Calculation of multilayer sieve classifiers for separating grained materials by size

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Abstract. From the standpoint of stochastic Poisson processes, the separation of granular materials by size on multi-tiered sieve classifiers is studied. Taking into account the selected separation scheme, a system of stochastic differential equations is drawn up for the distribution function of particles along with the sieve of the classifier. The coefficients of the differential equations are determined depending on the probability of sifting particles into the sieve cells. On the basis of the constructed mathematical model and the posed optimization problem, a technological calculation of the sieve classifier is carried out to determine its design and operating parameters, depending on the shape, size, and other characteristics of the material being separated.
Keywords. Mathematical model, separation, granular material, screening probability, multi-criteria optimization.

1 Introduction

In various technological processes, in particular, in the production of building materials, chemical technology, agriculture, the enrichment of minerals, and other areas of industry, it is often necessary to separate dispersed material according to some criteria into certain fractions. One of the most common transport and technological machines for separating bulk materials by size is multi-tiered classifiers. The working body of the multi-tier classifier is several oscillating screening surfaces made in the form of a sieve, which, depending on the characteristics of the separation, can be located in one plane or one above the other, forming tiers. The free-flowing mixture, which is characterized by the density of particle size distribution, is fed from the receiving side of the sieve channel and, under the influence of vibration, moves to the outlet end, being divided into through and outflow fractions.

The use of vibrating apparatus for the separation of dispersed (granular) materials is because vibration converts the dry friction forces characteristic of the interaction of particles of a granular mixture into forces such as viscous friction. As a result of vibration, conditions are created for the manifestation of differences in the separation parameters. Vibration increases the intensity of the separation process, makes it possible to separate particles with slightly different separation parameters.

The classification process on sieves can be represented as a combination of three qualitatively different types of vibration movement, namely, the movement of the material

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layer on the surface of the sieve, the movement of particles of the through-flow fraction towards the sieve through the layer of granular material and the passage of particles through the openings of the sieve.

The calculation of the sieve classifier is reduced to the determination of its design and operating parameters depending on the shape and size of the material to be separated. The technological calculation consists of following stages. The first stage includes the selection of such a shape and size of the holes, the length of the screening surface, which will ensure separation according to a given boundary grain with the required efficiency, as well as the width of the screening surface, taking into account the specific productivity. The second stage includes determination of the optimal values of the operating parameters of the apparatus, for the selected shape of the trajectory of oscillations, taking into account the maximum productivity. The classifiers use several modes of vibration movement with a constant average speed. The selection of the mode mainly depends on such characteristics as the shape, density, linear dimensions (the largest values of the longitudinal and transverse dimensions) of the particles.

In the paper [1], the motion of particles in an oscillating medium was studied, various models of vibrational motion were considered, and dependences were obtained for the average velocity and segregation velocity. Depending on the hydrodynamic properties of the material to be separated, particle size distribution, the shape of individual particles, the presence of specific properties, etc. the motion of granular materials along a vibrating surface can be modeled both in the approximation of a single material point and on the basis of methods of mechanics of heterogeneous media [2-7].

In the paper [8], the process of separation of granular media on sieve classifiers was considered as a diffusion process and a change in the concentration of the number of passage particles along the thickness of a layer of granular material depending on time in the direction of the vibrating surface was studied from the standpoint of Markov processes and described by the Kolmogorov-Fokker-Planck equation. In the papers [2, 9], the process of isolating target products on multistage classifiers was also studied using the theory of random processes.

The study of particles segregation in a fluidized bed based on Markov chains was the subject of the paper [10]. In the paper [11], the basic principles of the organization of technological processes with controlled segregated flows that arise during the processing of granular materials are given.

The Monte Carlo simulation screening probability was described in the papers [12-13]. The probability of sifting particles into sieve cells was studied in the papers [2, 14].

The results of the study of the processes of separation of granular materials on multi-tiered sieve classifiers by the authors of this article were published in [2, 9, 15-17]. The work [2] considers a mathematical model of the process of thin-layer separation of granular materials on multi-tiered sieve classifiers based on the theory of Markov processes. During separation, the coarse fraction is removed from the upper tier, and the product for separation on the next top i^{th} tier is particles sifted from the upper $(i\text{-}1)^{th}$ tier. To determine the distribution function of the number of particles on the surface of the sieves with the selected separation scheme from coarse to fine, a system of stochastic differential equations is constructed. The use of this approach for modelling makes it possible to describe the process of separating granular material into a small number of fractions. This is due to computational difficulties in solving a system of high-dimensional stochastic differential equations. It should also be noted that this equipment has relatively low productivity, but high separation purity and can be used to separate specific granular materials.

The likelihood of sieving per cell is the main parameter by which the separation effect is achieved. Works [15-16] studied the probability of sifting a particle into a cell, depending on the relative speed of its movement on a vibrating surface, the shape, and size of the sieve hole, and particles of the separated material.

In [9], a mathematical model of the process of separation of granular materials on multistage sieve classifiers based on the theory of Poisson processes is considered. To describe the position of particles on the sieves of a multistage classifier, the process of random walk of particles on a plane is considered, which is described by a system of difference equations. The solution of this system is a discrete Pascal distribution, based on which the probability distribution function of sifting particles into the cells of the lower tier and its continuous analogue from the family of gamma densities are constructed. Based on the constructed mathematical model of the separation process, an optimization problem is posed and numerical calculations are carried out. It should be noted that having a fairly high performance, this equipment has relatively low efficiency. Therefore, it can be successfully used for the primary processing of granular materials and for the separation of «tailings» from the main product.

The purpose of this work is, from the standpoint of stochastic Poisson processes, to study and build a mathematical model of the separation of granular materials for calculating a multi-tiered sieve classifier to determine its design and operating parameters depending on the shape, size, and other features of the material being separated, taking into account the selected separation scheme.

2 Materials and methods

In contrast to works [2, 9], mathematical modelling of the process of separation of granular materials on multi-tiered sieve classifiers, taking into account the selected separation scheme from coarse to fine fraction, we will be carried out on the basis of the theory of Poisson processes.

Let us denote by ΔX_i^j – the distance that a particle of the *j*-th fraction passes when moving on the surface of the i^{th} sieve of the classifier, $i=\overline{1,m}$; $j=\overline{1,m+1}$, we call a jump (random event) sifting a particle from the upper sieve to the lower one, the intensity of events λ is the number of transitions that the particle commits from one sieve to another per unit of its length. Let us construct the distribution function $F_i^j(x)$ for the ΔX_i^j values: $F_i^j(x) = P\{\Delta X_i^j \le x\}$ is the probability that the particles of the j^{th} fraction will make *i* jumps, i.e., will be on the surface of the $(i+1)^{th}$ sieve. Let us denote by $P_i^j(x) = P\{\Delta X_i^j > x\} = 1 - F_i^j(x)$ – the probability that the particles of the j^{th} fraction, without sifting, leave the i^{th} sieve. The theory of stochastic processes, to which the separation process on sieves can also be attributed, requires the distribution to be exponential [18].

Since the sieves work together, the probabilities $F_i^j(x)$ for $i \ge 2$ will be conditional. Taking into account that the mathematical expectation of the number of particles passing through the sieve depends not only on the length of the section, but also on its position on the O_x axis, we will consider sieving on the sieve as a non-stationary Poisson process. In view of the non-stationarity of the flow of events, the ΔX_i^j distribution law will depend on where the first event is located on the O_x axis. It can be assumed that the first event occurs at the point $x = 0$, when a particle from the feed hopper enters the beginning of the first sieve. Each event is represented by a point on the coordinate axis, so consider some random placement of points on the O_x axis.

Let us determine the probability $P_1^1(x) = P\{\Delta X_1^1 \ge x\}$ for the first sieve – the probability that is in the section $(0; x)$ with a width of $2b$ (cell size in the transverse direction) in the direction of motion. There is no sifting of particles of the first (coarse) fraction from the first sieve, i.e., particles of the first fraction remain on the surface of the first sieve (no events will appear). Taking into account the coordinate independence of the process changes on non-intersecting intervals (no aftereffect for a Poisson process), $P_1^1(x)$ must satisfy the equation:

$$P_1^1(x+\Delta x) = P_1^1(x)P_1^1(\Delta x)\,. \tag{1}$$

where interval $(0; x)$ is long, Δx is short.

The probability of the absence of an event can be represented as:

$$P_1^1(\Delta x) = P\left\{\min \Delta X_1^1 > \Delta x\right\} = P\left\{\Delta X_1^1(1) > \Delta x\right\} \cdot P\left\{\Delta X_1^1(2) > \Delta x\right\} \cdots P\left\{\Delta X_1^1(n) > \Delta x\right\} =$$

$$= \exp(-(\lambda_1^1(1) + ... + \lambda_1^1(n)) \cdot \Delta x) = \exp(-n \cdot \overline{\lambda_1^1} \cdot \Delta x) = \exp(-\lambda_0^1 \cdot x \cdot \overline{\lambda_1^1} \cdot \Delta x)$$

where $\lambda_0{}^1$ is the rate of entry of particles of the first fraction into the area under consideration, $n = \lambda_0{}^1 \cdot x$ is the number of particles of the first fraction in the interval (0; x), $\overline{\lambda_1{}^1} = p_1{}^1/2a_1$ is the rate of sifting of particles of the 1st fraction into the cells of the 1st sieve, p_i^j is the probability of sifting of particles of the jth fractions into the cells of the ith sieve, $2a_i$ – the step of the ith sieve.

In the first approximation, taking into account the non-stationarity and ordinariness of the flow of events, the probability of the absence of an event in the Δx interval can be estimated by the equation: $P_1^1(\Delta x) \approx 1 - \lambda_0^1 \cdot x \cdot \overline{\lambda_1^1} \cdot \Delta x = 1 - C_1^1 \cdot x \cdot \Delta x$, the probability of one event is $C_1{}^1 x \Delta x$, where the parameter $C_1^1 = \lambda_0^1 \cdot \overline{\lambda_1^1}$, which, like the intensity $\overline{\lambda_1{}^1}$, depends on the probability of sifting into the cell p [9].

Then the difference equation for the probability $P_1{}^1(x)$ taking into account (1) has the form: $P_1^1(x + \Delta x) = (1 - C_1^1 x \Delta x) P_1^1(x)$. Passing to the limit at $\Delta x \to 0$, we can write the following stochastic differential equation: $P_1^{1\prime}(x) = -C_1^1 x P_1^1(x)$. Its solution, taking into account the initial condition $P_1{}^1(0) = 1$ (at the beginning of the sieve, there is no sieving) has the form:

$$P_1^1(x) = \exp(-C_1^1 x^2 / 2). \quad (2)$$

Further, in order to isolate the second largest fraction from the material, it is necessary to perform two successive events: first, the particles of the second-largest fraction must be sieved from the first sieve (event A); secondly, if the particles in question end up on the surface of the second sieve, then they must leave it without sifting (event B). The probability of the first event is determined by the equation: $P(A) = 1 - \exp(-C_1^2 x^2 / 2)$, and the probability of the second event can be calculated by the formula: $P(B) = \exp(-C_2^2 x^2 / 2)$. Thus, the probability that the particles of the second fraction, without sifting from the second sieve, will leave it, is determined by the formula:

$$P_2^2(x) = \exp(-C_2^2 x^2 / 2) \cdot (1 - \exp(-C_1^2 x^2 / 2)). \quad (3)$$

For the rest of the fractions, $j = \overline{3, m}$, the probabilities of their separation from the separated material are determined in a similar way. In order to isolate the jth fraction from the material to be separated, it is necessary to perform two successive events. First, the particles of the j-th fraction must be on the surface of the jth sieve, i.e. must make a (j-1) jump (event A). Then, if the particles are on the surface of the jth sieve, then without sifting, they must leave it. The probability of the first event $P(A)$ is determined by solving a system of stochastic differential equations :

$$F'^{j}_{j-1}(x) = -C^j_{j-1} x F^j_{j-1}(x) + C^{\,j}_{j-2} x F^j_{j-2}(x)\,; \; j = \overline{3, m}, \quad (4)$$

the probability of the second event is determined by the equation : $P(B) = \exp(-C_j^j \cdot x^2 / 2)$. Then the probability of allocation of the j-th fraction is determined by the formula:

$$P_j^j(x) = P(A) \cdot P(B).$$

In the general case, the system of equations (4) can be solved by approximate methods. In some cases, for example, for a multistage classifier in [4], an analytical solution is given in the form : $F^j_{j-1}(x) = 1 - \exp(-Cx^2/2) \cdot \sum_{i=0}^{j-2} (Cx^2/2)^i / i!$, where $C = \lambda_0 \overline{\lambda}$ (on a multistage classifier, all sieve cells have the same dimensions).

It should be noted that the Poisson process can be viewed from various points of view. Poisson's law governs points distributed randomly along the coordinate axis, but Poisson ensembles of points distributed randomly on a plane can also be considered, providing that x is interpreted as an area.

The obtained solutions $P_i^i(x)$ allow one to determine all the characteristics of interest of the process under consideration, in particular, for the selected fractions to calculate the recovery factor and separation efficiency. To do this, it is necessary to determine the probabilities of sifting into cell p_i^j, to identify the constructed model and set an optimization problem to determine the optimal parameters of the classifier.

To determine the probabilities of separating the selected fractions $P_i^j(x)$, it is necessary to determine the probability of sifting particles of the j-th fraction into the cells of the i^{th} sieve, since the coefficients of the differential equations (4) are determined by the expression $C_i^j = \lambda_0^j \cdot p_i^j / 2a_i$, i.e. depend on the likelihood of sifting into the cell. In the first approximation, the sifting probability can be represented as the product of the probabilities of two independent events [6]: $p = p_г \cdot p_v$, where $p_г$ is the geometric probability depending on the size and shape of the sieve opening and the particles of the separated material and p_v is the probability depending on the particle velocity, which is determined according to the formula

$$p_v = 2 - (\Phi(z) + \Phi(z_0)), \quad (5)$$

where $z = (V_a - V_k)/\sigma$, $z_0 = V_k/\sigma$; V_k, σ are the parameters of the normal law, are determined from experimental data in the process of identifying the constructed models, V_a is the amplitude of the particle velocity relative to the sieve, $\Phi(x)$ is the standard normal distribution function. These issues are considered in [15].

Depending on the physical and mechanical properties of the material to be separated, particle size distribution, the shape of individual particles, the presence of specific properties, etc. the movement of dispersed materials on a vibrating surface can be modeled both in the approximation of a single material point and based on the methods of mechanics of heterogeneous media. Modeling the motion as the motion of a material point can be carried out with a thin-layer motion of the separated materials [15]. In this case, the forces of dry friction arising between a vibrating surface and a particle sliding along it are idealized in the form of Coulomb friction. Under certain conditions, in addition to the dry friction force, it is also necessary to take into account the force of viscous resistance to particle motion in the environment. For this, it is necessary to estimate the ratio of this force to the dry friction force, and consider its value, for example, proportional to the relative velocity of the particle. Viscous resistance is considered for relatively small and relatively light particles. In separating machines, it is advisable to use a regular mode of two-way movement with instant stops without tossing, which allows the most complete use of the residence time of the separated material on the surface of the sieve, regular modes of movement with a constant average speed are used. The issues of vibrational motion of granular materials are considered in work [2, 15].

Further, in the process of identifying the constructed models, it is necessary to determine the parameters of the speed modes V_k and σ depending on (5). For this, the calculated values of the recovery factor are compared with the experimental values obtained under certain well-defined high-speed modes of operation of the apparatus. Estimated values of recovery factors are determined based on decisions $P_i^j(x)$.

The recovery factor (descent fraction) of the j-th fraction from the i-th sieve is determined by the formula: $\eta_i^j = \exp(-C_i^j \cdot L_i^2/2), i = \overline{1,m}, j = i, i+1$, where $j = i$ is the coarse (descent) fraction, $j = i + 1$ is the next largest (continuous) fraction, taking into account the separation of the granular material from the coarse fraction to the fine one. The recovery factor is used to calculate the length of the L_i sieve : $\eta_i^i \geq \eta_i^{1*}$, $0 < \eta_i^{i+1} \leq \eta_i^{2*}$, $i = \overline{1,m}$ .The η_i^{1*}, η_i^{2*} values are selected taking into account the requirements for the final separation products, for example, recovery and purity.

The separation efficiency on the i-th sieve is determined by the dependence:

$$E_i = \eta_i^i (1 - \eta_i^{i+1}) \times 100\%, i = \overline{1,m},$$

where η_i^{i+1} is the convergent fraction of the fine fraction in the coarse one, which for the i^{th} fraction is considered as the fraction of impurities in the target product. The separation efficiency is used to calculate the size of the holes of the cells D_i and to match the shape of the cells. The shape of the hole and its size are selected taking into account the maximum efficiency from several types of shapes, for example, round, rectangular, etc. These issues are considered in work [15].

According to the experimental data for each sieve of the classifier, statistical estimates of the probability of sifting are determined:

$$\overline{P}_i^{j,s} = -\frac{2a_i}{L_i}\cdot\frac{2}{\lambda_0^j\cdot L_i}\cdot\ln\overline{\eta}_i^{j,s},\ i=\overline{1,m};\ j=i,i+1,\ s=\overline{1,k_s}, \quad (6)$$

where k_s is the number of experimental points included in the sample, carried out at different speed conditions.

The experimental values of the recovery factor $\eta\overline{_i^j},2$ – (the descent fraction of the jth fraction from the *i*-th sieve at high-speed mode with the serial number *S*) are determined by the debris from the sieves. The choice of the speed regime must be carried out taking into account the requirements for the final products of the separation of the first and second-largest fractions. The first fraction should go off the sieve without sieving, and the second should be sieved from it, leaving an insignificant waste residue, which is used to calculate parameters V_k and σ.

Based on statistical estimates of the probability of sieving (6) for the selected speed modes, the observed values of p^- quantiles of the speed are determined (dimensionless value of the speed at which the speed probability is p_v) :

$$\overline{U}_i^{j,s} = \Phi^{-1}(1-\overline{P}_i^{j,s}/P_{\varepsilon i}^j),\ i=\overline{1,m};\ j=i,i+1;\ \overline{s=1,k_s},$$

where $\Phi^{-1}[*]$ is the computation of the inverse of the standard normal distribution with a probability of p_v.

The calculated value of the p^- quantile of speed can be found by the formula:

$$U_i^{j,s} = \Phi^{-1}(\Phi(z_i^{j,s})+\Phi(z_{0i}^j)-1),\ i=\overline{1,m};\ j=i,i+1;\ s=\overline{1,k_s}, \quad (7)$$

where $z_i^{j,s} = (V_{ai}^s - V_{ki}^j)/\sigma_i^j$, $z_{0i}^j = V_{ki}^j/\sigma_i^j$, V_{ai}^s is the amplitude of the relative velocity of particles on the i^{th} sieve.

The unknown parameters of dependence (7) V_{ki}^j, σ_i^j are determined by comparing the observed and calculated values of the p^- quantile of the velocity:

$$\sum_{s=1}^{k_s}(\overline{U}_i^{j,s}-U_i^{j,s})^2 \to \min_{V_{ki}^j,\,\sigma_i^j}\ . \quad (8)$$

For small fractions that do not have residues from sieves, the distribution parameters are determined from the approximating dependence obtained using the previously found parameter values for large fractions with residues according to formula (8).

For a wicker sieve, the dependence of parameters V_k and σ on the cell and particle sizes can be represented as [2]:

$$V_k(D,l)=\sqrt{\frac{g}{2r}}\left(A_V D - B_V l\right),\ \sigma(D,l)= A_\sigma V_k(D,l)+ B_\sigma\ , \quad (9.1)$$

where r is the radius of the side face of the braided sieve, l is the characteristic size of the j^{th} fraction, D is the size of the opening of the cell of the i^{th} sieve.

For penetrating sieves, the parameters of the velocity probability can be found from the following approximating dependence [1]:

$$V_k(D,l)=\sqrt{\frac{g}{l}}\left(A_v D - B_V l\right)\cdot \quad (9.2)$$

Coefficients of dependencies (9.1) and (9.2) A_v, B_v, A_σ, B_σ are determined by comparing the calculated values of parameters V_k and σ with their previously found optimal values from (8) by the least squares method from the relations:

$$\sum_{i=1}^{n}\sum_{j=i}^{i+1}(V_k(D_i,l_j)-V_{ki}^j)^2 \to \min_{A_V,B_V},\ \sum_{i=1}^{n}\sum_{j=i}^{i+1}(\sigma\ (D_i,l_j)-\sigma_i^j)^2 \to \min_{A_\sigma,B_\sigma}.$$

The form of dependences (9.1) and (9.2) is consistent with the theoretical formulas known in the scientific literature [19].

3 Results and discussions

To establish the optimal design and operating parameters of the classifier, the optimization problem is posed in a multicriteria formulation. The performance of the apparatus and the coefficients of the separation efficiency on sieves are considered as criteria:

$$\max Q(A,\omega,\alpha,\beta,h,B)=\rho_c hBV_{\text{cp}},$$
$$\max E_i(A,\omega,\alpha,\beta,D_i,L_i,\delta_i)=\eta_i^i(1-\eta_i^{i+1})\times 100\%,\ i=\overline{1,m},\tag{10}$$
under conditions:
$$x_j^{\min}\le x_j\le x_j^{\max},\ \varphi_k^{\min}\le\varphi_k(A,\omega,\alpha,\beta)\le\varphi_k^{\max},$$

where x_j^{min}, x_j^{max} are the smallest and largest values of the components of the optimization parameters $\overline{x}=(A,\ \omega,\ \alpha,\ \beta,\ D,\ L,\ h,\ \delta)$, φ_k – are the functional limitations associated with the selected speed mode, ρ_c is the bulk density, B is the width of the sieve, L_i is the length of the sieve, V_{cp} is the average speed of movement of the granular material on the sieve, h is the thickness of the layer of granular material at the beginning of the first sieve, δ_I is the requirement for the end products of separation, for example, for each i^{th} fraction the value is $\eta_i^{i+1}<\delta_i$.

The multicriteria problem (10) is solved in stages [15]. At the initial stage, the optimal design parameters of the apparatus are determined: length L_i, width B, mesh sizes of sieves D_i. For a fixed value of the amplitude of the relative speed V_a and the selected shape of the sieve hole (square, rectangular, round) with a constant value of the area of the holes in the light, the dependence of the separation efficiency is established on the size of the hole [15]. The optimum hole diameter is determined taking into account the maximum separation efficiency. The length of the L sieve is determined taking into account the requirements for the recovery factors: for the coarse (waste) fraction, the recovery factor is $\eta_i^i\ge\eta_i^{1*}$; for fine (continuous) fraction – $0<\eta_i^{i+1}\le\eta_i^{2*}$, where η_i^{1*}, η_i^{2*} are specified requirements for recovery and separation purity. The coarse fraction should leave the sieve without sifting, and the fine one should have time to sift, leaving an insignificant waste residue, which is used to calculate the parameters of model V_k, σ. The probability of sifting particles of the j^{th} fraction into cell p depends on the intensity of their arrival at the beginning of the first sieve from the top λ_0^j, which can be determined from the thickness of the layer h, therefore the parameters V_k, σ depend on the thickness of the layer h at the initial section. Therefore, the layer thickness h must be maintained within a certain interval. To ensure maximum performance, it is necessary, in turn, to ensure the highest average speed. An increase in the average speed of vibration movement can be achieved by increasing the amplitude of acceleration $A\omega^2$.

For the «no tossing» mode, the maximum average speed is reached at the border of this mode [2]. Therefore, the tilt angle α is selected when all the conditions for the existence of the selected motion model are met. The vibration angle β and the vibration frequency ω, or the amplitude A are to be determined, since the optimal acceleration amplitude $A\omega^2$ should be optimal.

Thus, the values of the tuning parameters, for example, A,α, can be selected before operation, and the optimal values of the control parameters ω,β are selected by the decision maker from the solution of the optimization problem (10), for example, from a set of Pareto optimal solutions [20].

4 Conclusion

The theory of random processes, in particular Poisson processes, supplemented by experimental studies to determine the parameters of the model, makes it possible to build mathematical models of the process of separating granular materials by size on sieve classifiers, which are the basis for optimizing and carrying out the technological calculation of the sieve classifier to determine its design and operating parameters depending on the shape, size and other features of the material being separated.

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