

Seasonal temperature waves in the ground, non-periodic case

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Abstract. Seasonal fluctuations in soil temperature at a depth of several meters are considered. It is assumed that the temperature of the earth surface changes strictly periodically. Then, according to the Fourier law, the soil temperature at depth will also change periodically with a smaller amplitude and time lag. What happens if we let the temperature on the surface deviate from the strict periodicity at some point in time? How will the nature of soil temperature fluctuations change at depth? Two types of deviations of the surface temperature from the periodic law are considered: 1) A sharp cold snap. For 30 days, the temperature of the earth surface is -30°C and 2) Warm winter. It is assumed that the temperature of the earth surface is zero during the winter months. Graphs of temperature changes at different depths in both cases are plotted. Conclusions are drawn about the duration of the period of noticeable deviations and the magnitude of the temperature deviation from the normal value.

Keywords. Temperature fluctuations; soil temperature; extreme weather fluctuations.

1 Introduction

The temperature of the earth surface changes periodically. At the same time, there are daily and annual fluctuations. If the surface temperature periodically changes for a long time, then temperature fluctuations with the same period (temperature waves) are established in the soil. This phenomenon was studied by Fourier. Although it was a long time ago, the study of temperature fields in the ground continues to this day.

This is due to the complexity of the issue, the unpredictability of the deviation of the surface temperature from the correct harmonic oscillations, the variety of the structure of the soil and the processes occurring in its thickness.

For builders, the bearing properties of the soil are interesting. They can change dramatically when the soil is frozen and thawed. Considering recent works, these issues are addressed in the work [1]. The following phenomenon is being investigated.

In permafrost zones along a linearly extended object, thawing halos appear, leading to a change in its spatial position and deformation. On the basis of experimental data obtained over a long period in the process of field measurements of soil temperature along the route of a linearly extended object, the following factors have been determined: the actual depth of the zone of zero annual temperature fluctuations; average monthly soil temperatures at different depths; a family of models that predict cyclical changes in ground temperature.

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The following article [2] examines one cross-section of permafrost on the northern slope of the Bayan Khar Mountains (China). Findings. Temperature is a key factor affecting the hydrological process of soils in the active layer in permafrost regions. Soil temperature and soil moisture showed significant exponential linear correlation at different depths.

The article [3] substantiates a new method of geocontrol and monitoring of soil stability at the base of the foundations of buildings and structures in zones of seasonal and long-term frozen soil.

The next article [4] examines the issue of permafrost stability. Information on susceptibility to melting is important for predicting the behavior of permafrost as an engineering substrate. We compare apparent conductivity versus ground displacement measurements obtained with synthetic aperture differential interferometric radar to establish apparent conductivity as an indicator of susceptibility to melting for regional characterization of terrain stability and permafrost conditions.

The article [5] presents the results of monitoring the temperature of the surface layers of earth. The zone of fluctuations in the temperature of the soil massif depending on the temperature of the outside air has been revealed.

The downhole heat exchanger can be used in heating and cooling systems, since the ground temperature is usually higher than the average air temperature in winter and lower in summer.

The performance of the device is affected by soil temperature, which can have significant seasonal variations. The article [6] presents a new approach to an internal source for accounting for (seasonal) fluctuations in soil temperature during the operation of shallow wells.

The article [7] investigated a steep loess slope with a height of 70 m. Cyclic deformation of the slope associated with seasonal changes in the temperature of the soil and the environment was found. A quantitative relationship was found between weather cycles and slope thermal deformation.

Based on the latest series of data on the homogenized surface temperature and surface air temperature for China, a detailed analysis of the trend of differences between the two homogenized series in 1961-2016 was carried out in [8]. The differences, referred to in this study as surface air temperature difference (SATD), were separately averaged over months, seasons and years. Long-term and spatial changes in SATD trends were investigated.

Underground ventilation in a coal mine comes from surface air, so seasonal fluctuations in air temperature inevitably affect its climate.

In the article [9], a simplified mathematical method for calculating the radial and axial temperature fields was obtained.

Soil temperature (ST) is closely related to surface air temperature (AT), but other factors can influence their relationship. A study [10] found significant effects of AT on the underlying ST, and the time required to spread down to 320 cm can be up to 10 months. Besides AT, ST is also susceptible to memory effects, namely its thermal preconditions. At deeper depths (e.g., 320 cm) the AT effects from a particular season may be outweighed by the memory effects of the soil from the previous season.

The temperature regime of peat and mineral soils is significantly different. The article [11] proposes an approach that allows modeling the temperature distribution in soils with a complex soil profile, for example, bog ecosystems (peat deposit and a layer of mineral soil under it). Peatlands in the permafrost region are large accumulations of carbon sensitive to global warming.

The article [12] studies how temperature changes at different depths affect the rate of release of CO₂ and CH₄.

Alpine wetlands play an important role in the global carbon cycle during ongoing climate warming, but the timing of carbon dynamics derived from long-term ground-based in situ observations remains unclear. In the course of long-term experiments, the article [13] studied the relationship between temperature and the amount of CO₂ produced.

The highly dynamic nature of snow requires frequent observations to study its various properties. In [14] georadar (GPR) is used to measure the characteristics of the snow cover. A significant correlation was found between field-measured snow depth and data obtained using GPR. This means that GPR can be used to remotely investigate snow cover.

In [15] the spatial and temporal changes in the air temperature, precipitation and snow cover have been quantitatively illustrated for the main regions of northern Eurasia for the period of 1966-2011. Steady warming is registered widely, regional variability of precipitation and snow cover have low trend values – with the exception of the territory of the Russian Far East. Values of regional anomalies of the characteristics under study have been calculated and categorized. The contribution of the cumulative values of anomalies to the total variability of the amount of sample values has been defined for all the characteristics. As a result, statistical distribution of the property values and the type of long-term trends have been refined, particularly for cases with not significant coefficients of the linear trend.

The rate of global warming depends on the amount of carbon entering the atmosphere. One of the most important sources of carbon is soil processes, and the amount of carbon depends on the temperature of the soil. The article [16] analyzes temperature changes over 20 years. Impact of long-term tillage on seasonal fluctuations in soil conditions [17].

2 Materials and methods

The article deals with the problem of heat propagation through the soil. This problem is reduced to solving the equation:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0, \tag{1}$$

Under the boundary condition:

$$u|_{x=0} = \psi(t), t \geq 0, \tag{2}$$

And the initial condition:

$$u|_{t=0} = w(x), x > 0, \tag{3}$$

here $u = u(x, t)$ is the temperature of the soil at a depth of $x(m)$ at time t (days).

If the temperature on the surface repeats from year to year, i.e., the function $\psi(t)$ is periodic; the solution to the problem was obtained by Fourier.

A description of the solution can be found in the book by Tikhonov and Samarsky [18] in the section «temperature waves». This theoretical model has the following disadvantages: 1) When compiling the heat balance, heat consumption during phase transitions is not taken into account; 2) the thermal insulation of the soil, which is created by a layer of snow on the surface, is not taken into account; 3) the surface temperature changes according to exactly the same law in different years, i.e., the boundary condition is given by a periodic function.

In our work, we build a computer model that allows us to answer the question of how the temperature will change at depth if the function $\psi(t)$ deviates from the periodic law.

We considered two variants of deviation 1) Sharp cooling at the end of the year; 2) «abnormally warm winter», when the surface temperature throughout the winter is zero.

Let us apply the algorithm used to solve the problem of «heat propagation in a semi-bounded rod» [18, p. 459]. In this case, the solution is sought in the form $u = v + w$, where v and w are solutions to the following problems:

$$(I) \begin{cases} \frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2} \\ v|_{x=0} = 0 \\ v|_{t=0} = \varphi(x) \end{cases} \quad (II) \begin{cases} \frac{\partial w}{\partial t} = a^2 \frac{\partial^2 w}{\partial x^2} \\ w|_{x=0} = \psi(t) \\ w|_{t=0} = 0 \end{cases} \tag{4}$$

The solution to Problem 1 is determined by the formula:

$$v(x, t) = \frac{1}{2a\sqrt{\pi t}} \int_0^\infty \varphi(\xi) \left[e^{-\frac{(\xi-x)^2}{4a^2 t}} - e^{-\frac{(\xi+x)^2}{4a^2 t}} \right] d\xi. \tag{5}$$

For problem 2, we obtain the solution by the formula:

$$w(x, t) = \frac{x}{2a\sqrt{\pi}} \int_0^t \frac{\psi(\tau)}{(t-\tau)^{\frac{3}{2}}} e^{-\frac{x^2}{4a^2(t-\tau)}} d\tau. \tag{6}$$

Maxima software is used to calculate these integrals and plot the graphs.

Let $\psi(t) = h + A \cos \omega t$, where $\omega = 2\pi/365.25$.

In this case, the solution of the problem of heat propagation, at least up to the moment of time t_0 , will have the form (Tikhonov Samara, p. 247):

$$u(x, t) = h + A e^{-\sqrt{\frac{\omega}{2a^2}} x} \cos\left(\sqrt{\frac{\omega}{2a^2}} x - \omega t\right). \tag{7}$$

It corresponds to the initial condition:

$$u(x, 0) = h + A e^{-\sqrt{\frac{\lambda}{2a^2}} x} \cos\left(\sqrt{\frac{\lambda}{2a^2}} x\right) = \varphi(x). \tag{8}$$

For $t > t_0$, let the function $\psi(t)$ no longer be periodic. In this case, we will perform further calculations using formulas (5) and (6). When calculating the integral (5), the following difficulties arise. Firstly, it cannot be calculated analytically, and secondly, it is taken over an infinite interval. This means that we will have to calculate the incorrect integral using approximate methods. Replace the integral over the infinite interval with the integral over the segment $[0, 50]$.

In this case, the error is equal to the integral from 50 to ∞ . We are interested in the behavior of the integrand on the interval of $1 \leq t \leq 700$. We will assume that $\xi \in [50, \infty), x \in [0.5], 0.04 \leq a^2 \leq 0.09 \Rightarrow 0.2 \leq a \leq 0.3, |\varphi(\xi)| \leq 25$. Then one can estimate the integral:

$$\begin{aligned} & \frac{1}{2a\sqrt{\pi t}} \int_{50}^\infty \varphi(\xi) \left[e^{-\frac{(\xi-x)^2}{4a^2 t}} - e^{-\frac{(\xi+x)^2}{4a^2 t}} \right] d\xi \leq \\ & \leq \frac{1}{2 \cdot 0.2 \cdot \sqrt{\pi} \cdot 1} \int_{50}^\infty 25 \cdot \left(e^{-\frac{(\xi-5)^2}{4 \cdot 0.09 \cdot 700}} - e^{-\frac{(\xi)^2}{4 \cdot 0.09 \cdot 700}} \right) d\xi \leq \\ & \leq 35.26 \cdot [8 \cdot 10^{-4} + 1 \cdot 10^{-4}] = 0.032. \end{aligned} \tag{9}$$

This means that in formula (1) the improper integral can be replaced with an integral from 0 to 50 with satisfactory accuracy.

Let's make the calculation under the boundary condition:

$$u(0, t) = \psi(t) = B - A \cdot \cos(\omega t). \tag{10}$$

And the initial condition:

$$u(x, 0) = \varphi(x) = B - A \cdot e^{-\lambda x} \cdot \cos(\lambda x). \tag{11}$$

Parameter values: $A = 16$ degrees – amplitude of annual temperature fluctuations; $B = 5.7$ degrees – the average annual temperature of the earth surface;

$\omega = 2\pi / 365.25 = 0.0055$ is the frequency of annual fluctuations;

$a^2 = 0.08 \text{ m}^2/\text{day}$ – thermal diffusivity coefficient.

Consider the fragments of the program for calculating the function $v(x, t)$ by Eq (3).

Definition of the integrand:

$$\begin{aligned} vv(x, t, ksi) &:= 1/(2 \cdot a \cdot \text{sqrt}(\%pi \cdot t)) \cdot \text{phi}(ksi) \\ &\cdot \left(\exp\left(-\frac{(ksi-x)^2}{4 \cdot a^2 \cdot t}\right) - \exp\left(-\frac{(ksi+x)^2}{4 \cdot a^2 \cdot t}\right) \right). \end{aligned}$$

3 Results

We will call the temperature field corresponding to the initial and boundary conditions (2) and (3) standard. Let the temperature field be determined by the standard conditions up to a

certain moment. And then the surface temperature deviates sharply from the usual values. How much will the soil temperature change at depth then? We consider two options for deviating from the standard values: «severe frost» and «warm winter».

3.1 Sharp cold snap

For the beginning of the time scale, we take the moment $t = 0$ corresponding to January, 15 (mid-winter). Let the temperature of the earth surface at $320 \leq t \leq 350$ (i.e., from December, 1 to December, 30) be $-30\text{ }^\circ\text{C}$. The rest of the time, it is determined by the boundary condition (10). The calculation results at different depths are presented in the case of a sharp drop in temperature shown in fig. 1.

The magnitude of the deviation from the usual temperature field is shown in fig. 2. It can be seen that the duration of freezing at depth is significantly longer than the duration of freezing at the surface. So, at a depth of 1m, a deviation of two degrees or more is observed for 69 days.

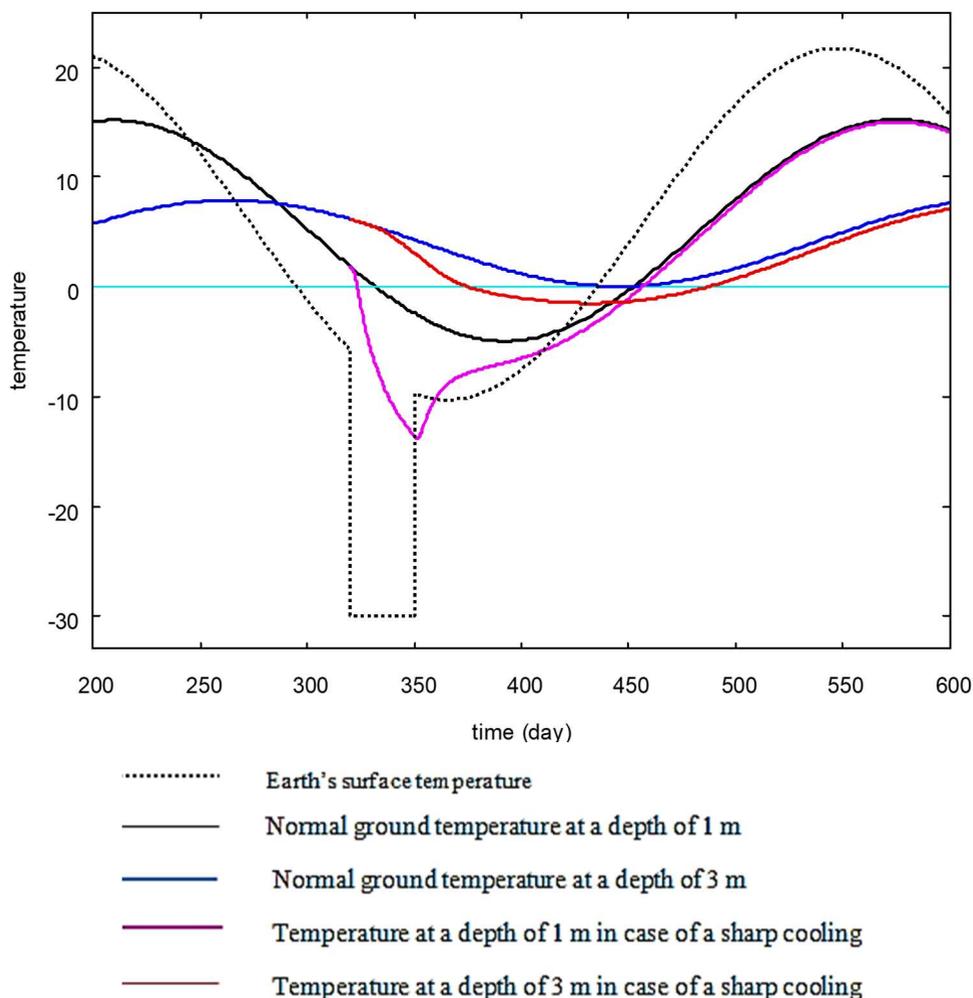


Fig. 1. Comparison of the stationary temperature field and soil temperature as a result of a sharp cooling, up to $30\text{ }^\circ\text{C}$ from 1.12 to 30.12.

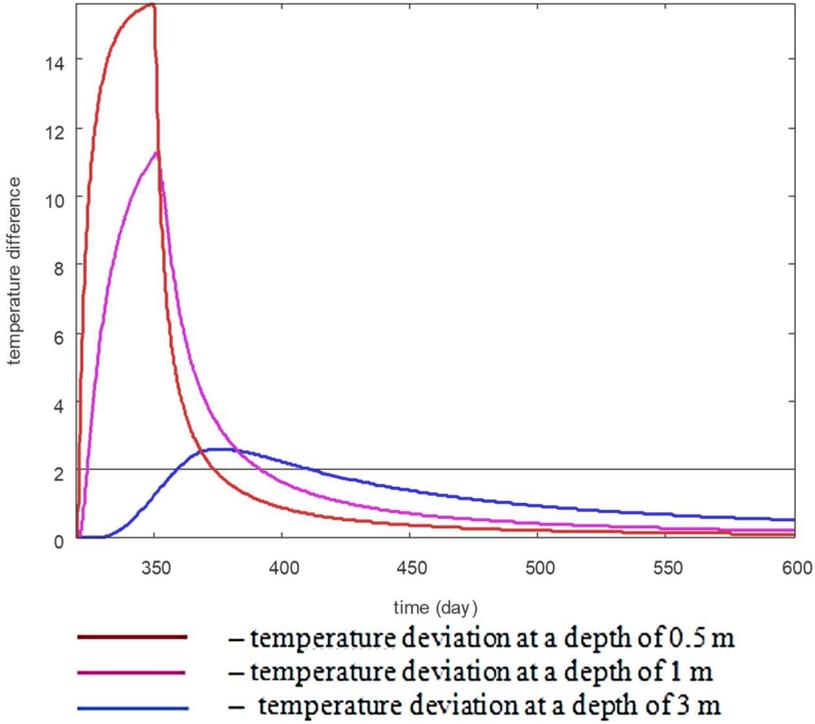


Fig. 2. Difference between the standard temperature field and the perturbed temperature distribution after a cold snap in the time interval from 330 days to 360 days.

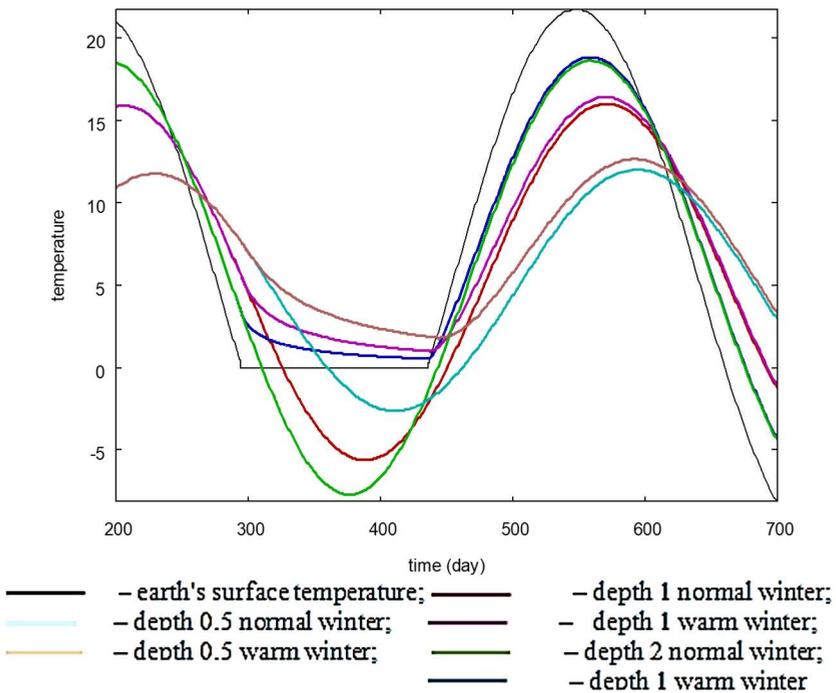


Fig. 3. Temperature plots at different depths in the case of a normal winter and a warm winter.

3.2 Warm winter

The second type of disturbance, i.e., deviations from the standard periodic law of surface temperature will be called «warm winter». Zero temperatures are assumed to be observed throughout the winter. Temperature fields in case of warm winter and normal winter are shown in Fig. 3. Differences between temperatures at depths of 0.5 m, 1m, and 2 m during warm and normal winters are shown in Fig. 4.

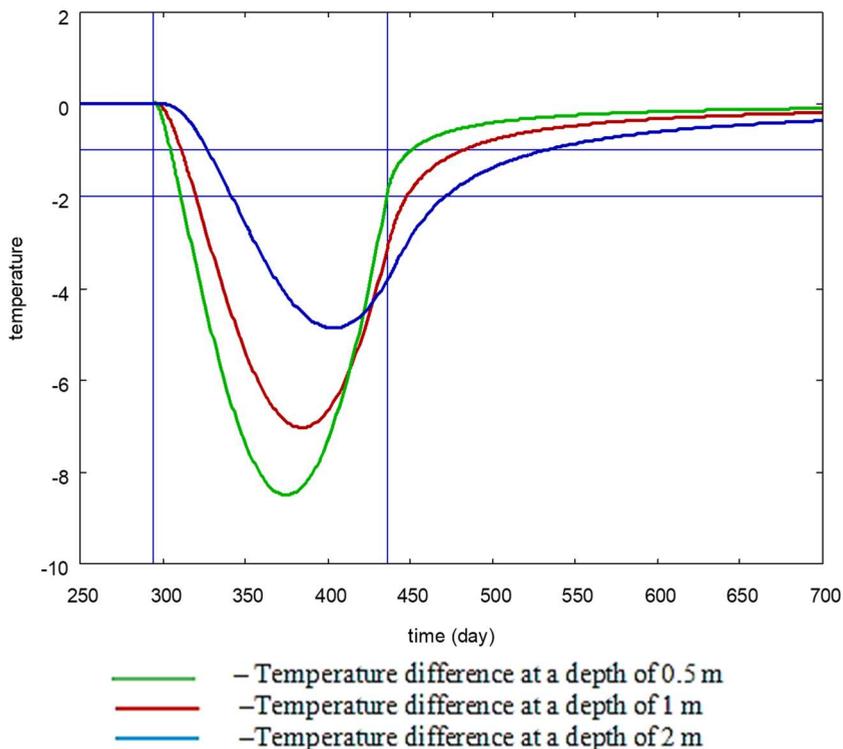


Fig. 4. The difference between the usual ground temperature and the temperature during a warm winter.

4 Discussion

The article considered two examples of calculating soil temperatures with extreme weather deviations from average values. Moreover, in the event of a sharp cold snap at the beginning of winter, there may not be a layer of snow of noticeable thickness. The soil is assumed to be dry. When wet soil is frozen, additional heat is released due to the phase transition.

For Kazan Freezing depth for loams and clays is $m = 1.4$ m; Freezing depth for sandy loams, fine and silty sands is $m = 1.7$ m; Freezing depth for gravelly, coarse and medium-sized sands is $m = 1.9$ m; Freezing depth for coarse soils is $m = 2.1$ m.

We can consider our results as an attempt to estimate from above the unfavorable consequences of extremely weather fluctuations.

Such studies are of interest to specialists in the construction industry, in connection with survey work in the design process, as evidenced by the publications of Russian authors [19, 20] as well as foreign authors [21, 22].

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