Application of pressure head theory to determine the aerodynamic characteristics of the helicopter main rotor

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Abstract. A method is proposed for calculating main aerodynamic characteristics of the main rotor of the Mi-26 helicopter: a lift, a reactive moment, a required power of the propulsion system, a speed of the inductive air flow, a throttle response coefficient, a speed of the horizontal flight. This method based on the pressure propeller theory. The parameters of the Mi-26 helicopter and the propeller of the experimental setup calculated according to the pressure theory of the main rotor are experimentally confirmed. The percentage of the discrepancy between the statistical data on the aerodynamic characteristics of the Mi-26 helicopter rotor calculated with the pressure theory does not exceed 1.5%.

1 Introduction

At present, the aerodynamic calculation of the helicopter main rotor is theoretically described by the classical theory of the helicopter main rotor, developed back in the fifties of the last century by M. Mil and B. Yuriev. However, this theory has some assumptions and does not provide an accurate explanation of the dependence of the rotor thrust on the kinetic energy and the speed of the inductive flow, it includes many approximate coefficients that are refined after experimental verification. The pressure head theory of the propeller (called in this text as the PHTP theory) appeared as a result of combining the experimental and statistical data of the aerodynamic characteristics of the helicopter main rotor of three types by Mi-2, Mi-24, Mi-26 helicopters and the propeller of an experimental aerodynamic installation. The pressure-applied method of determining the main aerodynamic characteristics of a helicopter blade, based on the PHTP theory, is a simple, reliable and fairly accurate method for calculating the main parameters of the helicopter main rotor of the Mi-26 helicopter. The PHTP theory is a good addition to the vortex, impulse and classical theories of helicopter main rotor and for a designer who designs any aircraft that has helicopter main rotor, it is a good help in his creative work. The PHTP theory can be used not only to determine the aerodynamic characteristics of a helicopter propeller, but also to determine the parameters of the propeller of a subsonic aircraft.

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2 Theoretical research

To consider, the PHTP theory, we will start from the idealized representation of the velocity head of the blade (called in this text as VHB), in which all parameters and characteristics of the helicopter main rotor are calculated in the most energy-intensive mode of its operation - hovering, which is associated with its safety in all modes work [1]:

$$Q_i = \frac{\rho \left[\left(\frac{R_b + R_m}{2} \right) \cdot 2\pi \cdot n \right]^2}{2}, \quad (1)$$

where:

Q_i – ideal velocity head of the blade, N

ρ – mass density of air at the calculated height, kg/m³;

n – speed of rotation helicopter main rotor, s⁻¹;

R_b – distance from the axis of rotation of the helicopter main rotor to the end part of the blade, m;

R_m – the distance from the axis of rotation of the helicopter main rotor to the tail part of the blade, m.

According to the PHTP theory, the velocity head of the blade is determined by the formula:

$$Q_r = \frac{\rho (0.5 U_{kl})^2}{2},$$

where:

Q_r – real velocity head of the blade, N/ m²

U_{kl} – circumferential velocity of the end of blade, s⁻¹.

Velocity head of the blade is a component of the lift force, that we can calculate the value of the lift force of the helicopter main rotor [1,2]:

$$L = C_{l\,mean} \frac{\rho \left[\left(\frac{R_b + R_m}{2} \right) \cdot 2\pi \cdot n \right]^2}{2} \cdot S_b, \quad (2)$$

where:

$C_{l\,mean}$ – the mean value of the lift coefficient of the blade of the helicopter main rotor;

S_b – area of the feathered part of the blade, m².

To verify the validity of formula (2), we calculate the lift of the helicopter main rotor of the Mi-26 helicopter using fragmentary method (Fig. 1).

To begin with, we divide the feathered part of the blade of the helicopter main rotor into 4 parts, with the length of each fragment being 3.325 m with an area of 2.7 m². Then the distance from the axis of rotation of the helicopter main rotor to the middle of the feathered part of the first fragment will be 14.3 m, the second is 11 m, the third is 7.7 m, the fourth is 4.4 m. The distance to the center of feathered part of the blade is 9.35 m. Length feathered blades and 13.3 m. Chord of the blade over the entire length is 0.8 m. Frequency of rotation of the helicopter main rotor is 2.2 s⁻¹.

The mean value of the lift coefficient $C_{l\,mean}$ for each fragment are 0.5; 0.55; 0.85; 0.95 respectively [2,16].

Fig. 1. Determination of the lift force of the main rotor blade of a helicopter by the fragmentary method according to the PHTP theory.

Then the thrust of the one blade of the helicopter main rotor is:

$$T_{frag1} = 0,5 \cdot \frac{0,125[6,28 \cdot 14,3 \cdot 2,2]^2}{2} \cdot 2,7 = 3293,48 \text{ kgf},$$

$$T_{frag2} = 0,55 \cdot \frac{0,125[6,28 \cdot 11 \cdot 2,2]^2}{2} \cdot 2,7 = 2143,77 \text{ kgf},$$

$$T_{frag3} = 0,85 \cdot \frac{0,125[6,28 \cdot 7,7 \cdot 2,2]^2}{2} \cdot 2,7 = 1623,24 \text{ kgf},$$

$$T_{frag4} = 0,95 \cdot \frac{0,125[6,28 \cdot 4,4 \cdot 2,2]^2}{2} \cdot 2,7 = 592,44 \text{ kgf},$$

$$T_{blade} = 3293,4 + 2143,77 + 1623,24 + 592,44 = 7652,9 \text{ kgf}.$$

The mean value of the lift coefficient of the blade of the helicopter main rotor is:

$$C_{l \text{ mean}} = \frac{0,5+0,55+0,85+0,95}{4} = 0,71.$$

Then the thrust of the blade of the helicopter main rotor is:

$$T_{blade} = 0,71 \cdot \frac{0,125[6,28 \cdot 9,35 \cdot 2,2]^2}{2} \cdot 10,64 = 7878,9 \text{ kgf}.$$

The error between the fragmentary and general calculation of the lifting force of the blade of the helicopter main rotor by Mi-26 is:

$$\Delta T = 100 - \frac{7652,9 \cdot 100}{7878,9} = 2,9\%.$$

The error of 2.9% is quite small, so it can be ignored, because you can set more accurate values of the C_l for each fragment of the blade.

The maximum take-off weight of the Mi-26 helicopter is 56000 kgf, which means that each of the eight blades must have a lifting force of 7000 kgf. The difference between the calculated and statistical data arose from the fact that the calculation of the lifting force of the blade was made according to the formula for determining the thrust using the ideal

velocity head of the blade (see formula 1), which does not take into account the loop and end loss of lift force. The value of the coefficient of loop and end losses is in the range of 0.85...0.9 [1].

Take the value $X = 0.89$, then:

$$T_{blade} = 7878.9 \cdot 0.89 = 7012.2 \text{ kgf} \approx 7000 \text{ kgf},$$

which almost completely coincides with the statistical value of the lifting force of one blade.

Velocity head of the blade according to the formula (1) allows the calculation of lift force helicopter main rotor due to the fact that the distance from the axis of rotation of helicopter main rotor to the middle of the feathered part of the helicopter's blade is 0.56-0.58 to helicopter main rotor radius. If excluded from the calculation of lift force value $X=0.85 \div 0.9$, then it becomes possible to calculate the value $(0.5 U_{kl})$ which is the peripheral speed of rotation of the middle radius of the blade, while $C_{l\text{mean}}$ is determined by the angle of attack of the blades in the middle of the feathered part of the blade, taking into account their geometrical twist.

Now, we will determine the main aerodynamic characteristics of the main rotor of the Mi-26 helicopter according to the classical theory and the PHTP theory and conduct a comparative analysis. For the convenience of calculating the main aerodynamic characteristics of the main rotor of the Mi-26 helicopter according to the PHTP theory, it is proposed to use the real blade velocity head, which is determined by the formula (1).

1. Lift force of the helicopter main rotor (thrust of the helicopter main rotor).

Traditionally, the lift force of the helicopter main rotor (thrust) T is determined by the formula (3) [2-4, 10,11]:

$$T = C_t S_{HMR} \frac{\rho(\omega R)^2}{2}, \quad (3)$$

where,

C_t – thrust coefficient of the helicopter main rotor;

S_{HMR} – area of swept surface of the helicopter main rotor;

ρ – air density;

ωR – the peripheral speed of the blade tip.

In the formula (3), C_t is determined as

$$C_t = \frac{\sigma}{3} C_{l0.7},$$

where:

$\sigma = \frac{k \cdot F_n}{S_{HMR}}$ – the rotor solidity ratio of the helicopter main rotor.

$C_{l0.7}$ – the lift coefficient of the blade in the section as $0.7R$ of the main rotor.

According to the PHTP theory, the helicopter main rotor thrust is determined as:

$$T = C_{l\text{mean}} \frac{\rho(0.5U_{kl})^2}{2} \cdot S_b, \quad (4)$$

where,

$C_{l\text{mean}}$ – mean value of the lift coefficient of the helicopter main rotor blade; $C_{l\text{mean}} = 0.71$;

S_b – area of the feathering part of the blades of the helicopter main rotor; $S_b = 85,12 \text{ m}^2$.

According to the PHTP theory, the $C_{l\text{mean}}$ coefficient is determined using tables of aerodynamic profiles for the angle of attack of the blade through the angle of its installation in the middle of the feathering part [3,10]:

$$\alpha_{mean} = \varphi \cdot f_n, \quad (5)$$

where:

α_{mean} – mean the angle of attack of the helicopter main rotor blade;
 φ – installation angle in the middle of the feathered part of the helicopter main rotor blade;
 f_n – conversion factor from the installation angle to the angle of attack ($f_n = 0,55 \div 0,75$);

Then, the thrust of the helicopter main rotor by Mi-26 is:

$$T = 0,86 \frac{0,125(0,5 \cdot 221)^2}{2} \cdot 85,12 = 55864,33 \text{ kgf}.$$

2. Reactive moment. The reactive moment of the helicopter main rotor according to the classical theory of the helicopter main rotor is defined as the torque of the power plant with the opposite sign [3-5,12-14]:

$$M_R = m_b S_b \frac{\rho(\omega R)^2}{2} \cdot R_{HMR}, \quad (6)$$

where:

R_{HMR} – radius of the helicopter main rotor;
 m_b – blade activity factor, determined by the formula:

$$m_b = \frac{\sigma}{4} C_{d0.7} + \frac{\sigma}{3} C_{l0.7} (\overline{V_1})_{0.7},$$

where:

$C_{d0.7}$ – profile drag coefficient of the helicopter main rotor blade in a section as 0.7R of the main rotor.

$(\overline{V_1})_{0.7}$ – air flow speed before the helicopter main rotor blade in a section as 0.7R of the main rotor.

According the PHTP theory, the reactive moment of the helicopter main rotor is determined as:

$$M_R = C_{dmean} S_b \frac{\rho(0,5U_{kl})^2}{2} \cdot R_{HMR}, \quad (7)$$

where:

C_{dmean} – mean profile drag coefficient of the helicopter main rotor blade; $C_{dmean} = 0,072$;
 C_{dmean} of the blade is determined using tables of aerodynamic profiles by the angle of attack of the blade through the angle of its installation in the middle of the feathered part.
then:

$$M_R = 0,072 \frac{0,125(0,5 \cdot 221)^2}{2} \cdot 85,12 \cdot 16 = 74832 \text{ kgf} \cdot \text{m}$$

3. Required engine power. The required power of the engine N_R in hover mode according to the classical theory of the helicopter main rotor is determined through the coefficient of the engine torque [3-5,11]:

$$N_R = m_b \cdot S_{HMR} \frac{\rho(\omega R_{HMR})^3}{150}, \quad (8)$$

According to the PHTP theory, the required power of the engine is determined as [5]:

$$N_R = T \cdot U_{kl} \cdot C_{dmean}, \quad (9)$$

where:

T – the lift force of the helicopter main rotor (thrust) expressed in Newtons,
then:

$$N_R = 560000 \cdot 221 \cdot 0,072 = 8910,720 \text{ (kW)}$$

4. The speed of the inductive air flow. The speed of the inductive air flow (V_{i1}) in the plane of rotation of the helicopter main rotor in the hover mode according to the classical theory is determined as [3-5]:

$$V_{i1} = \sqrt{\frac{T}{2\rho S_{HMR} \cdot X}} \quad (10)$$

where:

X – root and tip losses factor.

According to the PHTP theory, the speed of the inductive air flow in the plane of rotation of the helicopter main rotor is determined as [6,7]:

$$V_{i1} = U_{kl} \cdot \sigma_1, \quad (11)$$

where:

σ_1 – load factor main rotor blade, defined as the ratio of the load on one blade to the load of the main rotor.

The speed of the inductive air flow behind the helicopter main rotor is determined as twice speed of the inductive air flow in the plane of rotation:

$$V_{i2} = 2V_{i1}$$

Because coefficient σ_1 is constant any conditions, then in the PHTP theory speed of the inductive air flow (V_{i1}) can be determined through the rotor solidify ratio (σ) of the helicopter main rotor as [8]:

$$V_{i1} = U_{kl} \cdot \sigma \quad (12)$$

The speed of the inductive air flow behind the helicopter main rotor is determined also:

$$V_{i2} = V_{i1} \cdot 2 \quad (13)$$

5. Thrust pick-up ratio. Thrust pick-up ratio (Z_t) of the propeller and the helicopter main rotor is a value that determines the relationship between the weight air flow rate and the thrust developed by the propeller at the selected operating mode. Thrust pick-up ratio is a variable value that changes when the propeller operating mode changes. Thrust pick-up ratio to decrease its value with increasing load on the propeller, i.e [4]:

$$T/S_{HMR} > T/S_{PR} \rightarrow Z_{iHMR} < Z_{iPR}$$

where:

S_{HMR} - area of the swept surface of the helicopter main rotor;

S_R - area of the swept surface of the propeller.

The lower the load on the engine power, the lower the thrust pick-up ratio:

$$T/N_{HMR} < T/N_{PR} \rightarrow Z_{iHMR} < Z_{iPR}$$

Since the value of Z_t is variable, then for the characteristics of the helicopter main rotor, this value must be indicated as extremely small, in this case the relationship between the maximum thrust of the helicopter main rotor and its power-to-weight ratio will be seen. The higher the power-to-weight ratio of the helicopter, the lower its thrust pick-up ratio.

According to the PHTP theory, the thrust pick-up ratio is determined as [6]:

$$Z_t = \frac{\theta_m}{T}, \quad (14)$$

where:

θ_m – weight air flow of the helicopter main rotor. Its determined as:

$$\theta_m = \theta_v \cdot \gamma$$

where:

γ – density of the air; $\gamma = 1,25 \text{ kg/m}^3$;

θ_v – volumetric air flow rate of the helicopter main rotor. Its determined as:

$$\theta_v = S_{HMR} \cdot V_{i2}$$

We will calculate the thrust pick-up ratio of the helicopter main rotor for the Mi-26 helicopter.

Data source [3,11]:

- $T = 56000 \text{ kgf}$;
- $R_{HMR} = 16 \text{ m}$;
- $U_{kl} = 221 \text{ m/s}$;
- $\sigma_I = 0,1$.

Then:

1. Speed of the inductive air flow V_{i1} и V_{i2} :

$$V_{i1} = 221 \cdot 0,1 = 22,1 \text{ m/s};$$

$$V_{i2} = 22,1 \cdot 2 = 44,2 \text{ m/s}.$$

2. Area of the swept surface of the helicopter main rotor of the Mi-26:

$$S_{HMR} = \pi R_{HMR}^2$$

$$S_{HMR} = 3,14 \cdot 16^2 = 804 \text{ m}^2.$$

3. Volumetric air flow rate of the helicopter main rotor of the Mi-26:

$$\theta_v = S_{HMR} \cdot V_{i2}$$

$$\theta_v = 804 \cdot 44,2 = 35536,8 \text{ m}^3/\text{s}.$$

4. Weight air flow of the helicopter main rotor of Mi-26:

$$\theta_m = 35536,8 \cdot 1,25 = 44421 \text{ kgf} (\sim 444210 \text{ N}).$$

5. Thrust pick-up ratio of the helicopter main rotor of Mi-26:

$$Z_t = \frac{44421}{56000} = 0,79 \text{ s} \approx 0,8 \text{ s}.$$

To develop a thrust of 56 tons, the helicopter main rotor of Mi-26 must throw 444210 N of air in the direction opposite to the thrust vector within one second, while the inductive air flow at a speed of 44.2 m/s will have kinetic energy:

$$A = \frac{\theta_m \cdot V_{i2}^2}{2}$$

$$A = \frac{444210 \cdot 44,2^2}{2} = 43391,3 \text{ kJ}$$

6. The speed of the horizontal flight. The speed of the horizontal flight V_{hf} of the helicopter is determined by the classical method using the formula [3,4]:

$$V_{hf} = 270 \frac{N_e}{G} \cdot L/D \cdot \xi_p \cdot \eta, \quad (15)$$

where:

N_e - effective power of the power plant;

G – weight of the helicopter,

L/D – lift/drag ratio,

ξ_p – the utilization rate of power,

η – efficiency of the propeller.

According to the PHTP theory, the speed of the horizontal flight is determined by the formula:

$$V_{hf} = \frac{N_A \cdot C_{l\ mean}}{G \cdot C_{d\ mean}} \Omega, \quad (16)$$

where:

N_A - available power of the helicopter main rotor ($N_A \geq N_R$)

Ω – the coefficient of correction of the main rotor thrust, which depends on the angle between the main rotor axis and the thrust vector relative to the longitudinal axis of the helicopter, as well as on the aerodynamic drag of the fuselage. $\Omega = 0 \div 0,5$.

Let us determine the speed of the horizontal flight of the Mi-26 helicopter according to the PHTP theory.

Data source [3,4]:

1. Total power of the two engines – 23000 *h.p.* (horsepower)
2. Weight of the Mi-26 helicopter – 560000 *N*
3. Lift coefficient of the helicopter main rotor – 0,86.
4. Drag coefficient of the helicopter main rotor – 0,072.
5. Traction correction factor – 0,28.
6. Power losses for the drive of the units and the tail rotor – 20 %.

Calculation:

1. Power going to the drive of the units and the tail rotor (20% of engine power):

$$X = 23000 \cdot 0,2 = 4600 \text{ h.p.}$$

2. Available power on the helicopter main rotor:

$$N_A = 23000 - 4600 = 18400 \text{ h.p. or } 13529400 \text{ W.}$$

3. Speed of the horizontal flight:

$$V_{hf} = \frac{13529400 \cdot 0,86}{560000 \cdot 0,072} \cdot 0,28 = 80,8 \text{ m/s} = 290,8 \text{ km/h.}$$

The results of calculations of the main aerodynamic characteristics of the propeller according to PHTP theory were tested on an experimental setup developed in the laboratory.

The thrust of the experimental unit's propeller:

$$T = 0,74 \frac{0,125(0,5 \cdot 81)^2}{2} \cdot 0,792 = 60,08 \text{ kgf}$$

The reactive moment of the propeller of the experimental plant:

$$M_{R\ propel} = 0,082 \frac{0,125(0,5 \cdot 81)^2}{2} \cdot 0,792 \cdot 1,85 = 12,316 \text{ kgf} \cdot \text{m}$$

Required engine power of the propeller of the experimental plant:

$$N_{R\ propel} = 608 \cdot 81 \cdot 0,082 = 4,038 \text{ kW}$$

All statistical, experimental and calculated according to the PHTP theory parameters of the helicopter main rotor of the Mi-26 helicopter and the propeller of the experimental plant are listed in the Table 1.

Table 1. Statistical, experimental and calculated according to the PHTP theory parameters of the helicopter main rotor of the Mi-26 helicopter and the propeller of the experimental plant.

Object	Initial parameters of the main rotor of the Mi-26 helicopter and the propeller of the experimental plant	Statistic data of the main rotor of the Mi-26 helicopter and the propeller of the experimental plant	Calculated data according to PHTP theory	Calculation error (%)
Propeller of the experimental plant	$n_{prop} (s^{-1}) - 7$	$P (kN) - 0.6$	$P (kN) - 0.608$	1.3
	$R_{prop} (m) - 1.85$	$M_R (kN \cdot m) - 0.125$	$M_R (kN \cdot m) - 0.123$	1.4
	$L_{prop frag} (m) - 1.65$	$N_{R propel} (kW) - 4.1$	$N_{R propel} (kW) - 4.038$	1.5
	$B_{prop} (m) - 0.16$	$V_{i2} (m/s) - 11.5$	$V_{i2} (m/s) - 11.34$	1
	$Z_{blade} (pieces) - 3$	$Z_t -$	$Z_t - 2.5$	—
Main rotor of the Mi-26 helicopter	$n_{HMR} (s^{-1}) - 2.1$	$P (kN) - 560$	$P (kN) - 558.64$	0.24
	$R_{HMR} (m) - 16$	$M_R (kN \cdot m) - 750$	$M_R (kN \cdot m) - 748.32$	0.22
	$L_{HMR frag} (m) - 13.3$	$N_R (kW) - 8455.8 \times 2$	$N_R (kW) - 8910.7$	—
	$B_{blade} (m) - 0.8$	$V_{i2} (m/s) - 44$	$V_{i2} (m/s) - 44.2$	0.45
	$Z_{blade} (pieces) - 8$	$Z_t -$	$Z_t - 0.8$	—

3 Conclusions

The proposed calculation method based on the pressure head theory of the propeller allows, with an error of no more than 1.5%, to determine the main aerodynamic characteristics of the Mi-26 helicopter main rotor (lift, jet torque, required power of the power plant, inductive air flow velocity, thrust response coefficient, speed horizontal flight). And it can be used when calculating the aerodynamic characteristics of any propellers.

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