

Force index computation for a magnetic separator based on permanent magnet

Fatima Ezzahra Kassid^{1,*}, Essaâdia Azelmad¹, Lahbib Bousshine¹, and Abdelmajid Berdai²

¹National Higher School of Electricity and Mechanics of Casablanca, Hassan II University, Material and Structure Mechanics Team, Casablanca, Morocco

²National Higher School of Electricity and Mechanics of Casablanca, Hassan II University, Electrical Systems and Energy Laboratory, Casablanca, Morocco

Abstract. Magnetic separation devices are widely used to separate tramp of iron from a specific feed of materials. Many industries rely on those devices and the variety of technological solution depend mainly on the characteristic of the tramp to be separated. In this paper, main characteristics of low intensity magnetic separation devices based on permanent magnets for dry feed are considered to evaluate force index computation.

1. Introduction

For a suspended magnetic separator based on permanent magnet, its capacity to generate enough force index and torque of rotation are essential for assessing its efficiency [1]

The torque must rotate the particle following its axe of magnetisation and the force index should attract it to the separator. Both factors depend on magnetic induction generated by the permanent magnet as it's expressed in equation (1).

$$\vec{f_m} = \nabla \left(\frac{\vec{m} \cdot \vec{B}}{\mu_0} \right) \quad (1)$$

Understanding factors impacting magnetic induction flux is very essential to move forward with understanding force index mechanism. Many studies discussed explicitly those factors, like:

shape and configuration of the magnet : flat magnets produce less B than bar magnets .If magnets are put in an assembly, the produced B depends on the configuration type: attraction or repulsion. [2].this help identifying the location of the “zero force” in the separator design.

Orientation of the field: Field flux shape depends on the orientation of the magnetisation inside the magnet. Particle should be surrounded by the magnetic induction flux, in order for the force to be applied .It was proved for rectangular magnets, that orientation through their thinnest direction help increasing the produced B and give a higher energy [3]

Also, uniformity of the magnetic field should be taking seriously into consideration. If H is not uniform, the particle has to be pushed to a smaller distance, from an initial energy state to a final energy state. [4].

This paper, by means of computing method using finite element, aims to propose a program evaluating the

required force index to pull the particle to the magnet, starting by magnetic induction flux calculation.

The program can be used in the design phase of the magnetic separator by choosing the optimum dimensions and precise magnet parameters. It can also be used in the final stage by simulating the behaviour of the separator towards the particle.

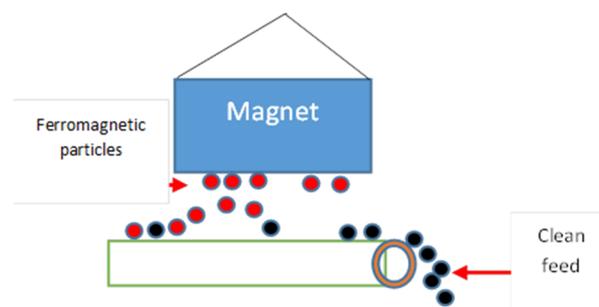


Fig. 1. Suspended magnet separator.

As vector potential method appears to be more effective method to calculate magnetic induction than scalar potential for this application [5]. This method is used in two dimensions with triangular elements. Permanent magnet is chosen as parallelepiped magnet, where its dimensions are shown in “Fig.2”;

Firstly, a mathematical formulation based on vector potential is presented, and then boundary conditions are discussed. Afterwards, finite element method is used in order to convert the obtained equation into matrixes.

Secondly, a brief presentation of the program generated by MATLAB is given in order to implement the data using the finite element method.

Thirdly, an analytical formula based on the shape of the magnet is also considered as to compare the results obtained by the finite element method and the analytical formulas.

Furthermore, three study cases are presented to evaluate and test the reliability of the program and to explore its features.

2. Theoretical background

2.1. Introduction

Magnetic field vectors can be expressed in terms of either the magnetic field strength H or the magnetic induction B [6]. The field vectors are related by:

$$\mathbf{B} = \mu_m \cdot \mathbf{H} \quad (2)$$

With μ_m : the magnetic permeability constant.
 In a vacuum, the permeability has the value of

$$\mu_m = \mu_0 \quad (3)$$

When a magnetic field passes through a material, the material acquires an induced magnetization M given by:

$$\mathbf{H} \cdot \chi_m = \mathbf{M} \quad (4)$$

Where χ_m the magnetic susceptibility of the material.
 The magnetic induction can also be expressed as

$$\mathbf{B} = \mu_0 \cdot (\mathbf{H} + \mathbf{M}) \quad (5)$$

Eq (2),(4) and (5) show that

$$\mu_m = \mu_0 \cdot (1 + \chi_m) \quad (6)$$

Which relates the permeability to the susceptibility.

Following Maxwell's equation for magnetostatic, magnetic field is expressed as follow:

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (7)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (8)$$

Where j is the density charge.

Via a magnetic vector potential approach. The flux density may be written in terms of the vector potential A as:

$$\mathbf{B} \times \nabla = \mathbf{A} \quad (9)$$

Where A is the vector potential

By using Lorentz gauge, the Alembert equation was found as follow:

$$\mu \mathbf{J} = \Delta \mathbf{A} - \mu \frac{\partial^2 \mathbf{A}}{\partial t^2} \quad (10)$$

In quasistationary terms:

$$\mu \mathbf{J} = \Delta \mathbf{A} \quad (11)$$

And finally, vector potential is expressed via Poisson's equation formulation

2.2. Boundary conditions

The permanent magnet is a suspended one in the air, its flux lines are parallel to the boundary edge. Hence the Dirichlet conditions are the best to express this problem.

$A = 0$ at the boundary of the space of study of the permanent magnet

3. Finite element formulation

3.1. Introduction

The boundary value problem under consideration is defined by the second order PDE:

$$\mu \mathbf{J} = \Delta \mathbf{A} \quad (12)$$

3.2. Domaine discretization

Triangular elements are used in this case, with the following interpolation equation:

$$\phi^e(x, y) = \sum_{j=1}^3 \mathbf{N}_j^e(x, y) \phi_j^e \quad (13)$$

By using variation formulation, we get the expressions of the matrices

3.3. Final formulation

Set of matrices are represented as follow:

$$\mathbf{K}_{i,j}^e = \int \left(\left(\frac{\alpha dN_i^e}{dx} \frac{dN_j^e}{dx} \right) + \beta \mathbf{N}_i^e \mathbf{N}_j^e \right) dx \quad (14)$$

$$\mathbf{b}_i^e = \int \mathbf{N}_i^e \mathbf{f} dx \quad (15)$$

$$\mathbf{g}_i^e = \alpha \mathbf{N}_i^e d\phi / dx \quad (16)$$

$$(\phi^e) = \begin{pmatrix} \phi_1^e \\ \phi_2^e \end{pmatrix} \quad (17)$$

$$(\mathbf{K})(\phi) = (\mathbf{b}) + (\mathbf{g}) \quad (18)$$

4. Finite element program

4.1. Introduction

For its simplicity and reliability of results, MATLAB is used to program the finite element formulation of matrices.

The basic structure of this program is split to many sections.

Global declaration section: where the model is presented, characteristic of the environment, magnet and particle are defined like number of elements, nodes, magnetic induction, type of elements. This section can be updated with each specific study case.

Main function: this section is considered the heart of the program. It includes all equations and formulas related to calculating all necessary parameters.

Sub-program function: this section is only optional as it can be included in the Main function section. On the other hand, it's considered essential to create a clean and a visible program. Especially if type of elements and dimensions changed (ex: quadratic instead of triangular). It includes all sub-functions required to calculate the final equation like: Stiffness matrix and source matrix.

Results view section: in this section, graphs, tables and curves can be obtained to visualise the output of the program.

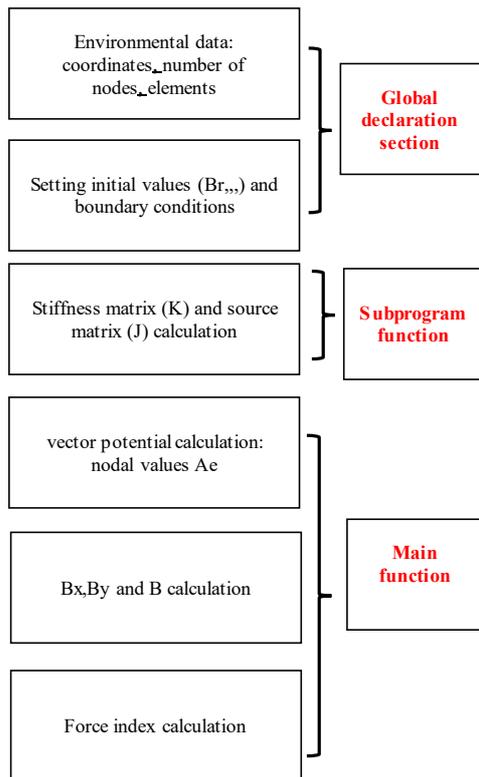


Fig. 2. Main sections of MATLAB program.

4.2. Magnetic induction Formulation

Fig below “2” describes set of steps related to the main sections of the program as discussed above.

Following steps mentioned below allow to calculate the nodal values of force index in each node. In case where global force index is desired, vector potential and magnetic induction values should be calculated for each element, to allow creating a global matrix structure for magnetic induction and it's gradient.

5. Analytical model

5.1. Introduction

Several previous publications had already discussed the behaviour of magnetic field of a rectangular permanent magnet. Elaboration of mathematical formulation

depends mainly on type of model (spring model, charge model....) and number of dimensions (two or three) [6, 7, 8].

In this work, the charge model method is taken into consideration as a reference to compare the obtained results. According to this model, magnet is considered as a set of distribution of equivalent “magnetic charge” [8].

Expressions are presented in three dimensions, nevertheless they will be customized to fit two dimensions model.

Considering magnet having two surfaces, expressions of the magnetic induction will be separated based on top (Z=0) and the bottom (Z=L).

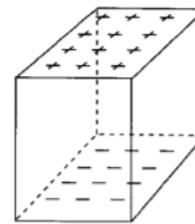


Fig. 3. Rectangular magnet: equivalent Charge model.

5.2. Formulation

For the top surface of the magnet (z=0), B(Z) is:

$$B_z(z) = (\mu_0 * M_s / \pi) * \left[\frac{\pi}{2} - \frac{\tan^{-1} z * \sqrt{a^2 + b^2 + z^2}}{a * b} \right] \quad (19)$$

For the bottom of the magnet (Z=L), B(Z) is:

$$B_z(z) = (\mu_0 * M_s / \pi) * \left[\frac{\tan^{-1}(z+l) * \sqrt{a^2 + b^2 + z(z+l)^2}}{a * b} - \frac{\tan^{-1} z * \sqrt{a^2 + b^2 + z^2}}{a * b} \right] \quad (20)$$

a and b are the length and the width of the magnet
 Z is the distance between the magnet and the particle.
 Ms: Magnetization of the magnet

6. Simulation and computing

6.1. Case 1: comparing the magnetic potential

6.1.1. Introduction

To create a model of the permanent magnet. FEMM was used with the following data:

- 2a = 2 mm
- 2b = 2mm
- L = 1 mm
- Br = 1.4T according to Y.

Both Femm [9] and Marc were used to compute the magnet, yet only Marc's results were taken into consideration, as it allows more precision and variation of results based on the orientation of the magnetization.

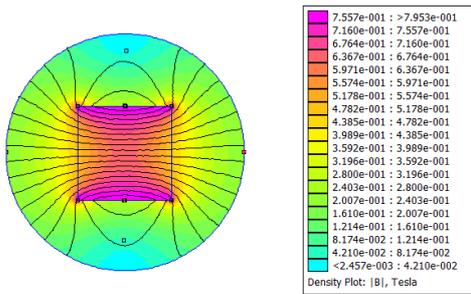


Fig. 4. Computation of magnetic induction flux -Femm.

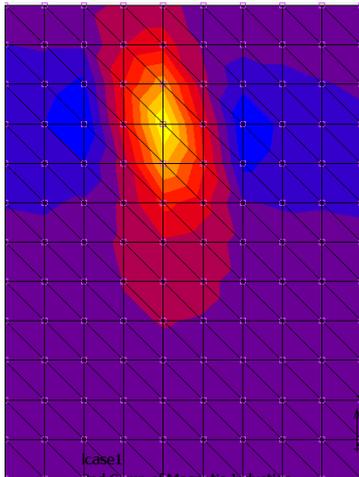


Fig. 5. Computation of magnetic induction flux -MARC.

6.1.2. Results

Fig 4 and 5 visualize flux lines of the magnet. As it can be seen, flux lines tend to reach the terminal and the pole spacing.

Fig. 6 shows the results of the variation of nodal magnetic vector potential obtained by the Marc and MATLAB methods for 36 nodes. As it can be seen, the difference between the two curves is hardly perceptible, because the estimated margin of error for Marc is less than 3%. As stated in its official publication [10] and it can therefore be considered identical to the results given by MATLAB.

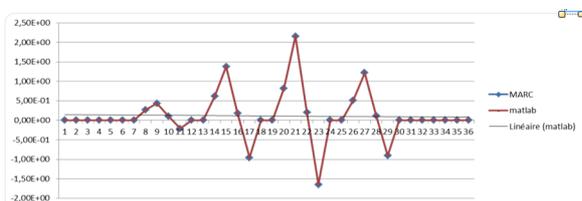


Fig. 6. Values of vector potential for 36 nodes. Node 21 present the maximum value of vector potential, afterword's the values start decreasing.

The table below N “1” helps understanding variation of values of vector potential generated by the magnet following Y direction.

Nodes are taking following Y direction. It is clear that as the distance from the magnet increases, the values of the potential vector drop decrease.

As the magnetic vector potential can be interpreted as potential energy per unit element of current, so direction of vector potential will determine the needed energy required by magnet to create an attraction force with the particle (fig 7)

Table 1. Values of potential vector.

NODE	MARC	MATLAB
19	2,05E-10	0
20	0,821356	0,8214
21	2,15875	2,1588

As it can see in “fig 6 and 7”, the difference between both results is hardly perceptible. Error margin is less than 6%, this part of the program can be considered validated.

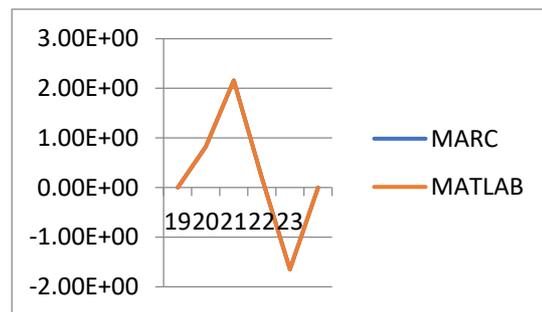


Fig. 7. Variation of vector potential.

6.2.Case 2: comparing magnetic induction

For this case, same data as the first example will be used to calculate the external magnetic induction generated by the magnet.

Table below “2” illustrate values of magnetic induction starting from the outer surface of the magnet to the location of the particle.

The analogy between values of vector potential and magnetic induction is very similar. Once Again, the more the distance from the magnet increases, the more the magnetic induction values decrease.

To validate the programme, an analytical calculation has been developed. Figure 8 shows that the values can be considered almost identical for the first two values, which is logical given the change of medium between the magnet and the air.

Table 2. Variation of magnetic induction.

No	B(Z) Analytical model (T)	B(Z) by Matlab (T)
0.0000	0.1470	0.15170
1.0000	0.0693	0.07146
2.0000	0.0298	0.03073
3.0000	0.0141	0.01451
4.0000	0.0071	0.00730
5.0000	0.0037	0.00380
6.0000	0.0019	0.00190
7.0000	0.0008	0.00080
8.0000	0.0003	0.00029

The understanding of the behaviour of magnetic induction outside the magnet is a major factor in the choice of magnet material. This means that each magnet has a unique internal energy and induction field range.

6.3. Case 3: distance choice between particle feeds and magnet based on force index.

Following this case, impact of burden on force induction will be explored. Likewise, same data will be taken into consideration. As for the particle data:

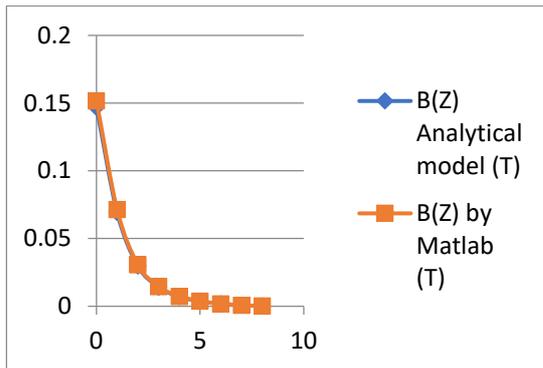


Fig. 8 Variation of magnetic induction.

- $\mu_p=1000$
- $\rho_P= 7800 \text{ Kg/M}^3$
- $P_b= 800 \text{ Kg/M}^3$
- $\Phi= 10 \text{ mm}$
- $N= 0,33$
- $G=9,8$
- $\mu_0= 4\pi \cdot 10^{-7}$

In general, “burden” is a material with a different permeability covering the ferromagnetic particle required to be separated (figure 9).

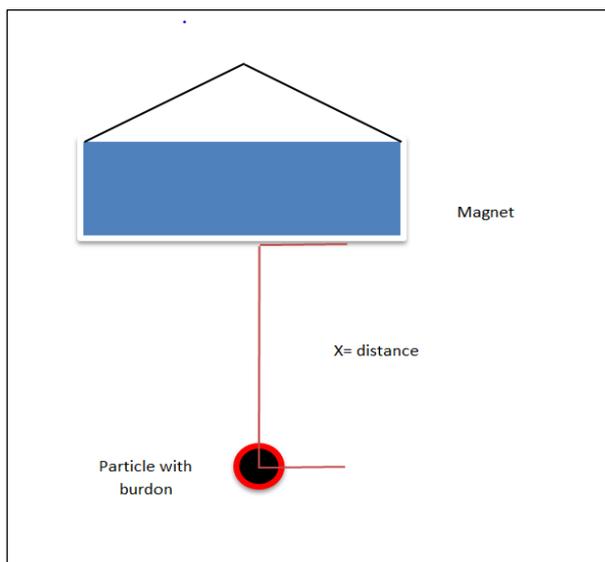


Fig. 9. Separation with burden.

Force index required to achieve the separation is:

- without burden : $f_m= 0,0318 \text{ T}^2/\text{m}$
- with burden : $f_m= 0,0374 \text{ T}^2/\text{m}$

As it can be seen, force index required to attract a particle covered by burden is more important than the value required for a particle alone.

SvoBoda in his work [1], explained explicitly the impact of the burden on the required force index depending on the dimensions of the burden. Force index increases steeply with increasing depth of the burden.

He stated also that force index depends solely on the shape of the particle size and not on its geometry, like it's the case with the magnetic force.

Therefore, this factor must be taken seriously during the separator design process to get precise data.

By considering results of magnetic induction in case 2, the optimum distance between the magnet and the particle feed should be 1 mm to achieve a field gradient superior than $0,037^2$ required to achieve the value of force index.

The further we get from the magnet, the more force index is required to achieve the separation process. Consequently, the bigger the magnet dimensions should be.

When dealing with real process, it is essential to take into consideration the distance between the suspended magnet and the conveyor band including the feed as one of the main factor for the magnetic separator choice to get the optimum design.

7. Conclusion

A finite element program for calculation of the force index is presented. Numerical simulation of the vector potential, magnetic induction and force index have been carried out in 2D Cartesian form.

This program can be used for modelling the permanent suspended magnetic separation and analyse the behaviour of different flux. As it was shown in case 1 and 2.

It can also be used to assist in the design phase of the separator to obtain optimal dimensions while simulating its force index behaviour towards the magnetised particle under different circumstances as it was shown in case 3.

Given the impact of the burden surrounding the particle on the required force index, as it was shown in case 3, most of separator designers, take into consideration this factor. As the force index required increases depending on the burden's volume.

The logic to elaborate the force index has been presented following the different stages of the program. Starting with the vector potential and ending with the calculation of the induction field gradient.

The program allows to communicate the results of each step of the simulation and to compare them with the analytical results as presented above (cases 1-3).

The results showed that the estimated error between computed and analytical values is between 3% and 6 %

In addition to all the features of this program, it can Also interact with different platforms to allow calculus for large numbers of nodes (ex 13000 Nodes).

Furthermore, duo to its subprogram function section, the program allows a quick switch to a different type of elements and give the possibility to compare results for each simulation.

It worth mentioning also that although the used approach has been applied to a rectangular magnet, it's equally applicable to the analysis of the magnetic field and force index a calculation of a cylindrical magnet, by taking into account the correct formulas.

This allows for the program to cover more types of magnetic separators and to help better understanding the real process of separation.

References

1. J. Svoboda, *Magnetic Techniques for the Treatment of Materials* (Kluwer Academic Publishers, 2004)
2. A. Gassner, M. Abonnenc, H. Chen, J. Morandini, J. Jossierand, J. Rossier, J. Busnel, H. Girault, RSC Lab on a Chip **9**, 2356-2363 (2009)
3. T. Morcos, The straight attraction. Motion control (2000)
4. A.W. Stradling, I.J.O.M.P **39**, 1-18 (1993)
5. E. Knoepfel, Magnetic Field, 79-82 (2000)
6. Z. Yuejin, X. Guodong, T. Guanzhen, J. O. Shan. Univ. 3 237-241 (1997)
7. Z.Q. Zhu, D. Howe, E. Bolte, EEE **29**, 124-135 (1993)
8. E P. Furlani, *Permanent magnet and eletromechanical devices*, pp. 134-135
9. D. Meeker, Finite Element Method Magnetics. <https://www.femm.info>. Accessed 21 Mar 2021
10. *Electromagnetic analysis* (Marc tutorial)