Clustering Algorithm Based on Density of Data

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Abstract: The k_means clustering algorithm has very extensive application. The paper gives **out_in clustering algorithm based on density**. The algorithm combines distance with data density to adapt to data distribution. It can effectively solve the clustering of data. **Out_in clustering based on density** reduce distortion by move out and move in. Simulation results show that out_in clustering algorithm is more effective than the k_means clustering algorithm.

1 Introduction

Clustering algorithm have very extensive application in pattern recognition. There are several ways to achieve clustering, such as Gas Ref[1-4], Clustering Algorithm based on Sofm Ref[5-8]. Ref[9] gives a clustering algorithm based on density. DBSCAN clustering algorithm is also density-based, and it is typical based on different data densities, it is an effective method to solve the clustering problem of different data distributions. The paper gives **Out_in clustering algorithm based on density**. The algorithm can effectively solve clustering problem of different data distributions, the local density of data is taken as the probability of data element in this paper. The advantage of mean clustering algorithm is that the mean is the optimal clustering center based on Euclidean metric. Average is widely used Ref[10-11]. Howerer, Euclidean distance alone is not enough. Based on probability of data and probability of clusters, the paper proposes a distortion function. And the distortion function is minimized by local strategy. **Out_in clustering algorithm based on density** uses local density as probability of data element and uses density of clusters as probability of clusters. According to the statistical characteristics of data, the probability distribution between clusters should be different. In **out_in clustering algorithm based on density**, Euclidean distance combined with probability distributions, at the same time, mean is still the clustering center, thus adapting to different data distributions. The simulation results show that **Out_in clustering algorithm based on density** converges quickly. Clustering algorithm is widely used in vector quantization image compression[12-16], **Out_in clustering algorithm based on density** is same.

2 Text

Theorem 1: Given data distribution: $p\{X=x_k\} = p_k \ k=1,$

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$2 \dots n$. p_k is probability of x_k . $p_1 + p_2 + \dots + p_n = 1$. Mathematical expectation $E = p_1 x_1 + p_2 x_2 + \dots + p_n x_n$. Base on the Euclidean metric, the optimal cluster center of the given data distribution is $E = p_1 x_1 + p_2 x_2 + \dots + p_n x_n$.

Proof: Let C is the clustering center of the given data distribution, the distortion $S = p_1 (x_1 - C)^2 + p_2 (x_2 - C)^2 + \dots + p_n (x_n - C)^2$.

According to the extreme value theorem, $dS/dC = 0$,
 $-2p_1 (x_1 - C) - 2p_2 (x_2 - C) - \dots - 2p_n (x_n - C) = 0$

So, $C = (p_1 x_1 + p_2 x_2 + \dots + p_n x_n) / (p_1 + p_2 + \dots + p_n)$

$p_1 + p_2 + \dots + p_n = 1$

$C = p_1 x_1 + p_2 x_2 + \dots + p_n x_n$

So, the optimal clustering center of the given data distribution is $E = p_1 x_1 + p_2 x_2 + \dots + p_n x_n$.

Q. E. D.

Theorem 1 gives a meaningful result about mathematical expectation. For multidimensional data, **Theorem 1** is the same holds true. Proof is needed to use partial derivative. For the equal probability data distribution, the optimal clustering center of given data distribution is $E = (x_1 + x_2 + \dots + x_n)/n$, it is average value. For multidimensional data, the optimal clustering center of given data distribution is average vector. Of course, need to assume that components of vector are independent.

Theorem 2: If (Ω, ψ, P) is probability space (Ω is sample space, ψ is event domain, $P(\cdot)$ is the probability defined in (Ω, ψ)), $B_1, B_2, \dots, B_n$ is pairwise incompatible positive probability events, $B_i B_j = \Phi$, $i, j = 1, 2, \dots, n$; $P(B_k) > 0$, $k = 1, 2, \dots, n$; A is subsubset of $B = \bigcup B_k$, then

$$P(A) = \sum P(A | B_k) P(B_k)$$

Proof: $\because A$ is subsubset of $B = \bigcup B_k$

$\therefore A = AB = A(\bigcup B_k) = AB_1 \cup AB_2 \cup \dots \cup AB_n$

$\because B_i B_j = \Phi$, $i, j = 1, 2, \dots, n$

$\therefore (AB_i)(AB_j) = \Phi$, $i, j = 1, 2, \dots, n$

$$\begin{aligned} & \because P(B_k) > 0, k=1, 2, \dots, n \\ & \therefore P(A) = P(AB_1) + P(AB_2) + \dots + P(AB_n) \\ & = P(A | B_1)P(B_1) + P(A | B_2)P(B_2) + \dots + P(A | B_n)P(B_n) \\ & = \sum P(A | B_k)P(B_k) \end{aligned}$$

Q. E. D.

Theorem 2 show that total probability theorem in Ref[17-18] is inference of **Theorem 2**.

The clustering problem: The given data distribution: $p\{X=x_k\} = p_k, k=1, 2, \dots, n$. p_k is the probability of x_k , $p_1 + p_2 + \dots + p_n = 1$. Divide $x_1, x_2, \dots, x_n$ into m clusters.

The general situation is discussed below. The optimal solution of the problem is almost impossible to calculate. The strategy of finding the local optimal solution is adopted. Although k-means algorithm adopts local strategy, the actual effect is very good. **Out_in clustering algorithm based on density** also uses a local strategy to find the optimal solution from the current clustering result. Of course, in most case the local optimal solution is found. A distortion function is given below, according to the **Theorem 2**. Here, adopt euclidean metric. The distortion function is related to distance and probability distribution, therefore clustering is not limited to spherical clusters. The process can be understood as such a random event: (1) In $x_1, x_2, \dots, x_n$, select an element x_k . (2) Put x_k to a cluster. (1) and (2) are independent. According to **Theorem 2**,

$J = \sum q_k \sum p_t \sum (x_{ij} - c_{kj})^2$ x_{ij}, c_{kj} is the j th component of x_t, c_k

Adopts local strategy,

$$\text{Min } J = \sum q_k \sum p_t \sum (x_{ij} - c_{kj})^2 \quad (1)$$

Probability of cluster k is Q_k , $q_k = Q_k / \text{num}$, num is number of elements in cluster k , c_k is the clustering center of cluster k , x_t is in cluster k , Probability of x_t is p_t . **Out_in clustering algorithm based on density** uses local density as probability of data. In **out_in clustering algorithm based on density**, for any data element, **move out** first from the cluster; then to select the cluster with the least distortion, **move in** data element to the cluster. In order to simplify the calculation, do the following appropriate derivation, suitable for fast iteration:

If x_t selects cluster k , to **move in**. Be divided into two conditions: (1) x_t is not in cluster k , (2) x_t is in cluster k .

(1) Let $s_k = p_1 \sum (x_{1j} - c_{kj})^2 + \dots + p_h \sum (x_{hj} - c_{kj})^2$, distortion function $J = \sum q_k s_k$. In the above model, based on the local strategy, for cluster k , assume that components of vector are independent, at the same time, assume that clusters are independent. The optimal clustering center is $c_k = (p_1 x_1 + \dots + p_h x_h) / (p_1 + \dots + p_h)$, similarly to the proof of **Theorem 1**. If x_t selects cluster k , then new clustering center of cluster k is $C_k = (p_1 x_1 + \dots + p_h x_h + p_t x_t) / (p_1 + \dots + p_h + p_t)$, p_t is probability of x_t , q_k is the probability of cluster k .

$$\text{Let } S_k = p_1 \sum (x_{1j} - C_{kj})^2 + \dots + p_h \sum (x_{hj} - C_{kj})^2 + p_t \sum (x_{tj} - C_{kj})^2$$

$$S_k - s_k = p_1 \sum (x_{1j} - C_{kj})^2 + \dots + p_h \sum (x_{hj} - C_{kj})^2 + p_t \sum (x_{tj} - C_{kj})^2 - \sum p_i \sum (x_{ij} - c_{kj})^2$$

$$\begin{aligned} & \because (p_1 x_1 + \dots + p_h x_h) = (p_1 + \dots + p_h) c_k \\ & \therefore = (p_1 + \dots + p_h) \sum (c_{kj} - C_{kj})^2 + p_t \sum (x_{tj} - C_{kj})^2 \\ & = (p_1 + \dots + p_h) p_t^2 / (p_1 + \dots + p_h + p_t)^2 \sum (x_{tj} - c_{kj})^2 + \\ & p_t \sum (x_{tj} - C_{kj})^2 \\ & = p_t^2 (p_1 + \dots + p_h) / (p_1 + \dots + p_h + p_t)^2 \sum (x_{tj} - c_{kj})^2 + p_t (p_1 + \dots + p_h)^2 / (p_1 + \dots + p_h + p_t)^2 \sum (x_{tj} - c_{kj})^2 \\ & = p_t (p_1 + \dots + p_h) / (p_1 + \dots + p_h + p_t) \sum (x_{tj} - c_{kj})^2 \\ S_k & = s_k + p_t (p_1 + \dots + p_h) / (p_1 + \dots + p_h + p_t) \sum (x_{tj} - c_{kj})^2 \quad (2) \end{aligned}$$

Let $Q_k = p_1 + \dots + p_h$ is probability of cluster k , $q_k = Q_k / h$, then the increase distortion of the cluster k is:

$$I J_k = ((Q_k + p_t) / (h+1)) S_k - (Q_k / h) s_k \quad (3)$$

(2) If x_t is in cluster k , similarly,

$$\begin{aligned} \text{Let } s_k & = p_1 \sum (x_{1j} - c_{kj})^2 + \dots + p_h \sum (x_{hj} - c_{kj})^2 + p_t \sum (x_{tj} - c_{kj})^2, \\ c_k & = (p_1 x_1 + \dots + p_h x_h + p_t x_t) / (p_1 + \dots + p_h + p_t) \\ C_k & = (p_1 x_1 + \dots + p_h x_h) / (p_1 + \dots + p_h) \\ S_k & = p_1 \sum (x_{1j} - C_{kj})^2 + \dots + p_h \sum (x_{hj} - C_{kj})^2 \\ S_k - s_k & = p_1 \sum (x_{1j} - c_{kj})^2 + \dots + p_h \sum (x_{hj} - c_{kj})^2 + p_t \sum (x_{tj} - c_{kj})^2 - \\ & p_1 \sum (x_{1j} - C_{kj})^2 - \dots - p_h \sum (x_{hj} - C_{kj})^2 \\ & = (p_1 + \dots + p_h) \sum (c_{kj} - C_{kj})^2 + p_t \sum (x_{tj} - c_{kj})^2 \\ & = p_t^2 (p_1 + \dots + p_h) / (p_1 + \dots + p_h + p_t)^2 \sum (x_{tj} - c_{kj})^2 + \\ & p_t \sum (x_{tj} - C_{kj})^2 + p_t \sum (x_{tj} - c_{kj})^2 \\ & = (p_t^2 / (p_1 + \dots + p_h)) \sum (x_{tj} - c_{kj})^2 + p_t \sum (x_{tj} - c_{kj})^2 \\ & = p_t (p_1 + \dots + p_h + p_t) / (p_1 + \dots + p_h) \sum (x_{tj} - c_{kj})^2 \end{aligned}$$

$$S_k = s_k - p_t (p_1 + \dots + p_h + p_t) / (p_1 + \dots + p_h) \sum (x_{tj} - c_{kj})^2 \quad (4)$$

Let probability of cluster k is $Q_k = p_1 + \dots + p_h + p_t$, $q_k = Q_k / (h+1)$, then the increase distortion of the cluster k is:

$$I J_k = (Q_k / (h+1)) s_k - ((Q_k - p_t) / h) S_k \quad (5)$$

Out_in clustering algorithm based on density uses local density as probability of data. This can be achieved by grid division. The data space is divided into grids. Count the number of data elements in the grid, the density of each grid unit is determined. uses the density of each grid unit as the probability of the grid. The size of grid affects the probability distribution of data. Therefore, the appropriate grid edge length should be selected to reflect the probability distribution of data. In **Out_in clustering algorithm based on density**, the order in which grid elements select clusters has an impact on the clustering results. So, the operation is performed in the order from small probability grid element to large probability grid element. In this way, give small probability grid elements a chance to cluster. Here, a grid element used as data in clustering algorithm. Specifically, the grid is replaced by the average value of data elements in the grid. Uses the probability of grids in the cluster as the probability of the cluster.

Algorithm 1 (Out_in clustering algorithm based on density):

1 To select randomly m unrepeated data from $x_1, x_2 \dots x_n$, to become the initial clustering centers: $c_1, c_2 \dots c_m$.

2 The remaining data elements select the cluster according to formula (3), to **move in**.

Any one x_i , $I J_o = \min \{I J_k\}, k=1, 2 \dots m$. Cluster o is selected.

3 Any one $x_i, t=1, 2, \dots n$; To **move out** first.

Then to **move in**,

If x_i is in cluster k

Compute $I J_k$ according to formula (5)

Else

Compute $I J_k$ according to formula (3)

End if

$I J_o = \min \{I J_k\}, k=1, 2 \dots m$. cluster o is selected.

Compute cluster parameters: total probability, number of elements in the cluster, mean value.

4 If the convergence condition is not satisfied

GoTo **3**

Else

End

End if

Theorem 3: **Algorithm 1** converges to extreme point.

Proof: In each iteration of the **Algorithm 1**. The distortion J is monotonically decreasing.

The distortion J is monotonically decreasing and is bounded,

So **Algorithm 1** converges.

Let the currently clustering centers is the optimal clustering centers,

The distortion J is monotonically decreasing, according to **Algorithm 1**.

Algorithm 1 converges to the optimal clustering centers,

So, **Algorithm1** converges to extreme point.

Q.E.D.

3 Simulation

The software of the simulation is MATLAB 2012a. Vector quantization of image is applied to simulations. The simulation original image is LENNA (256*256), and **Algorithm 1** is compared with k-means algorithm. The standard uses mean square error (MSE), the edge length of grid is 8. The number of clusters is 256. Because the initial clustering center has random factors, the simulation was done 5 times. The simulation results are given in table 1. **Algorithm 1** converges quickly. The result of simulation explains that **Algorithm 1** is more effective compared with the k-means algorithm. Simulation results show that **Out_in clustering algorithm based on density** is more effective than the k-means clustering algorithm for clustering problem, average performance improve 165.3%, as shown in TABLE 1. At the same time, simulation results show that the distortion function is effective.

TABLE 1: MSE of reconstructed image

Number of times	1	2	3
Algorithm 1	38.40	38.26	37.46
K-means	45.28	45.24	44.74
Number of times	4	5	average
Algorithm 1	36.81	37.55	37.70
K-means	45.36	45.21	45.17

4 Conclusion

Simulation results show that **Out_in clustering algorithm based on density** can process clustering problem. **Out_in clustering algorithm based on density** adapt to different data distribution, while retaining the characteristics of kmeans clustering algorithm. The clustering method base on kmeans algorithm is spherical clustering method, which can only solve the convex clustering problem. In **Out_in clustering algorithm based on density**, probability distribution is introduced, so **Out_in clustering algorithm based on density** can also solve the non_convex clustering problem. At the same time, due to the introduction of probability distribution in Out_in clustering algorithm based on density, result of clustering is different to the convex clustering problem. This paper deals with the clustering problem by mathematical statistics method, and obtains a meaningful conclusion. In **out_in clustering algorithm based on density**, Euclidean distance combined with probability distributions, can deal with both convex and non convex data clustering problems. Analysis on clustering of non convex data will be given in the following paper.

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