1,2,...\right\} \tag{7}$$

2.2. Space $D_+$ of trial finite functions $\phi(t)$ with compact support in $R_+^1$, its topology being induced by the countable family of semi-norms like (7) .

2.3. Space $G_+$ of trial finite functions $\phi(t)$of complex variable with compact support in $R_+^1$

$$G_+ = \left\{\phi(t) : \gamma_{k,m}(\phi) = \sup_{t>0}[(1+|t|^{2m})\ |D^k\phi(t)|] < \infty, m, k = 1,2,...\right\}$$

2.4. Space $Z_+$ of entire functions $\psi(z)$ as follows:

$$Z_+ = \psi(x+\text{iy}) : |\psi(x+\text{iy})| \le A\exp(B|y|), \psi(x+\text{iy}) \in G_{+,}\ \forall x = x_0$$

2.5. Space $S_\delta(\Gamma), \delta > 0$ of functions in the strip $\Pi$, containing the real axis $R^1$ and contour $\Gamma$:

$$\|X\|_{S_\delta(\Gamma)} = \sup_{z\in\Gamma}\left| X(z)z^\delta\right|, \lim_{|z|\to\infty}\left| X(z)z^\delta\right| = 0$$

2.6. Space $c_\gamma(a,b), 0 < \gamma < 1$ of functions bounded together with the polynomial weight

$$\|f\|_{c_\gamma(a,b)} = \sup_{a<r<b} |f(r)(r-a)^\gamma(b-r)^\gamma|$$

In Sobolev-Slobodetsky spaces $W_2^\gamma(a,b)$the norm is defined as usual. In the sequel, the adjoint space for some ones listed above will be indexed up by prime.

Theorem 2.2. To satisfy the equality treated in distributions' adjoint space $D'_+$

$$\int_0^\infty K_{-i\tau}(\kappa_1\rho)F_1(\tau)d\tau = \int_0^\infty K_{-i\tau}(\kappa_2\rho)F_2(\tau)d\tau, \rho \in R_+^1, \kappa_{1,2} > 0 \tag{8}$$

it is necessary and sufficient to satisfy the correlation

$$\kappa_1^{-i\tau}F_1(\tau) = \kappa_2^{-i\tau}F_2(\tau), \tau \in R_+^1 \tag{9}$$

treated in distributions' adjoint space $Z'_+$ .

To prove the necessity, to consider the Mellin's transform $Mf$ of the distribution $f(t) \in D'_+$ defined by the equality of inner products:

$$(Mf, \psi) = (f, M^{-1}\psi), \quad \psi \in Z_+ \tag{10}$$
$$Mf = \int_0^\infty f(t)t^{-z}\,\text{dt}$$

The latter fulfills the isomorphism $D'_+$ to $Z'_+$. Transformation of integrals (10), operations under distributions and use the correlation (6) lead to (8).

The proof of the sufficiency of the equality (9) points out by its direct substitution in (8) and subsequent using the transform (10).

## 3 Results

To solve the boundary value problem (1), to put $\kappa_{1,2} > 0$ temporary. Obeying the interface conditions along the line $L_1$ dividing the media 1, 2 we establish the displacement and stress transformations have the saltus via the $L_1$ on the strength of the Theorem 2.2, the next equalities being taken place :

$$\kappa_1^{-i\tau} \begin{pmatrix} \bar{u}_{11}(\tau) \\ \bar{\sigma}_{11}(\tau) \end{pmatrix} = \kappa_2^{-i\tau} \begin{pmatrix} \bar{u}_{21}(\tau) \\ \bar{\sigma}_{21}(\tau) \end{pmatrix} \tag{11}$$

The equality (11) may be transformed by means of (4), (5) to the form:

$$P^{-1}(\tau|\phi_2,\mu_2) \begin{pmatrix} \bar{u}_{22}(\tau) \\ \bar{\sigma}_{22}(\tau) \end{pmatrix} = \frac{\kappa_1^{-i\tau}}{\kappa_2^{-i\tau}} \begin{pmatrix} \bar{u}_{11}(\tau) \\ \bar{\sigma}_{11}(\tau) \end{pmatrix} = \left(\frac{\kappa_1}{\kappa_2}\right)^{-i\tau} P(\tau|\phi_1,\mu_1) \begin{pmatrix} \bar{u}_{10}(\tau) \\ \bar{\sigma}_{10}(\tau) \end{pmatrix}$$

There results the linear correlation connected the displacements and stresses transformations on the exterior boundaries of the two-components wedge-shaped medium

$$\begin{pmatrix} \bar{u}_{22}(\tau) \\ \bar{\sigma}_{22}(\tau) \end{pmatrix} = \left(\frac{\kappa_1}{\kappa_2}\right)^{-i\tau} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1) \begin{pmatrix} \bar{u}_{10}(\tau) \\ \bar{\sigma}_{10}(\tau) \end{pmatrix}$$

almost everywhere with the matrix- propagator:

$$P(\tau|\phi_1+\phi_2,\mu_1,\mu_2) = \left(\frac{\kappa_1}{\kappa_2}\right)^{-i\tau} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1)$$

Let us, in part, the lower plane $\phi = 0$ is stiffly connected with the non-deformable and unmoved foundation:

$$\begin{pmatrix} \bar{u}_{10}(\tau) \\ \bar{\sigma}_{10}(\tau) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \bar{\sigma}_{10}(\tau) = e_2 \bar{\sigma}_{10}(\tau)$$

Going over to the medium 2 providing the same operations and applying the results of the Theorem 2.2 when obeying the interface conditions and eliminating the unknown $\bar{\sigma}_{10}(\tau)$ we yield the expression (*T*- transpose operation):

$$\bar{u}_{22}(\tau) = \frac{e_1^{\mathrm{T}} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1) e_2}{e_2^{\mathrm{T}} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1) e_2} \bar{\sigma}_{22}(\tau)$$
$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{12}$$

Going over from displacement and stress transformations in (12) to its originals by the inverse Kontorovich-Lebedev transform it may be obtained the BIE of the mixed boundary value problem for the two-components composed elastic wedge stiffly connected by its lower boundary:

$$Kq = \int_a^b k(r,\rho)\ q(\rho)\ d\rho = f(r)\,, a < r < b \tag{13}$$
$$k(r,\rho) = \frac{1}{\pi}\int_{-\infty}^{\infty} I_{-\mathrm{iz}}(\kappa_2\rho) K_{-\mathrm{iz}}(\kappa_2 r) K(z) z\, \mathrm{dz}\,, \kappa_2 = -\mathrm{ik}_2$$
$$K(z) = \frac{e_1^{\mathrm{T}} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1) e_2}{e_2^{\mathrm{T}} P(\tau|\phi_2,\mu_2) P(\tau|\phi_1,\mu_1) e_2}\,, q(\rho) = \sigma_{22}(\rho)/\mu_2$$

Multiplication of matrix-propagators in the numerator and denominator of $K(z)$ results to its expression

$$K(z) = \frac{\mathrm{th}\phi_2 z + \lambda \mathrm{th}\phi_1 z}{z(1+\lambda\ \mathrm{th}\phi_2 z\ \mathrm{th}\phi_1 z)}\ , \lambda = \mu_2/\mu_1\ , \kappa_2 = -\mathrm{ik}_2$$

All assertions have been provided above under assumption $\kappa_{1,2} > 0$ and then the passage to the initial case $\kappa_{1,2} = -\mathrm{ik}_{1,2}$ is provided by the analytical continuation principle since all functions are analytical with respect to $\kappa$ in the domain $\mathrm{Re}\kappa \geq 0, \kappa \neq 0$ of the complex plane $\kappa$, where, in part, the points $\kappa_{1,2} = -\mathrm{ik}_{1,2}$ are located [11].
To investigate the solvability problems for the BIE system the next theorem is established.

Theorem 4.1 Operator $K$ of the left hand side for the BIA (11) is uniquely inverted as operator acting in vector function spaces :

$$\mathrm{K}: H(a,b) \rightarrow W_2^{12}(a,b)$$
$$H(a,b) \subset W_2^{-12}(a,b)$$

where $W_2^{\gamma}(a,b), \gamma = \pm 12$ are Sobolev-Slobodetsky spaces of fractional smoothness, $H(a,b)$is the generalized solutions' space for the BIA.

To prove the theorem, to put $\kappa > 0$ temporary. Then the operator $K$ is positively defined and induces the space $H(a,b)$ of generalized solutions of the equation (13) by the norm:

$$\|q\|_{H(a,b)} = \left(\int_0^\infty K(z)\ |\bar{q}(z)|^2 \mathrm{dz}\right)^{12}$$
$$\bar{q}(z) = \sqrt{\mathrm{zsh}\pi\, z} \int_0^\infty q(r) K_{-\mathrm{iz}}\,(\kappa r)\mathrm{dr}$$

with the inner product (* means complex conjugation ):

$$(q_1, q_2)_H = \int_0^\infty K(z)\, \bar{q}_1(z) \bar{q}_2\ \ (z)\mathrm{dz}$$

Furthermore it may be used the method [11] and to obtain the solvability condition for BIE (13) as follows

$$\int_0^\infty |\bar{f}(z)|^2\, K^{-1}(z)\mathrm{dz}\ < \infty\, ,$$

It points out the existence of the unique solution $q \in W_2^{-12}(a,b)$ for any right hand side $f \in W_2^{12}(a,b)$ and the imbedding $H(a,b) \subset W_2^{-12}(a,b)$. This result is in accordance with well-known ones about boundary properties of functions belonging to Sobolev space $W_2^1(a,b)$ in which the solution of boundary value problems of the dynamic elasticity is searched . The passage to the required case $\kappa = -ik$ may be fulfilled by the analytical continuation principle used above [11,12].

The solution structure of the BIE (13) may be presented by [13,14] in the modified form:

$$\rho q(\rho) = \frac{1}{\pi}\int_{\Gamma_1} K^{-1}(z)\bar{f}(z) K_{-\mathrm{iz}}(\kappa_2\rho)\mathrm{dz}\ + \tag{14}$$

$$+\frac{1}{\pi}\int_{\Gamma_2} \{X_1(z) I_{-\mathrm{iz}}(\kappa_2\rho) K_{-\mathrm{iz}}(\kappa_2 b) + X_2(z) K_{-\mathrm{iz}}(\kappa_2\rho) I_{-\mathrm{iz}}(\kappa_2 a)\} K_-^{-1}(z)\mathrm{dz}$$

where $I_{-\mathrm{iz}}(\kappa_2\rho), K_{-\mathrm{iz}}(\kappa_2\rho)$ are modified Bessel functions. The expression (14) leads to the imbedding $q \in c_{12}(a,b) \subset W_2^{-12}(a,b)$ provided $f \in W_2^{12}(a,b), X_{1,2} \in S_\delta(\Gamma_2), \delta > 12$, contours $\Gamma_{1,2} \subset \Pi$, where $\Pi \supset R^1$ is the regularity strip of function $K(z)$.Functional spaces mentioned are described in the item 2. The function $K_-(z)$is the result of factorization$K(z) = K_+(z) \cdot K_-(z), K_\pm(z) = O(|z|^{-12})$ with respect to real axis $R^1$[13,14]. Functions $X_{1,2}(z)$ are found from an integral equations' system uniquely resolved in the space $S_\delta(\Gamma_2)$ , $\delta > 12$ . Both the system and its solution are not submitted there because of their awkwardness. Solution (14) has the well-known power singularity on boundaries of the interval $(a,b)$ which is typical for elasticity contact problems.

The reconstruction the wave field in the whole heterogeneous elastic wedge-shaped medium $\Omega$is obtained by means of formulae (3) .Consideration of inverse problems for this medium may be fulfilled on the base of methods having been worked out in [15,16].

## 4 Conclusions

1. It is solved the problem of the dynamic elasticity for the piecewise homogeneous wedge-shaped medium being of interest both in machinery constructions exploitation reliability and seismic prospects problems.
2. The new method based on the distributions' integral transform technique for the resolve the problem mentioned above is offered.
3. The problem in question is reduced to boundary integral equation (BIA) investigated in details.
4. The solution structure of BIA is established and described the way of its obtained.
5. The method presented permits to reconstruct the wave field in the whole piecewise homogeneous elastic wedge-shaped medium and consider inverse problems for one.

## References

1. D.A. Pozharskii, Strip cut in the composed elastic wedge, J.Appl. Math. and Mech., **80(4)**, 489-495 (2018)
2. V.N. Berkovich, S.I. Builo, Amplitude Reconstruction of the Defect Radiation by Acoustic Emission Signals on the Unloaded Boundary of a Massive Body, Russian Journal of Nondestuctive Testing, **4**, 262-267 (2019)
3. S.I. Builo, *Physical, mechanical Statistical and chemical aspects of acoustic emission diagnostics*, 184p. (Rostov-on-Don, Southern Federal University Press, 2017)
4. V.I. Ivanov, V.A. Barat, *Acoustic emission diagnostics*, 368p. (Moskow. Publishing «Spectr», 2017)
5. S.I. Builo, *Physical, mechanical and statistical aspects of increasing the reliability of results of acoustic emission testing and diagnostics*, 192p. (Rostov-on-Don, Southern Federal University Press, 2008)
6. S.I. Builo, Diagnostics of the pre-destructive state of materials by amplitude parameters of signals of the concomitant acoustic radiation, Russian Journal of Nondestuctive Testing **11**, 32-45 (2012)
7. S.I. Builo, Physical, mechanical and statistical aspects of acoustic emission diagnostics. In: *Physics and Mechanics of New Materials and Their Applications*, 444 p. (New York: Nova Science Publishers. Chapter 15, 2013)
8. M.Yu. Kashtalyan, J.J. Rushchitsry, General Hoyle-Youngdahl and Love solutions in the linear inhomogeneous theory of elasticity, Int.Appl.Mech. **46(1)**, 1-17 (2010)

9. G. N. Watson, *A treatise of the theory of Bessel functions (Nobel press)* Web Res. 814p. (2011)
10. I.I. Daniluck, *Nonregular boundary value problems on the plane*, 296p. (Science press, Moskow: LitRes, Web Res, 2019)
11. V.N. Berkovich, D.A. Pozharski, V.I.Poltinnikov, A.V. Morozova, On a mathematical model of dynamics of the elastic wedge-shaped medium with radiating defect. E3S Web Conf. **175** (2020)
12. XIII International Scientific and Practical Conference “State and Prospects for the Development of Agribusiness – INTERAGROMASH 2020” DOI: http//doi.org 10.1051/e3sconf/202017505015
13. V.N. Berkovich, On a correctness of the mathematical modelling method of acoustic radiation by linear defect in the angular domain, Int. Res.J., **2** (92), part 1, 19-24 (2020) DOI: http//doi.org/10.23670/IRJ. 2020.92.2.003.
14. V.N. Berkovich, A.V. Babkin, Vibrations of elastic half-space containing two radiating defects of finite length, Int. Res. J., **58(23)**, 101-104 (2017) DOI: http//doi.org/10.23670/IRJ. 2017.58.042.
15. V.N. Berkovich, V.I.Poltinnikov, On a problem of the elastic wedge-shaped body oscillations generated in its rib zone. ERSME (2020)
16. IOP Conf. Series: Materials Science and Engineering 1001, 012052 (2020) DOI:10.1088/1757-899X/1001/1/012052
17. A.O. Vatulyan, To the theory of inverse coefficient problems in linear mechanics, Appl. Math.& Mech. **6**, 911-918 (2010)
18. A.O. Vatulyan, O.A. Belyak , D.Yu. Suhov, O.V.Yavruyan, *Inverse and incorrect problems,* 232p. (Rostov-on-Don: Southern Federal University Press, 2011)