

General case of reinforced concrete rod elements calculation using the diagram method

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Abstarct. In BC 63.13330 the general calculation case for the diagram method is oblique off-center compression, which takes into account only three components of internal force factors in the cross section: the longitudinal force - N_z - and two bending moments relative to the corresponding axes - M_x and M_y . The other three components-the Q_x and Q_y transfer forces and the M_z torque - are left out of consideration. In addition, for this case, the search in the available literature, including the founders of the diagram method, for the output of calculation formulas, was not successful - in all sources they are given in ready-made form without evidence. This article is intended to try filling in these gaps. For this purpose, based on the expressions for rod displacements that are generally accepted in mechanics, in particular on the Mora integral of displacements, the resolving expressions of the diagram method are obtained in the most general form.

1 Introduction

BC 63.13330 also includes the diagrammatic method as an alternative to the ultimate force method for the reinforced concrete rods' design. Its distinctive feature lies in the possibility of calculating for both groups of limiting states according to the same formulas, as well as obtaining a stress-strain state (SSS) of the considered elements at all loading stages: from zero to failure. In this set of rules, the general design case for this method is oblique off-center compression, which takes into account only three components of internal force factors in the section: longitudinal force - N_z - and two bending moments about the corresponding axes - M_x and M_y . The other three components are shearing forces Q_x and Q_y and torque M_z - remain out of consideration. In addition, for oblique eccentric compression, the search in the available literature, including the founders of the diagram method [1] - [3], the calculation formulas' derivation was not crowned with success - in all sources they are already presented in a finished form without proof. This article is intended to try filling in these gaps.

2 Models and Methods

$$\varepsilon_z = \varepsilon_z^{(1)} + \varepsilon_z^{(2)} + \varepsilon_z^{(2)} = \varepsilon_0 + \chi_y \cdot x - \chi_x \cdot y$$

$$\begin{aligned} \sigma_z &= E\varepsilon_z = E(\varepsilon_0 + \chi_y x - \chi_x y) \\ N_z &= \int_A \sigma_z dA = E(A\varepsilon_0 + \chi_y S_y - \chi_x S_x) \\ M_x &= -\int_A \sigma_z y dA = E(-S_x \varepsilon_0 - \chi_y I_{xy} + \chi_x I_x) \\ M_y &= \int_A \sigma_z x dA = E(S_y \varepsilon_0 + \chi_y I_y - \chi_x I_{xy}) \end{aligned}$$

In the special case of oblique eccentric compression, the governing equations for the rod shown in Fig. 1, a, are known. For example, in [4] such expressions are given:

$$\begin{Bmatrix} N_z \\ M_x \\ M_y \end{Bmatrix} = E \begin{bmatrix} A & -S_x & S_y \\ -S_x & I_x & -I_{xy} \\ S_y & -I_{xy} & I_y \end{bmatrix} \times \begin{Bmatrix} \varepsilon_z \\ \chi_x \\ \chi_y \end{Bmatrix} = [D] \times \begin{Bmatrix} \varepsilon_z \\ \chi_x \\ \chi_y \end{Bmatrix}, \tag{1}$$

where ε_z is a longitudinal relative deformations of the rod; χ_x, χ_y are the curvature of the longitudinal axis of the rod relative to the corresponding axes; E is the material deformation modulus; $A, S_x, S_y, I_x, I_y, I_{xy}$ show the geometric characteristics of the section: area, static moments, moments of inertia, centrifugal moment of inertia.

If the adopted coordinate axes coincide with the main central ones, then the static moments are equal to zero S_x, S_y and centrifugal moment of inertia I_{xy} , and the stiffness matrix $[D]$ turns into a diagonal one.

For a reinforced concrete section, the formula (1) can be rewritten as:

$$\begin{Bmatrix} N_z \\ M_y \\ M_x \end{Bmatrix} = \begin{bmatrix} D_{11} & -D_{12} & D_{13} \\ -D_{21} & D_{22} & -D_{23} \\ D_{31} & -D_{32} & D_{33} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_z \\ \chi_x \\ \chi_y \end{Bmatrix} = [D] \times \begin{Bmatrix} \varepsilon_z \\ \chi_x \\ \chi_y \end{Bmatrix}, \tag{2}$$

where for the section shown in Fig. 1b, the components of the stiffness matrix are:

$$\begin{aligned} D_{11} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k}, \\ D_{12} = D_{21} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} y_{bi} + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k} y_{s,k}, \\ D_{13} = D_{31} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} x_{bi} + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k} x_{s,k}, \\ D_{22} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} y_{bi}^2 + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k} y_{s,k}^2, \\ D_{23} = D_{32} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} x_{bi} y_{bi} + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k} x_{s,k} y_{s,k}, \\ D_{33} &= \sum_{i=1}^{n-1} \sum_{j=1}^{m-1} v_{b,ij} E_{b,ij} A_{b,ij} x_{bi}^2 + \sum_{k=1}^p v_{s,k} E_{s,k} A_{s,k} x_{s,k}^2, \end{aligned} \tag{3}$$

where V_b, V_s are the coefficients of the secant modulus of deformation (according to Karpenko N.I.) or the coefficients of elastic-plastic deformations (according to Murashev V.I.), respectively, of concrete and reinforcement; the rest of the (geometric) values are clear from Fig. 1, b.

Comparing the obtained expressions with the formulas (8.39) - (8.47) BC 63.13330, we find a lot of similarities. The difference between the set of rules is, firstly, in the rotation of the coordinate axes Oxy about the axis Oz on 90° clockwise, and secondly, in the complete absence of minus signs in the stiffness matrix, which is apparently connected with the rotation of the axes. The third difference lies in the indexing of bending moments: in this article, the generally accepted in the resistance of materials is used - the subscript at the moment denotes the coordinate axis relative to which rotation occurs (for example, the moment M_x causes rotation about the axis Ox); in BC 63.13330 – otherwise: the subscript at the moment indicates the coordinate axis along which the moment acts. At the same time, although the plane in which a particular moment acts is concretized in BC, without this specification, rotation relative to any of the two remaining axes can occur, which causes confusion. Therefore, in our works we use the designations generally accepted in the strength of materials and structural mechanics.

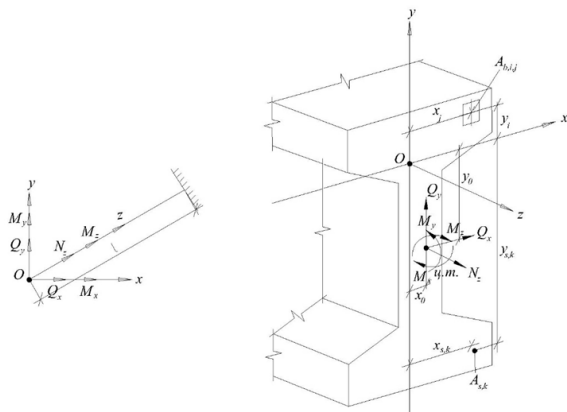


Fig. 1. Design scheme: *a* - rod; *b* - cross section

The solution to the system of equations (2) will be:

$$\begin{Bmatrix} \varepsilon_z \\ \chi_x \\ \chi_y \end{Bmatrix} = \begin{bmatrix} D_{11} & -D_{12} & D_{13} \\ -D_{21} & D_{22} & -D_{23} \\ D_{31} & -D_{32} & D_{33} \end{bmatrix}^{-1} \times \begin{Bmatrix} N_z \\ M_y \\ M_x \end{Bmatrix} = [D]^{-1} \times \begin{Bmatrix} N_z \\ M_y \\ M_x \end{Bmatrix}, \quad (4)$$

To derive the general case of calculating the rod (Fig. 1, a) by the diagrammatic method, we write the Mohr displacement integral [4]:

$$\Delta_{1P} = \int_0^l \left(\frac{\bar{N}_{z1} N_{zP}}{D_z} + \frac{\bar{M}_{x1} M_{xP}}{D_x} + \frac{\bar{M}_{y1} M_{yP}}{D_y} + \frac{\bar{M}_{z1} M_{zP}}{D_k} + k_x \frac{\bar{Q}_{x1} Q_{xP}}{D_Q} + k_y \frac{\bar{Q}_{y1} Q_{yP}}{D_Q} \right) dz \cdot \quad (5)$$

The formulas are valid if at the pole O the rod is set to the right coordinate system $Oxyz$: axis z – along the rod, axis x – from drawing plane, axis y – up. Moreover, the axes are the main central ones. The positive direction for the moments and angles of rotation is

counterclockwise as viewed from the positive end of that axis. For compactness, the moments and angles of rotation are designated as spin vectors.

Let us consider each of the six movements separately. So, in the action $\bar{N}_{z1} = 1$ and N_z (the rest of the force factors are equal to zero), using the rule of multiplying the diagrams, we obtain a displacement along the axis Oz : $u_z = \int_0^l \frac{N_z}{D_z} dz$. Pole rotation $Orod$ relative to the

axis Ox at the corner φ_x possible both from the action of the moments $\bar{M}_{x1} = 1$, M_x – they will cause displacement $\varphi_{x1} = \int_0^l \frac{M_x}{D_x} dz$, and from the action of cutting forces $\bar{Q}_{y1} = 1$, Q_y ,

which create a moment equal $Q_y z$ and the corresponding movement $\varphi_{x2} = -\int_0^l \frac{Q_y z}{D_x} dz$. Then

$\varphi_x = \varphi_{x1} + \varphi_{x2} = \int_0^l \frac{M_x}{D_x} dz - \int_0^l \frac{Q_y z}{D_x} dz$. The rest of the displacement components are obtained by analogy:

$$\begin{aligned} u_z &= \int_0^l \frac{N_z}{D_z} dz, \quad \varphi_x = \int_0^l \frac{M_x}{D_x} dz - \int_0^l \frac{Q_y z}{D_x} dz, \quad \varphi_y = \int_0^l \frac{M_y}{D_y} dz + \int_0^l \frac{Q_x z}{D_y} dz, \quad \varphi_z = \int_0^l \frac{M_z}{D_k} dz, \\ u_x &= \int_0^l k_x \frac{Q_x}{D_Q} dz + \int_0^l \frac{Q_x z^2}{D_y} dz + \int_0^l \frac{M_y z}{D_y} dz, \quad u_y = \int_0^l k_y \frac{Q_y}{D_Q} dz + \int_0^l \frac{Q_y z^2}{D_x} dz - \int_0^l \frac{M_x z}{D_x} dz, \end{aligned} \tag{6}$$

where: D_z is the axial stiffness of the rod, D_x , D_y – flexural, D_Q – shear, D_k – twisting; l – rod length; k_x , k_y are the shear shape factors (for a rectangular section $k_x = k_y = 1, 2$). More details about the stiffness will be given at the end of the article.

To find the shape coefficients during shear, the following expressions are used (reinforcement is neglected):

$$k_x = \frac{A_b}{I_{by} A_b} \int \frac{S_{by}^{omc} dA_b}{b^2}, \quad k_y = \frac{A_b}{I_{bx} A_b} \int \frac{S_{bx}^{omc} dA_b}{b^2}, \tag{7}$$

where A_b shows the concrete section area; b is a concrete section width; I_{bx} , I_{by} are the moments of inertia of a concrete section relative to the axes Ox and Oy ; S_{bx}^{omc} , S_{by}^{omc} define the static moments of the cut-off part of the concrete section relative to the corresponding axes.

The vector of forces applied at the rod pole O , we write it in the form:

$$\{P\}^T = \{N_z \quad Q_x \quad Q_y \quad M_z \quad M_x \quad M_y\}^T, \tag{8}$$

the corresponding displacement vector:

$$\{u\}^T = \{u_z \quad u_x \quad u_y \quad \varphi_z \quad \varphi_x \quad \varphi_y\}^T. \tag{9}$$

3 Research results and their analysis

In the expressions (6), we differentiate along the coordinate z linear displacements u_z once:

$\frac{\partial u_z}{\partial z} = \varepsilon_z$, linear displacements u_x, u_y – twice: $\frac{\partial^2 u_y}{\partial z^2} = \chi_x, \frac{\partial^2 u_x}{\partial z^2} = \chi_y$, angles of rotation $\varphi_z, \varphi_x, \varphi_y$ – once: $\frac{\partial \varphi_z}{\partial z} = \theta_z, \frac{\partial \varphi_x}{\partial z} = \chi_x, \frac{\partial \varphi_y}{\partial z} = \chi_y$, where θ_z is a relative angle of twist about the axis Oz . Now we get:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z}, \\ \chi_x &= k_y \frac{q_y}{D_Q} + \frac{q_y z^2}{D_x} + \frac{2Q_y z}{D_x} - \frac{Q_y z}{D_x} - \frac{M_x}{D_x} = q_y \left(\frac{k_y}{D_Q} + \frac{z^2}{D_x} \right) + \frac{Q_y z}{D_x} - \frac{M_x}{D_x}, \\ \chi_y &= k_x \frac{q_x}{D_Q} + \frac{q_x z^2}{D_y} + \frac{2Q_x z}{D_y} + \frac{Q_x z}{D_y} + \frac{M_y}{D_y} = q_x \left(\frac{k_x}{D_Q} + \frac{z^2}{D_y} \right) + \frac{3Q_x z}{D_y} + \frac{M_y}{D_y}, \\ \theta_z &= \frac{M_z}{D_k}, \chi_x = -\frac{Q_y z}{D_x} + \frac{M_x}{D_x}, \chi_y = \frac{Q_x z}{D_y} + \frac{M_y}{D_y}. \end{aligned} \tag{10}$$

In these formulas, we make the replacement: $Q_y z = -M_x, Q_x z = M_y$, then we will have:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z}, \chi_x = q_y \left(\frac{k_y}{D_Q} + \frac{z^2}{D_x} \right) - \frac{2M_x}{D_x}, \chi_y = q_x \left(\frac{k_x}{D_Q} + \frac{z^2}{D_y} \right) + \frac{4M_y}{D_y}, \\ \theta_z &= \frac{M_z}{D_k}, \chi_x = -\frac{M_y}{D_x} + \frac{M_x}{D_x}, \chi_y = \frac{M_x}{D_y} + \frac{M_y}{D_y}. \end{aligned} \tag{11}$$

The equations 2, 3 and, accordingly, 5,6 in the formulas (11) are identical - they determine one and the same quantity, therefore we discard, for example, 2 and 3. We obtain the following system of equations:

$$\varepsilon_z = \frac{N_z}{D_z}, \theta_z = \frac{M_z}{D_k}, \chi_x = \frac{M_x - M_y}{D_x}, \chi_y = \frac{M_y + M_x}{D_y}. \tag{12}$$

Or in a matrix form:

$$\begin{Bmatrix} \varepsilon_z \\ \theta_z \\ \chi_x \\ \chi_y \end{Bmatrix} = \begin{bmatrix} \frac{1}{D_z} & 0 & 0 & 0 \\ 0 & \frac{1}{D_k} & 0 & 0 \\ 0 & 0 & \frac{1}{D_x} & -\frac{1}{D_x} \\ 0 & 0 & \frac{1}{D_y} & \frac{1}{D_y} \end{bmatrix} \times \begin{Bmatrix} N_z \\ M_z \\ M_x \\ M_y \end{Bmatrix}, \tag{13}$$

The expressions obtained do not contain shearing forces and the shifts are taken into account implicitly. To take them into account explicitly, it is necessary to discard the last two expressions in the formulas (11) and put $z=l$, $q_x = \frac{3Q_x}{8l} \dots \frac{5Q_x}{8l} \approx \frac{2Q_x}{l}$, $q_y = \frac{3Q_y}{8l} \dots \frac{5Q_y}{8l} \approx \frac{2Q_y}{l}$. Then (11) will appear in the form:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z}, \theta_z = \frac{M_z}{D_k}, \chi_x = 2Q_y \left(\frac{k_y}{D_Q l} + \frac{l}{D_x} \right) - \frac{2M_x}{D_x}, \\ \chi_y &= 2Q_x \left(\frac{k_x}{D_Q l} + \frac{l}{D_y} \right) + \frac{4M_y}{D_y}. \end{aligned} \tag{14}$$

Or in matrix form:

$$\begin{Bmatrix} \varepsilon_z \\ \theta_z \\ \chi_x \\ \chi_y \end{Bmatrix} = \begin{bmatrix} \frac{1}{D_z} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{D_z} & 0 & 0 \\ 0 & 0 & \frac{2k_y}{D_Q l} + \frac{2l}{D_x} & 0 & -\frac{2}{D_x} & 0 \\ 0 & 0 & 0 & \frac{2k_x}{D_Q l} + \frac{2l}{D_y} & 0 & \frac{4}{D_y} \end{bmatrix} \times \begin{Bmatrix} N_z \\ Q_x \\ Q_y \\ M_z \\ M_x \\ M_y \end{Bmatrix}. \tag{15}$$

It should be noted that, although shear forces are taken into account in the expressions (14) and (15), in the deformation vector, shear deformations γ are absent, they are taken into account indirectly through shear stiffness D_Q , which depends on τ and γ .

We also note that in the structure of the formulas for curvatures, the first term is responsible for the pure shear strains effect on the curvature, the second term is for bending deformations caused by a shearing force, and the third term is for bending deformations caused by the moment action.

In addition, the proposed approach carries the development prospects due to taking into account, in addition to longitudinal deformations ε_z also transverse deformations ε_x and ε_y . Possible options for such clarifications are given in the works [5], [6], [7].

Prior to this, all formulas were derived with respect to the main central axes of the rod. With an arbitrary choice of the coordinate system $O'x'y'z'$ it is necessary to make adjustments to the stiffness characteristics of the section based on the following reasoning (Fig. 1, b). The longitudinal force has no eccentricity relative to the main central axes, $a_x=a_y=0$, and the centrifugal moment of inertia and the corresponding centrifugal stiffness of the rod section are equal to zero: $I_{xy} = I_{bxy} + I_{sxy} = 0$, $D_{xy} = 0$. The last value is determined by the last expression from the formulas (3): $D_{xy} = D_{33}$.

The static moments about the axes are also equal to zero Ox and Oy and the corresponding static stiffnesses: $S_x = S_{bx} + S_{sx} = 0$ and $S_y = S_{by} + S_{sy} = 0$, $D_{sx} = 0$ and $D_{sy} = 0$. The specified stiffnesses are determined by the expressions 2 and 3 of the formulas (3): $D_{sx} = D_{12} = D_{21}$, $D_{sy} = D_{13} = D_{31}$.

When specifying the coordinate system arbitrarily, it is necessary to take into account additional deformations of the section rotation caused by the eccentrically applied longitudinal force N_z , as well as additional turns from M_x about the axis Oy and from M_y about the axis Ox . This is achieved by including the formulas (12) or (14) to the first expression, the relative deformations caused by bending: $\varepsilon_z = \varepsilon_0 + \chi_y x - \chi_x y$, where $\varepsilon_0 = \frac{N_z}{D_z}$. And, in addition, it is necessary to replace the bending stiffnesses with the following values:

$$\begin{aligned} D_x &= \frac{D'_x + D'_y}{2} + \frac{1}{2} \sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sx}}{D'_z}, \\ D_y &= \frac{D'_x + D'_y}{2} - \frac{1}{2} \sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sy}}{D'_z}, \end{aligned} \tag{16}$$

where is the stroke «'» – means that this value is calculated with respect to arbitrary axes $O'x'y'z'$.

Wherein $D_z = D'_z$, $D_Q = D'_Q$ и $D_k = D'_k$.

It is also necessary to replace bending moments and shear forces with:

$$\begin{aligned} M_x &= \frac{(M'_x + N_z a_y) \cos(\alpha_{0M} - \alpha)}{\cos \alpha_{0M}} = \frac{N_z (e'_y + a_y) \cos(\alpha_{0M} - \alpha)}{\cos \alpha_{0M}} = \frac{N_z e_y \cos(\alpha_{0M} - \alpha)}{\cos \alpha_{0M}}, \\ M_y &= \frac{(M'_y + N_z a_x) \sin(\alpha_{0M} - \alpha)}{\sin \alpha_{0M}} = \frac{N_z (e'_x + a_x) \sin(\alpha_{0M} - \alpha)}{\sin \alpha_{0M}} = \frac{N_z e_x \sin(\alpha_{0M} - \alpha)}{\sin \alpha_{0M}}, \\ Q_x &= \frac{Q'_x \cos(\alpha_{0Q} - \alpha)}{\cos \alpha_{0Q}}, \quad Q_y = \frac{Q'_y \sin(\alpha_{0Q} - \alpha)}{\sin \alpha_{0Q}}, \end{aligned} \tag{17}$$

where a_x, a_y show distance between corresponding axes $O'x'y'$ and Oxy (taken with their own sign relative to the axes Oxy); $e'_y = \frac{M'_x}{N_z}$, $e'_x = \frac{M'_y}{N_z}$ – axis projection $O'x'y'$ eccentricity of longitudinal force application relative to the center O ; $e_y = \frac{M_x}{N_z}$, $e_x = \frac{M_y}{N_z}$ is an axis projection Oxy eccentricity of application of longitudinal force relative to the center O ; $\alpha = \frac{1}{2} \arctg\left(-\frac{2D'_{xy}}{D'_x - D'_y}\right)$ is a rotation angle of the axes $O'x'y'$ with respect to the axes Oxy ; $\alpha_{0M} = \arctg\left(\frac{M'_y}{M'_x}\right)$; $\alpha_{0Q} = \arctg\left(\frac{Q'_y}{Q'_x}\right)$ the angles between the vectors M' and Q' and horizontal axis $O'x'$ respectively.

For brevity, we denote $C_M = \frac{\cos(\alpha_{0M} - \alpha)}{\cos \alpha_{0M}}$, $S_M = \frac{\sin(\alpha_{0M} - \alpha)}{\sin \alpha_{0M}}$, $C_Q = \frac{\cos(\alpha_{0Q} - \alpha)}{\cos \alpha_{0Q}}$, $S_Q = \frac{\sin(\alpha_{0Q} - \alpha)}{\sin \alpha_{0Q}}$.

Thus, for the formulas (12) we will have:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z} + \chi_y x - \chi_x y, \quad \theta_z = \frac{M_z}{D_k}, \\ \chi_x &= \frac{N_z(-e'_x - a_x + e'_y + a_y)C_M}{\frac{D'_x + D'_y}{2} \pm \frac{1}{2}\sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sx}}{D_z}}, \\ \chi_y &= \frac{N_z(e'_x + a_x + e'_y + a_y)S_M}{\frac{D'_x + D'_y}{2} \mp \frac{1}{2}\sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sy}}{D_z}}. \end{aligned} \tag{18}$$

And we rewrite the formulas (14) as follows:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z} + \chi_y x - \chi_x y, \quad \theta_z = \frac{M_z}{D_k}, \\ \chi_x &= \frac{2Q'_y C_Q k_y}{D_Q l} + \frac{2Q'_y C_Q l - 2N_z(e'_y + a_y)C_M}{\frac{D'_x + D'_y}{2} \pm \frac{1}{2}\sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sx}}{D_z}}, \\ \chi_y &= \frac{2Q'_x S_Q k_x}{D_Q l} + \frac{2Q'_x S_Q l + 4N_z(e'_x + a_x)S_M}{\frac{D'_x + D'_y}{2} \mp \frac{1}{2}\sqrt{(D'_x - D'_y)^2 + 4D'_{xy}} - \frac{D'^2_{Sy}}{D_z}}. \end{aligned} \tag{19}$$

Let us consider a special case when the angle $\alpha=0$, that is, the axes $O'x'y'$ parallel Oxy . It means that $\cos \alpha = 1$, $D'_{xy} = 0$, then for (18) we will have:

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z} + \chi_y x - \chi_x y, \quad \theta_z = \frac{M_z}{D_k}, \quad \chi_x = \frac{N_z(-e'_x - a_x + e'_y + a_y)}{D'_x - \frac{D'^2_{Sx}}{D_z}}, \\ \chi_y &= \frac{N_z(e'_x + a_x + e'_y + a_y)}{D'_y - \frac{D'^2_{Sy}}{D_z}}. \end{aligned} \tag{20}$$

And for (19):

$$\begin{aligned} \varepsilon_z &= \frac{N_z}{D_z} + \chi_y x - \chi_x y, \quad \theta_z = \frac{M_z}{D_k}, \quad \chi_x = \frac{2Q_y k_y}{D_Q l} + \frac{2Q_y l - 2N_z(e'_y + a_y)}{D'_x - \frac{D'^2_{Sx}}{D_z}}, \\ \chi_y &= \frac{2Q_x k_x}{D_Q l} + \frac{2Q_x l + 4N_z(e'_x + a_x)}{D'_y - \frac{D'^2_{Sy}}{D_z}}. \end{aligned} \tag{21}$$

If the axes $O'x'y'$ now move to the center of the section gravity, then $D'_{Sx} = 0$, $D'_{Sy} = 0$, $a_x = 0$, $a_y = 0$, $e'_x = e_x$, $e'_y = e_y$, $D'_x = D_x$, $D'_y = D_y$, $\varepsilon_z = \varepsilon_0$ then we get the formulas (12) and (14).

It should be noted that the expressions (12), (14), (18) - (21) were obtained for the first time.

Let us supplement them with the boundary conditions in the form of deformation criteria for the onset of a particular limiting state. So, the cracks formation in the element occurs when the maximum deformation in the most stretched concrete fiber reaches:

$$\varepsilon_{bt}^{\max} = \varepsilon_{bt2} \tag{22}$$

When an element is destroyed, three cases are possible:

$$\begin{aligned} \varepsilon_b^{\max} &= \varepsilon_{b2} - \text{fragile destruction,} \\ \varepsilon_s^{\max} &= \varepsilon_{s0} - \text{"Plastic" destruction,} \\ \varepsilon_b^{\max} &= \varepsilon_{b2} \cup \varepsilon_s^{\max} = \varepsilon_{s0} - \text{destruction of the optimal section.} \end{aligned} \tag{23}$$

Let us list the advantages of the proposed approach:

1 – all six components of external force factors acting on the rod element are considered (in BC 63.13330 there are only three: N_z , M_x , M_y): the main vector of forces R – three (N_z , Q_x , Q_y), at the main point M – three (M_x , M_y , M_z);

2 – the calculations take into account the work of the stretched zone of concrete above the crack;

3 – to describe the relationship between stresses and deformations of concrete, in addition to the diagrams of deformation for uniaxial tension / compression, shear diagrams are used « τ - γ » [8], [9]. Due to this, it becomes possible to calculate the cross-sections of bending elements in the joint action zone $MandQ$ based on the diagrammatic method.

We will extend the proposed approach to a wider class of structural rod elements made of reinforced concrete. For this, we write down the formulas for determining stiffness in the most general form:

$$\begin{aligned} D_z &= \sum \sum v_b E_b A_b + \sum \sum k_{b,ad} v_{b,ad} E_{b,ad} A_{b,ad} + \sum k_s v_s E_s A_s + \\ &+ \sum k_{s,ad} v_{s,ad} E_{s,ad} A_{s,ad} + \sum \sum k_{r1} v_{s,r1} E_{s,r1} A_{s,r1} + \sum \sum k_{r2} v_{s,r2} E_{s,r2} A_{s,r2} \\ D_x &= \sum \sum v_b E_b I_{bx} + \sum \sum k_{b,ad} v_{b,ad} E_{b,ad} I_{bx,ad} + \sum k_s v_s E_s I_{sx} + \\ &+ \sum k_{s,ad} v_{s,ad} E_{s,ad} I_{sx,ad} + \sum \sum k_{r1} v_{s,r1} E_{s,r1} I_{sx,r1} + \sum \sum k_{r2} v_{s,r2} E_{s,r2} I_{sx,r2} \\ D_y &= \sum \sum v_b E_b I_{by} + \sum \sum k_{b,ad} v_{b,ad} E_{b,ad} I_{by,ad} + \sum k_s v_s E_s I_{sy} + \\ &+ \sum k_{s,ad} v_{s,ad} E_{s,ad} I_{sy,ad} + \sum \sum k_{r1} v_{s,r1} E_{s,r1} I_{sy,r1} + \sum \sum k_{r2} v_{s,r2} E_{s,r2} I_{sy,r2} \\ D_k &= \sum \sum v_b G_b I_{bk} + \sum \sum k_{b,ad} v_{b,ad} G_{b,ad} I_{bk,ad} + \sum k_s v_s G_s I_{sk} + \\ &+ \sum k_{s,ad} v_{s,ad} G_{s,ad} I_{sk,ad} + \sum \sum k_{r1} v_{s,r1} G_{s,r1} I_{sk,r1} + \sum \sum k_{r2} v_{s,r2} G_{s,r2} I_{sk,r2} \end{aligned} \tag{24}$$

$$D_Q = \sum \sum V_b G_b A_b + \sum \sum k_{b,ad} V_{b,ad} G_{b,ad} A_{b,ad} + \sum k_s V_s G_s A_s + \\ + \sum k_{s,ad} V_{s,ad} G_{s,ad} A_{s,ad} + \sum \sum k_{r1} V_{s,r1} G_{s,r1} A_{s,r1} + \sum \sum k_{r2} V_{s,r2} G_{s,r2} A_{s,r2}$$

where $E_b, E_{b,ad}, E_s, E_{s,ad}, E_{s,r1}, E_{s,r2}$ and $G_b, G_{b,ad}, G_s, G_{s,ad}, G_{s,r1}, G_{s,r2}$ are the initial deformation moduli and shear moduli, respectively, of the base concrete, added concrete reinforcement, main flexible rod reinforcement, added flexible rod reinforcement, rigid central reinforcement (steel core), rigid outer steel reinforcement (pipe shell); $A_b, A_{b,ad}, A_s, A_{s,ad}, A_{s,r1}, A_{s,r2}$ are the areas of the section corresponding components; $I_{bx}, I_{bx,ad}, I_{sx}, I_{sx,ad}, I_{sx,r1}, I_{sx,r2}$ and $I_{by}, I_{by,ad}, I_{sy}, I_{sy,ad}, I_{sy,r1}, I_{sy,r2}$ are the moments of inertia of the corresponding components of the section about the axes Ox and Oy ; $I_{bk}, I_{bk,ad}, I_{sk}, I_{sk,ad}, I_{sk,r1}, I_{sk,r2}$ are the torsional moments of inertia of the section corresponding components; $V_b, V_{b,ad}, V_s, V_{s,ad}, V_{s,r1}, V_{s,r2}$ are the secant modulus coefficients (otherwise, the elasto plastic deformation coefficients) of the corresponding components of the section; $k_{b,ad}, k_s, k_{s,ad}, k_{r1}, k_{r2}$ are the coefficients of collaboration. Stiffness summation is performed over elementary areas (horizontally and vertically).

Further refinement of the proposed approach will consist in taking into account the actual mechanism of the appearance and development of the inclined [10] and spiral cracks [11], respectively, under the action of shear forces and torque, in determining the parameters of the rod section components' joint operation (partly studies on the current reinforcement of reinforced concrete beams with a steel-fiber-concrete "jacket" are made in [12] - [14]), taking into account the cyclic static and dynamic loads, which is important when designing wind turbines of the tower type [15]-[17].

4 Conclusion

1. The diagrammatic method continuity for calculating rod reinforced concrete elements under oblique eccentric compression according to BC 63.13330 and the classical approach of material strength have been established.
2. For the first time, the resolving equations of the diagrammatic method, in which all six components of internal force factors are taken into account, have been obtained. This is done based on Mohr's integral of displacements.
3. The proposed approach has development prospects. Its further refinement will consist in taking into account the actual mechanism of the appearance and development of inclined and spiral cracks, respectively, under the action of shear forces and torque, in determining the parameters of joint operation of the components of the cross section of the rod, taking into account cyclic static and dynamic loads, which is important in the tower type wind power plants' design.

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