Synthesis and analysis of the spatial mechanism

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Abstract. Spatial mechanisms of various types are widely used in technology. Most of them have one or two ball pairs, the presence of which reduces the service life of the device (ball pairs are difficult to protect from dust and dirt. They are more difficult to manufacture and have a lower load-bearing capacity than rotational pairs). Replacing ball pairs with rotary ones, designed, for example, with standard bearings, will greatly simplify and reduce the cost of their manufacturing technology, increase the service life and expand the area of practical application. The difficulty and complexity of creating five-link and six-link mechanisms and devices with a special structure lies in the fact that the usual combination of links fails to achieve their operability (with rotatable links). Theoretically, this mechanism has been studied by many foreign and Russian scientists. However, Kazan scientists were the first to form and put into practice spatial mechanisms with rotational pairs. The paper presents synthesis and analysis of the spatial five-link mechanism by the degree of irregularity, axial displacement, elliptical law and by the reproduction of a linear function. The mechanism provides intensification and energy saving of mixing, separation, torque transmission of various drives.

Keywords. Spatial mechanism, hinge, unevenness, mixing, energy saving.

1 Introduction

Spatial mechanisms with a special structure (with rotary hinges) have interesting and useful properties for practice. A striking example of this is the Bennett mechanism, which has more than seven names of target groups [1-3]. It should be noted that this mechanism is the basic module for the formation of multi-link mechanisms of such type: five-link, six-link, seven-link and differential. This paper considers the synthesis of one of the spatial five-link mechanisms obtained by combining two Bennett mechanisms. Studies have shown that the mechanism also has a variety of functional properties that can be successfully applied in technology.

When forming a five-link mechanism, two Bennett mechanisms are combined so that the driven cranks with the hinges coincide, and the strut links are located sequentially on one straight line. Discarding the combined cranks and one of the common hinges, we get a five-link mechanism, in which the angle of the crank axes and the shortest distance are the same as those of the combined mechanisms, the distance between the axes of the strut hinges is equal to twice the length of the connecting rods of the Bennett mechanism.

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2 Materials and methods

For practical use, the mechanism can be presented in two versions. The first option in the form is shown in Fig. 1, where the rack is a longer link (combined).

In the formed five-link mechanism, the geometric axes of the hinges of the strut 5 are parallel, links 1 and 4 are cranks, and 2 and 3 are connecting rods.

In such a structural design, the mechanism has a number of interesting properties that are implemented at certain parameters. Let's consider the synthesis of the mechanism for each of the properties [4-6].

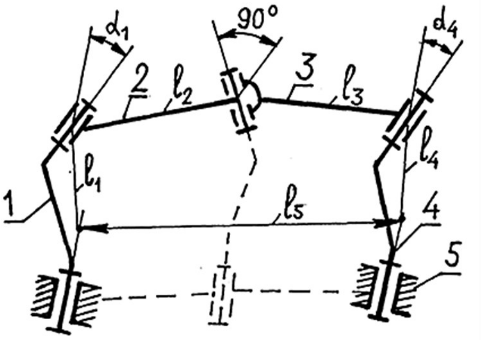

Fig. 1. Mechanism diagram.

2.1 Synthesis by the degree of uneven movement of the driven crank

This mechanism, as well as the Bennett mechanism, has the same property, when the driving crank rotates at a constant angular velocity, the slave has a variable angular velocity per revolution, determined by the expression:

$$\omega_{\text{в}} = \frac{\alpha_1}{1 + sin\ sin\ \alpha_{\ 1} - 2\ sin\ sin\ \alpha_1\ \ cos\ cos\ \varphi}\omega,$$

where α_1 – the angle of intersection of the axes of the crank joints;

ω – drive crank angular velocity;

φ – drive crank rotation angle.

The unevenness of rotation is estimated by the coefficient of unevenness, which is determined:

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_{\text{cp}}},$$

where ω_{max} – the maximum value of the angular velocity, at $\phi = 0^0$, i.e.

$$\omega_{min} = \frac{\alpha_1}{1 + sinsi\ \ \alpha_1 - 2sinsin\ \alpha_1} = \frac{\alpha_1}{(1 - sinsin\ \alpha_1\ )^2}\omega,$$

$\omega_{\min}$ – the minimum value of the angular velocity, at $\phi = 180^0$, i.e.

$$\omega_{min} = \frac{cos^2\alpha_1}{1 + si\ \ \ _1 + 2sin\ \ _1} = \frac{cos^2\alpha_1}{(1 + sin\alpha_1)^2}\omega.$$

Then the degree of non-uniformity is determined:

$$\delta\ = \ \pm 4\frac{tg\alpha_1}{cos\alpha_1}\ = \ \pm 4\ \frac{sin\alpha_1}{1 - sin^2\alpha_1}. \qquad (1)$$

It is required, for example, to transfer rotation between parallel shafts spaced at a distance l_5 with a given degree of unevenness δ. In this case, it is necessary to determine the angle α_l of crossing the axes of the hinges of the cranks, the shortest distance between them and the length of the connecting rods.

The angle α_1 of crossing the axes of the hinges of the cranks is determined from the expression (1):

$$\alpha_1 = arcsin\ arcsin\ \frac{-2 \pm \sqrt{4+\delta^2}}{\delta}. \qquad (2)$$

The shortest distance between the geometric axes of the crank joints is determined as

$$l_1 = 0.5 l_5\ sin\ sin\ \alpha_1.$$

The parameters of the connecting rods: the angle of intersection of the axes of the hinges is 90^0, and the distance between the geometric axes of the hinges $l_2 = l_3 = 0.5l_5$.

2.1.1 Example. To synthesize a mechanism for transferring rotation between parallel shafts of the mechanism with a coefficient of unevenness δ = 0.6 and a distance l_5 between the shafts of 0.5 m

The angle α_1 is determined by the formula (2):

$$\alpha_1 = \alpha_4 = arcsin\ arcsin\ \frac{-2 \pm \sqrt{4+0.6^2}}{0.6} = arcsin\ arcsin\ \frac{-2 \pm 2.088}{0.6}.$$

First angle value $\alpha_1 = \alpha_4 = arcsin\ arcsin\ \frac{-2+2.088}{0.6} = arcsin\ arcsin\ 0.146 = 8.43^0$.

Second angle value $\alpha_1 = \alpha_4 = arcsin\ arcsin\ \frac{-2-2.088}{0.6} = arcsin\ arcsin - 6.81 = no$.

Angle $\alpha_2 = \alpha_3 = 90^o$. Linear parameters:

$$l_1 = l_4 = 0.5l_5\ sin\ sin\ \alpha_1 = 0.5\cdot 500\ sin\ sin\ 8.43^o = 0.5\cdot 500\cdot 0.146 = 36.66 \text{ mm},$$

$$l_2 = l_3 = 0.5l_5 = 0.5\cdot 500 = 250 \text{ mm}.$$

In the same way, it is possible to synthesize the mechanism for other values of the motion unevenness coefficient.

A mechanism with this property can be used in devices that carry out various technological processes. For example, it is implemented to drive the web of a bar elevator in order to increase separation and reduce energy consumption and material injury, while the separation increases more than three times, and material injury is halved and energy consumption – by 50%.

When mixing materials in drum mixers with a drum drive by means of this mechanism, the process time is more than halved, energy consumption is reduced by 50-60%, and the homogeneity of the mixture increases to 97-98%.

The mechanism can also be recommended for other devices in which it is necessary to convert the constant angular velocity of the drive shaft into the variable speed of the driven shaft, with parallel shafts. Bennett's mechanism cannot be applied to this case [7-9].

2.2 Synthesis of the mechanism by the axial displacement of the working shaft from the middle position

In technology, it becomes necessary to carry out axial displacement from the middle position of one working shaft relative to the other. For example, when processing polymeric materials in the chemical industry, two worm mixers are used, which, however, do not provide the required processing quality due to the fact that the material is poorly processed in a constant inter-turn space by worm shafts rotating at a constant angular velocity. To improve the quality of materials processing by intensifying the mixing process in the inter-turn space, it is necessary to provide axial displacement of one worm relative to the other with simultaneous uneven rotation.

For these purposes, this five-link mechanism can be successfully used, the synthesis of which was carried out according to a given axial displacement ∇S of the turns. The parameters of the mechanism should be selected taking into account the given offset.

Depending on the physical and mechanical properties of the mixed materials, the parameters of the worm shafts are constructively assigned: t – step; ∇S – axial displacement of the worm turns; S – the thickness of the turns of the worm; l_5 – center distance of worm shafts.

The angle α_1 of crossing the axes of the crank joints is determined by the expression:

$$\alpha_1 = 90^0 \frac{t - 2\nabla S - 2S}{t}, \tag{3}$$

where t – the pitch of the turns of the worm;

∇S – given displacement of the worm turns;

S – the thickness of the turns of the worm.

With a two-thread worm, the angle α_1 is determined as $\alpha_1 = 90^0 \frac{t - 2\nabla\ - 2S}{2t}$.

At n – thread worm the angle is $\alpha_1 = 90^0 \frac{t - 2\nabla S - 2S}{nt}$.

The maximum displacement X of the worm from the middle position is determined by the expression:

$$X = \frac{t - \nabla S - S}{2}. \tag{4}$$

The shortest distance between the geometric axes of the crank joints is determined:

$$l_1 = l_4 = l_2 \sin \sin \alpha_1.$$

The length l_2 of the connecting rods will be equal to half the center distance, i.e., $l_2 = l_3 = 0.5 l_5$. The angle between the axes of the joints of each of the connecting rods is 90^0.

2.2.1 Example. Find the parameters of a two-worm mixer with an axial displacement of X turns of the worm from the middle position equal to 25 mm, the worm is single-threaded, the pitch of the turn is t = 70 mm, the center distance is 300 mm.

Based on the given displacement, according to the formula (4), one of the parameters ∇S or S is assigned and the other is being determined.

Assign the thickness S of the turns equal to 14 mm, then the minimum gap ∇S between the turns will be determined ∇S = t – S – 2X = 70 – 14 – 2 · 25 = 6 mm.

According to expression (3), the angle α_1 of crossing the axes of the crank joints will be

$$\alpha_1 = 90^0 \frac{70 - 2\cdot 6 - 2\cdot 14}{70} = 38^0 34'.$$

The shortest distance l_1 between the axles of the cranks is $l_1 = l_2 \sin \sin \alpha_1 = 150 \sin \sin 3\,8^0 34' = 150 \cdot 0.6234 = 93.52\ mm$.

The length $l_2 = l_3$ each of the connecting rods is equal to half the center distance of the worms, i.e., 150 mm.

Consequently, the specified axial displacement of the worm from the middle position, equal to 25 mm, is provided by the following parameters of the cranks and connecting rods: $\alpha_1 = \alpha_4 = 38^0 34'$, $l_1 = l_4 = 93.52$ mm and $l_2 = l_3 = 150$ mm, $\alpha_2 = \alpha_3 = 90^0$, $l_5 = 300\ mm$.

In the same way, the mechanism is synthesized for other values of displacement and other parameters of the turns.

2.3 Synthesis by the elliptical law of motion of the driven link

The mechanism with parallel axes of the driving and driven shafts of the cranks can be used instead of elliptical wheels, i.e., to reproduce the «elliptical» law of motion of the driven link. In this case, the transmission is greatly simplified; its accuracy, reliability and durability are increased. It is necessary to find the parameters of the mechanism that replaces elliptical wheels with a given eccentricity e.

According to the research of P.G. Mudrov [4] the kinematic dependence for the mechanism is determined by the ratio:

$$\frac{tg\,0{,}5\phi}{tg\,0{,}5\phi_4} = \pm \frac{0{,}5(\alpha_1 - \alpha_2)}{0{,}5(\alpha_1 + \alpha_2)} = \pm \frac{1+e}{1-e},$$

where *e* – eccentricity of the ellipse, which is defined for the mechanism by the expression:

$$e = \frac{sinsin\, \alpha_1\, sinsin\, \alpha_2}{1 \pm coscos\, \alpha_1\, coscos\, \alpha_2}. \qquad (5)$$

The minus is taken when the angles α_1 and α_2 are in different quadrants. From expression (5), the value of the angle α_2:

$$\alpha_2 = arcsin\, arcsin\, \left(-e\, sin\, sin\, \alpha_1\ \pm cos\, cos\, \alpha_1 \sqrt{1-e^2}\right). \qquad (6)$$

The plus is taken at $\alpha_1 < 90^0$, minus – at $\alpha_1 > 90^0$. The shortest distance between the axes of the crank joints is determined:

$$l_1 = l_4 = l_2 \frac{sinsin\, \alpha_1}{sinsin\, \alpha_2}. \qquad (7)$$

The synthesis is carried out in the following order. For example, it is necessary to provide an eccentricity with e = 0.2 m and center-to-center distance $l_5 = 0.3$ m.

The value of the angle α_1 is given constructively, for example, we take the angle α_1 = 250, then the angle α_2 is determined from expression (6):

$$\alpha_2 = arcsin\, arcsin\, \left(-0.2 \cdot 0.4226 + 0.9063\sqrt{1-0.2^2}\right) = 53^0 28^{'}.$$

The shortest distance l_1 is determined from the expression (7) considering that:

$$l_2 = l_3 = 0{,}5 l_5 = 0{,}5 \cdot 0{,}3 = 0{,}15 m,$$

$$L_1 = \frac{0.15 \cdot 0.4226}{0.8034} = 0.0788\, m = 78.8\, mm.$$

The angle of twisting α_3 of the connecting rod 3 is determined

$$\alpha_3 = 180^0 - \alpha_2 = 180^0 - 53^0 28' = 126^0 32'.$$

Thus, the parameters of an elliptical mechanism with an eccentricity e = 0.2 m are: $\alpha_1 = \alpha_4 = 25^0$, $\alpha_2 = 53^0 28'$, $\alpha_3 = 126^0 32'$, $\alpha_5 = 180^0$,

$$l_1 = l_4 = 78.8\, mm, l_2 = l_3 = 150\, mm, l_5 = 300\, mm.$$

2.4 Synthesis by reproducing a linear function

The mechanism in this structural design can reproduce a linear function in certain sections of the angle of rotation of the driven crank, the equation of motion of which is determined by the expression:

$$\psi = arcsin\, arcsin\, \frac{\alpha_1\, sinsin\, \phi}{1+\alpha_1 - 2 sinsin\, \alpha_1\, coscos\, \phi}.$$

The analysis of the formula revealed the values of the angle α_1 of crossing the axes of the crank joints, at which the linear dependence of the driven crank is ensured. It was found that with a decrease in the angle α_1, the graph of the dependence of the angle Ψ of rotation of the driven crank approaches quite accurately the linear dependence in individual sections of the graph.

In this case, the crossing angle lies in the range 12...3^0, the angle less than 3^0 is not desirable to take due to design considerations and the large value of the shortest distance between the axes of the hinges, as well as the operability of the mechanism. The counting of a linear function, for example, at a crossing angle α_1 of 3^0, starts from 90 to 270^0 and from 290 to 70^0, i.e., on one revolution of the driven crank. There are two sections of the linear function.

At an angle of $\alpha_1 = 10^0$, the first section lies within the angle 85 ... 265^0, the second section at the angle of rotation 285 ... 65^0.

The linear parameters of the mechanism (the distance l_5 between the axes of rotation of the shafts of the crooked spikes and the length $l_2 = l_3$ of the connecting rods) are assigned constructively, based on practical needs. The shortest distance between the axes of the crank joints is determined by the expression $l_1 = l_2\, sin\, sin\, \alpha_1$.

The accuracy of the linear function is within 2 ... 8%.

The mechanism can be presented in the second version, when one of the two connecting rods, for example, connecting rod 2, is taken as a fixed link (rack). In this case the nature of the movement of the links of the mechanism changes. So, the second of the connecting rods becomes a balancer (co-arm), and the former rack becomes a connecting rod.

3 Results and discussion

Spatial mechanisms have been used by men since ancient times. These mechanisms have in their structure ball socket joints, ball socket joints with fingers, cylindrical and rotatory hinges, combination of which permits to create mechanisms for necessity. There are no difficulties in the formation of such mechanisms. But the simplicity of the creation leads to the low period of service, by the limitations of the bearing ability, by the complicacy of making ball and socket joints ad by some other shortcomings [10-13].

In every respect it is preferably to have in mechanisms only rotating hinges, mounting by the standard bearings of swaying or bearings of sliding. It appeared so, that to use in spatial mechanism only rotating hinges was a very complicated and difficult task.

There is no other example in the history of mechanics being for such a long period of time in development and statement as spatial mechanisms with rotating hinges.

In the variant of crank-balanced performance the mechanism also can be used in different cases.

First, for connection of rectilinear motion on the direct and reverse motion of balancer, that can be used in instruments, scanning devices and so on.

Second, for reproduction of different movements, for instance, oars for boats, wings for ornithopter, brushes for furs and so on.

Third, for connection to working parts (the capacity of mixer, the blade of stirrer and so on) of rotary and return-rotary movement.

Fourth, it can be used in training equipment and devices for entertaining, for creation of a strong training and entertaining effects.

The group of spatial mechanisms with rotating hinges was only accomplished by many mechanisms within the past few years. It was clarified that such mechanisms possess interesting characteristics and are worth attention of scientists and professional experts, as on their deepened theoretical investigation and on usage in different areas of engineering [14-17].

As it is evident from the article's material, spatial mechanism possesses multifunctional characteristics, knowledge of which will accomplish the scope of scientists and professional experts.

4 Conclusions

The investigated spatial five-link mechanism with rotary joints can be synthesized and used in engineering in a wide variety of cases. Placed on a rack by a long link, the mechanism has two connecting rods and two cranks, the axes of rotation of which are parallel. In this case, the mechanism is synthesized according to four properties:

- by the degree of uneven rotation of the driven crank;
- by axial displacement of the working shaft from the middle position;
- by the elliptical law of motion of the driven link;
- by reproduction of a linear function.

The same mechanism, put on one of the two connecting rods, becomes crank-rocker (balance), in this case the synthesis is carried out according to two properties [15-21]:

- along the balancer;
- on the reproduction of a linear function on the forward and backward moves.

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