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About natural oscillations thin elastic wavy non-circular shell

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Abstract. The open profile thin elastic wavy shell natural oscillations problem is considered. The proposed method for determining the numerical values of the lowest frequencies and the corresponding forms of natural vibrations of shells with a complicated shape is based on the Rayleigh-Ritz energy method. The results of numerical calculation for a thin wavy shell with a rigidly pinched lower contour are presented.

1 Introduction

The experience of construction increasingly points to the prospects of using the shells of complex shapes, including such wavy shells as coatings for public, industrial, warehouse and agricultural buildings and structures.

The creation of new, more advanced engineering structures led to the need to develop a theory for calculating shells with complex structure: layered and corrugated, wavy and reinforced with a rod set.

The theory of calculating thin shells is well developed, so that it is fully possible to calculate and then design the constructions and structures in the form of shells with rather complex outlines.

However, in many cases, they are exposed to periodic and impulsive loads, especially in seismic areas, so it is very important to have a fairly simple mathematical apparatus for determining the natural frequencies and the corresponding forms of natural vibrations of shells with complex geometry.

Analyzing the scientific works devoted to the dynamics of shells, it is easy to notice the presence of a relatively small number of studies on the calculation of the natural oscillations of shells with non-classical shape.

A method for determining the numerical values of the low frequencies and the corresponding forms of natural vibrations of thin shells of complex shape by the energy method is proposed, which is acceptable for use in design practice.

This method gives an opportunity to replace the differential equations with a homogeneous system of linear algebraic equations, which greatly simplifies the dynamic calculation of shells of complex structure.

As a special case, this technique is used to calculate smooth shells with any boundary conditions.

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2 Problem statement

The problem of determining the lower frequencies numerical values and the corresponding forms of natural vibrations of a thin elastic wavy shell, rectangular in plan, with a rigidly pinched lower contour along the generatrix (Fig. 1)

The cross-section of the shell median surface (along the ridge) is outlined by the curve which equation is:

$$\frac{x^2}{a^2}+\frac{y^3}{b^3}=1 \tag{1}$$

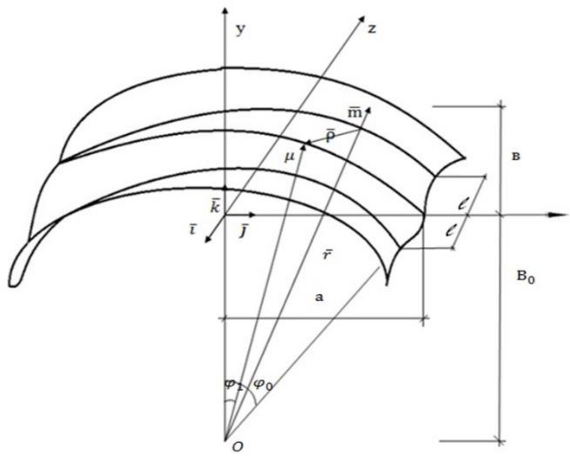

Fig. 1. Shell surface

The median surface of the shell is formed by the curve displacement

$$\alpha=-\Delta\,(1-\cos\pi\frac{z}{\ell}) \tag{2}$$

along two adjacent ridges of the shell and located in the normal plane to them.

Here Δ, 2ℓ are the amplitude and wavelength.

2.1 Geometric characteristics (shell dimensions)

Span – $2a$ = 18 m, height in the cross section passing through the crest of the shell – b = 5 m, wavelength – 2ℓ = 3 m, wave amplitude – Δ = 0.225 m, the pole distance – B_0 = 9 m, shell thickness – $2h$ = 0.05 m.

2.2 Physical characteristics

The modulus of elasticity of the shell material – E = 28 Gpa, the Poisson's ratio – μ = 1/6, the density of the shell material – ρ = 2500 kg/m^3.

3 Purpose of the work

Obtaining the numerical values of the lowest frequencies and their corresponding eigenvocation forms for a thin elastic wavy shell rectangular in plan (Fig. 1), with a rigidly pinched lower contour along the generatrix with the coordinate $\alpha_2 = \pm 1$ (Fig. 2).

The Rayleigh-Ritz energy method is the basis of the proposed method for determining the low frequencies and the corresponding forms of natural vibrations of shells with a complicated shape.

This method makes it possible to obtain the fairly accurate values of the low frequencies and the corresponding forms of natural vibrations for the shells with complex shape under arbitrary conditions of fixation and under any law of change in its geometric and physical characteristics. The main provisions of this method are described in [1] – [8].

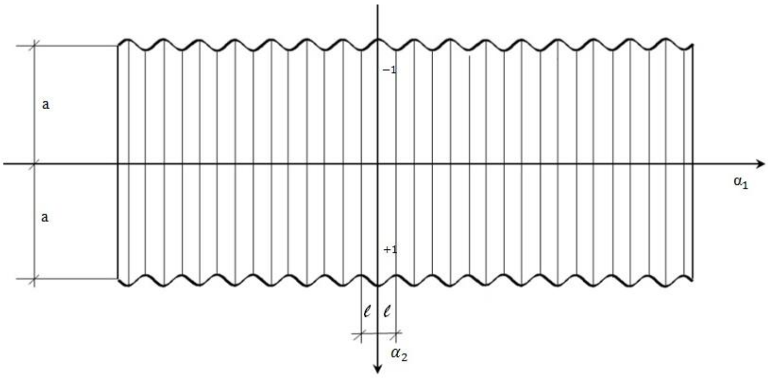

Fig. 2. Design scheme of the shell

The calculation is performed on the basis of geometric and physical linearity using the Kirchhoff-Love hypotheses.

4 The numerical example solution

It is assumed that the shell length along the axis Ƶ (Fig. 1) is quite large, so we can restrain it to considering only its middle sections, without taking into account the influence of the parts of the shell adjacent to its ends.

In matrix form, the basic equations and differential dependences describing the deformation of the shell and its parallel layer median surface are obtained, as well as the matrix formulas for calculating the shell deformation potential and kinetic energy amplitudes. A generalized secular equation is obtained for finding the frequencies and forms of the shell's natural oscillations, which is reduced to the classical secular equation [5].

The vector equation of the median surface of this wavy shell is derived. Formulas for calculating the parameters of the Lam and the Christoffel symbols of a given shell are obtained [6].

The functions approximating the shell median surface points displacements amplitudes along the curvilinear coordinate axes α_1, α_2 and α_3 in the form of double trigonometric series, satisfying the conditions of rigid fixation of the shell along the lower contour along the generatrix (Fig. 2), are selected, namely: $\alpha_2 = \pm 1$: $u_1^0 = u_2^0 = u_3^0 = 0$; $u_{3,2}^0 = 0$ [6].

According to the formulas given in [7], all the matrices necessary for determining the numerical values of the lowest frequencies and the corresponding forms of natural oscillations of a given wavy shell are obtained.

Control of the proposed algorithm numerical convergence for calculating the wavy 1increasing the number of the series terms approximating the solution.

In the first approximation, four terms ($m = n = 2$) were left in the double trigonometric series approximating the displacements, in the second approximation nine ($m = n = 3$), then sixteen ($m = n = 4$), twenty-five ($m = n = 5$), and finally thirty-six terms of the series ($m = n = 6$). Comparing the last three approximations at m = n = 4; 5; 6, we obtain a sufficient convergence of the results for the engineering calculations [8].

As a result of the calculations, the numerical values of the first three lower frequencies and the corresponding forms of natural oscillations of this wavy shell were obtained.

5 Calculation results

The results of calculating the first frequency and the corresponding shape of the natural oscillations of this wavy shell, rigidly pinched along the lower contour along the generatrix, are shown in Fig. 3-9.

The first frequency

ω_1 = 58.3 Hz

The first form of natural oscillations

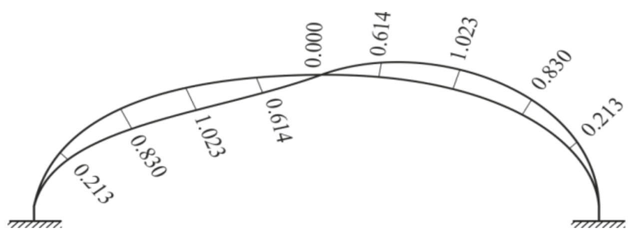

Fig. 3. Amplitude values of dimensionless displacements u_3^0 in the cross sections of the shell with the coordinates $\alpha_1 = 0$

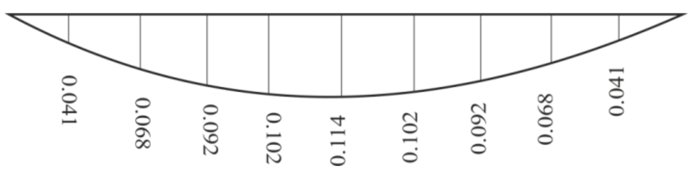

Fig. 4. Amplitude values of dimensionless displacements u_2^0 in the cross sections of the shell with the coordinates $\alpha_1 = 0$

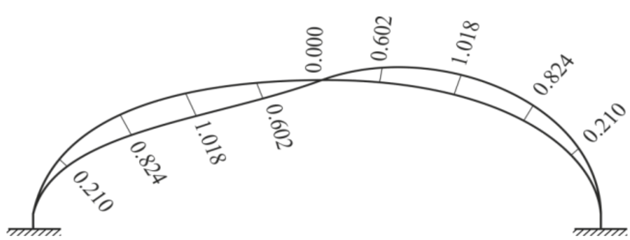

Fig. 5. Amplitude values of dimensionless displacements u_3^0 in the cross sections of the shell with the coordinates $\alpha_1 = \pm 0.5$

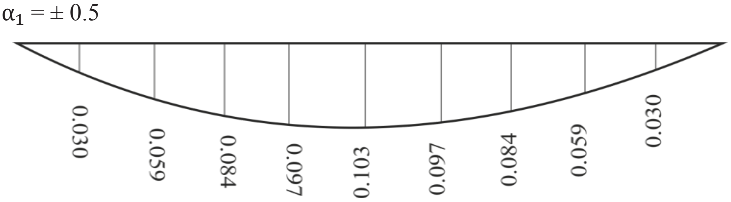

Fig. 6. Amplitude values of dimensionless displacements u_2^0 in the cross sections of the shell with the coordinates $\alpha_1 = \pm 0.5$

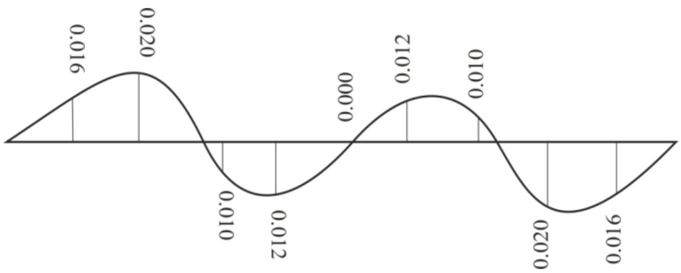

Fig. 7. Amplitude values of dimensionless displacements u_1^0 in the cross sections of the shell with the coordinates $\alpha_1 = 0.5$

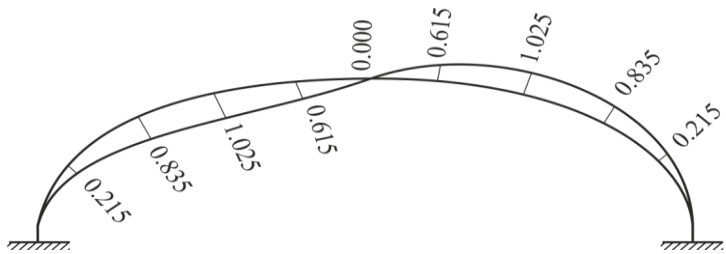

Fig. 8. Amplitude values of dimensionless displacements u_3^0 in the cross sections of the shell with the coordinates $\alpha_1 = \pm 1$

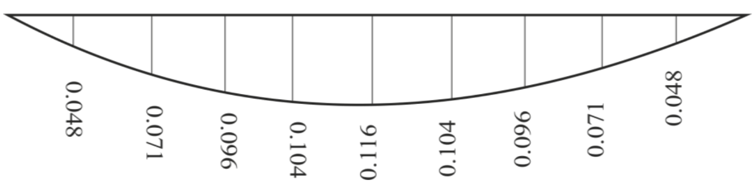

Fig. 9. Amplitude values of dimensionless displacements u_2^0 in the cross sections of the shell with the coordinates $\alpha_1 = \pm 1$

The amplitude values of dimensionless displacements u_1^0 in the cross-sections of the shell with the coordinates $\alpha_1 = 0$ and $\alpha_1 = \pm 1$ are zero.

On these charts

ω_1 is the first frequency of natural oscillations of a thin wavy shell;

u_1^0, u_2^0, u_3^0 are the shell median surface points dimensionless displacements amplitudes along the coordinate axes α_1, α_2 and α_3 respectively.

6 Conclusion

Analyzing the obtained numerical values of the natural oscillations' first three frequencies of a thin wavy shell with a rigidly pinched lower contour along the generatrix (Fig. 1), it can be noted that in the example under consideration, the fifth approximation (36 terms of the series) gives a possibility to determine the values of the lower frequencies with sufficient engineering accuracy.

The difference in the results between the fourth and fifth approximations for the first frequency is 2.9%, for the second – 3.5%, for the third – 4.1%.

The obtained values of the lowest frequencies of this thin wavy shell natural oscillations confirm the good convergence of the developed algorithm for determining the frequencies and the corresponding forms of natural oscillations of shells with a complicated shape.

In the case of the shell fixing under consideration, two transverse half-waves correspond to the main tone of the natural vibrations of this thin wavy shell.

The results of this shell numerical calculation allow us to recommend the use of the proposed method for determining the low frequencies and the corresponding forms of natural vibrations of the shells with a complex shape.

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