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Beyond dogmatic fundamentalism in mathematics

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Abstract. In this work we discuss the path from the initial stage of mathematical awareness to the understanding and critique of one's standing beliefs as new stages of conceptual realization occur. Initial interpretation of discourse at the beginning of mathematical education passes through several stages from naive to reflexive and critical. We arrive at the concept of “initiation” into the intersubjective system at the root of mathematics and the necessity of faith in its agreements at the initial building blocks of mathematical method. There is an urgent need to reform the introduction of abstract concepts in mathematics at an earlier stage in education as they turn out not to be linked to accepted ways of teaching mathematics in a historical setting. Education in natural science suffers from lack of progression in maths curriculum and to achieve a consistent programme of development the educational progression needs to be reformed. The current state of mathematical education leaves students 500 years behind the present day and is inadequate for further progression of science.

1 Introduction

English anthropologist E.B. Tylor asserts that prayers begin as spontaneous utterances and degenerate into traditional formulas [1]. The spontaneity of appearance and reduction to dogmatic thinking is connected with the view of this article in regard to initial stages of mathematical education. The initial connection to mathematics in a person's development often occurs intuitively or spontaneously. Then it's met with the tradition of mathematical education which can broadly be described as “**continuous initiation**” which is considered as central concept of the discussion.There is rigid repetition at the core of the initial set up of the system that can be said to be mathematical knowledge.

From first grade children are taught Maths as strict science, with concrete, unwavering and absolute foundation, further quoting Karl Friedrich Gauss[2]’ expression putting mathematics as the queen of sciences. However, in practice young minds instead of facing this absolute foundation encounter dogmatic repetition of a set of instructions and methods underpinned by the authority of the teacher. Basic rules of Maths (for instance, the commutative property of addition) appear as unquestionable laws, until in the end they become ingrained in consciousness and ethics. The ethics is touched upon in the sense that questioning the dogma becomes sinful and is not tolerated, and more importantly, not

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expected to be tolerated. Bertrand Russel [3] says: ”Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of anything, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is, of which it is supposed to be true“.

There is a situation where this dogmatic setting becomes the story’s end just at the stage where mathematics is ready to present itself, be it as an intrinsic part of the world or a cultural construction. The leap to abstract thinking proves to be elusive for some. Discussion on how best to introduce the more conceptual parts of mathematical education clearly needs to be conducted.

Most people have no realisation that its ability to take things on faith is the engine behind the initial grasp of mathematics. It is clear that people do not realise that the process constitutes belief at its core. The cornerstone of the issue is realisation that the belief in the laws of maths is not very much different from religious faith, yet this faith in commonality of experience is essential to exploration of maths.

Throughout their education people go through certain aspects of “initiation” that brings them closer to fundamentals of mathematics, yet only a handful learn of even if not about lack of those fundamentals, then of their artificial and imagined nature as well as about the openness of the problem of fundamentals of mathematics. Current system of education simply doesn’t allow most people to get to the required stage as it guides them through milenia of mathematical knowledge,trying to guide people from trivial to abstract understanding, to which few arrive at.

Continuation of analysis of the acquisition of mathematical knowledge follows. It aims to answer the question of whether an alternative path exists that allows to find mathematics as an abstract entity initially, circumventing the “initiation” stages, to defeat the dominance of the dogmatic fundamentalism in relation to maths in minds of many.

2 Analysis

Structure of the process of maths education with Russian Federation’s education was considered. Six stages of understanding mathematics and its foundations were outlined, mostly along with existing schooling stages. Boundary is not absolute here as a lot depends on the individual’s worldview, but teaching curriculums along the school stages allow for the overarching theme of these divisions to work.

2.1 Illiterate adults and children under 4

Here one is directed by intuitive mathematics, with the instruments available being the simple count of material objects. Here mathematics is a part of the natural world. In Piaget's theory of development there is a stage that humans pass where their self encompasses the whole of the world, and is all encompassing, syncretic and absolutely subjective, individualistic. People remaining at this level of thought possess mythological thinking apparatus as the simplest one to use as the base of one's worldview according to Claude Levi-Strauss. The lack of critical analysis, reflection doesn't allow for understanding of maths fundamentals.

Past this stage there is a differentiation of self and the outside world that places a strong window of opportunity to introduce abstract thinking as this separation is vital to understand intersubjectivity as introduced by Husserl [4] in its most rudimentary form. Perhaps before this separation there is only the teaching of count that is available as a person without individuality has no concept of subjective agreement between people that we assert lies at the root of modern mathematics. Once there is individuality a concept of following the axiomatic systems and agreements between mathematicians can be introduced into the developing mind. Piaget [5] puts the age of operational thought at some stage past 7 years old and abstract thinking into adolescence and adulthood. Yet the realization of self at this age allows for the introduction of modern mathematics as we assert there is no need for progression from operational to abstract and the two can in fact be taught simultaneously. The notion that there is imagination in kids, directly contradicts the perceived lack of abstract thought. Bertrand Russel in Principles of Mathematics states [6] that “What does not exist must be something, or it would be meaningless to deny its existence; and hence we need the concept of being, as that which belongs even to the non-existent”. It is clear that lack of self differentiation and the change to awareness must be regarded as something and acted on in the process of education.This is the basis of further work on the subject and critique of Piaget’s stages [7] of development where we attempt to justify the new educational method.

According to Kant, Husserl and those who continue their traditions such as Barbara Jaworski[8,10], intersubjectivity is required to even talk about math. In philosophy the intersubjectivity remains an open question, therefore only faith into universal basic constructs of maths, into identical recognition of its ideas by individuals allows math to exist.

2.2 Elementary school

This level of understanding has been touched upon in the introduction and here the main experience of the learner is the setting of mathematical habits for use at further stages of education. Maths teaching is based on tangible objects, with the utmost effort of the curriculum and teachers alike to conceal the abstract nature of objects under study. There is a conscious effort to link the maths education to the material as there is a belief that children aren’t able to work with abstractions. Mathematics is shown conceptually in its state from 2500 years ago, with little regard for its journey since then.

As it was previously mentioned, dogmatism takes the central role in the process. Due to undeveloped critical thinking methods, the faith in authority of an adult teacher means that children form a view that mathematical laws are objective and universal. Even the most meticulous kids, for example if he or she feels the doubt over the commutative property of addition of large numbers, with those numbers which the child is not able to count yet, is forced to settle on the notion that adults have most certainly checked every large number for this property.

Even at this level there forms a faith into the mind of a higher order, that has already checked, proved and justified all the properties, laws and rules. Those who are not able to go past this stage of mathematical and scientific understanding form a faith into some authority as a basis of one’s trust in journal articles or news items be those of true nature or falsified information.

2.3 Middle school

At this stage in the curriculum pupils first encounter something abstract - variables. They can not be touched or felt, can only be imagined to exist. With the introduction of variables kids are confronted with the abstract essence of mathematics. Some objects exist that can only be thought of, rather than touched. There is some degree of "initiation" at this stage of learning. Some confess that at this stage they lose grasp of the study of mathematics.

Yet, rules of engagement with new objects still come as a set of instructions from a figure of authority - the teacher, which still falls into dogmatic acceptance of mathematics. Thus even as abstract objects are introduced, the fundamental understanding remains at the level of dogmatic belief.

2.4 High school

High school level of education corresponds to use of Maths in conjunction with Natural Science which comes into play at this level of development. At this stage of learning, a student is aware of the existence of axioms, knows about induction and deduction, and is able to build a chain of reasoning. With successful completion of the curriculum of this stage these skills are mastered practically in full. A person is able to formulate, prove and disprove statements in the system of axioms that is present in the school curriculum. Yet there is a lack of understanding of the nature and necessity of axiomatic systems, and the lack of comprehension that at the base of these systems lies faith in intersubjectivity. As a result there is no “initiation” at this stage, and the change in comprehension of maths compared to Middle School has quantitative and not qualitative character.

Usable mathematical apparatus in physical sciences suddenly presents countless practical problems which are utilized in the process of understanding the natural world. The problem here is twofold: while presenting the practical side of scientific calculations it becomes apparent that one can not progress in Natural Sciences without appropriate mathematical training. It is a race, where a mathematical tool is sometimes used in the following lesson in physics class.

School physics curriculums are severely limited by the available mathematical material, as is chemistry. A deeper understanding of Maths will undeniably broaden the scope of available topics in Chemistry and Physics. Thus it's beneficial to consider ways in which maths teaching can be hastened in order to unlock further dive into Natural Sciences. There isn’t enough interdisciplinary structure. The current 7 years of Maths training in preparation for Physics and Chemistry classes are all quickly utilized fully, while advanced topics don’t come until university level where there is no longer a variety of subjects taught in conjunction. There is a situation where students in STEM fields lack the mathematical apparatus necessary for approaching cutting edge level of thinking in their fields despite 11 years of math training. This situation is preventable with a radical rethinking of mathematical education

2.5 STEM & Philosophy degrees

Once at the level of higher education those who made a decision to continue studying mathematics at the level of tertiary education will be shown the main mystery of mathematics and will understand their own power in conducting the conceptual inquiry into the systems under study. The main axiom is revealed in the form of: “we are able to set axioms”. Those beyond this level are able to construct their own axiomatic systems, developing theorems and build their own theory.

For most however, even at this stage not only the answer to the question is not revealed, it's the question of why people understand mathematics homogeneously and is that in fact a true statement remains elusive.

In addition at this stage most students encounter philosophical concepts that were developed to try to address the problem of the entrance point (the basic axiomatic statement), criterias of completion and incontradictibily of axiomatic systems and others. Coming across the experience of previous generations students learn about the secondary nature of their reasoning while learning about the thought through but ultimately unsuccessful attempts to resolve the stated problems. Hegel in his lectures regarding the history of philosophy was asking the question: “why look at a gallery of oddities?” puting the idea of studying mutually contradictory philosophical systems to question. He answered himself on the matter stating: “Ideas are developed through contradictions.” Only with familiarity with known contradictions can a resolution be found.

2.6 Postgraduate/Self-further education, reflection and philosophizing

At this stage a person is able to find and critique the faith into intersubjectivity. Some reach this stage at the previous level of academic education, and these people are usually philosophers as this discipline has a relation to root causes of any area of academia, mathematics in particular.

To be bold, there is no guarantee a PhD in mathematics will not be a certain marker that a person has started questioning the concepts underpinning mathematics as it is possible to operate in the mathematics space, in the technical middle ground and achieve the criterias for PhD eligibility.

Central to this process is an individual's relation to philosophy, reflection; without philosophising on the nature of one's discipline a scientist goes against his nature as the very nature of the thought process in an area of study is philosophizing over concepts at the cutting edge of research.

Only this level of understanding allows for full fluency in operation with mathematics, compared to operation in a native language, allowing for reconstruction of the fundamentals making them more solid and based as required. Conversely if the problem lies in its excessive fundamentalism one is free to deconstruct the base somewhat. Critique of the existence of fundamentals becomes accessible. A person becomes not just a professional in the field but an entity that shapes the field and the discussion within it.

3 Results

For estimation of the percentiles of people passing each stage of mathematical understanding a table has been drawn up with the information about the understanding of maths, understanding of theoretical apparatus, education stage and the percentage of people passing the stages. Even with the direct correlation between the concepts of maths stage and fundamental understanding principal difference between them directs that they are expressed separately. Data for education is taken on the basis of the population census of 2010. Percentiles are placed from the total population.

Table 1. Distribution of population gone through different stages of math.

Fundamental understanding	**Maths stage (acc. curriculum)**	**Education level**	**% of population gone through the stage [9]**
No understanding/ Intuitive operation	Maths operating extrinsically	No education/Kindergarten	100%
Dogmatic repetition	Basic introduction	Elementary school	93,7%
First steps in logical understanding and the process of reasoning	Decoupling of the tangible and the abstract	Middle school	84%
Further steps in logical understanding and	Working toolkit	High school education/Professional education	77%

the process of reasoning			
Fundamental understanding	Hierarchical system	STEM & Philosophy degrees	8,4%
Deep expertise	Place of mathematics in human information field	Postgraduate/Self-further education, reflection and philosophizing	Needs census Approx. <0.1%

As seen from the table, the depth of understanding of the discipline is required for understanding of its apparatus. It has to be noted that levels of understanding listed in the first column are attainable even without rigorous study of mathematical apparatus. How is this possible? Its due to the case that mathematical education follows mathematics development path from a historical perspective through its 2500 year history and leaves most at the 500 mark by the end of their relationship with mathematics. There is however, the potential to directly come in contact with the fundamentals from studying philosophy and self reflection.

As it was mentioned before there is a certain percentage(0.3% of the population) of illiterate people. They are stuck at mythological thinking level and couldn't go past the elementary schooling stage. Further 6% of the population is aged at the level of elementary school and are not yet ready for other levels of thought.

Percentage of people receiving some degree of familiarity with abstract thought, yet do not understand the fact they possess it and use these concepts stands at 84%, 77% have a slightly deeper conceptual realisation and a stronger mathematical apparatus. Therefore there is a sizable drop off at the stage between acquiring abstract concepts and having a small usable mathematical apparatus.

The proportion of people receiving degrees in the STEM fields is 8.4 percent. They have a strong grasp of mathematical apparatus as well as a course in philosophy which touches upon the problems in the fundamentals of maths and sciences. The drop off at this stage is 68.6% which is severe and explained by lack of tendency for reflective existance in the value system. This value systems are set by educational culture.

Those in the 0.1% of the population achieving the final understanding stage are the authority of those who are at the level of dogmatic understanding. These people are the reformers of science.

Diagram 1 has been constructed for representation of these concepts, there is a obvious divide between high school and STEM levels mentioned previously.

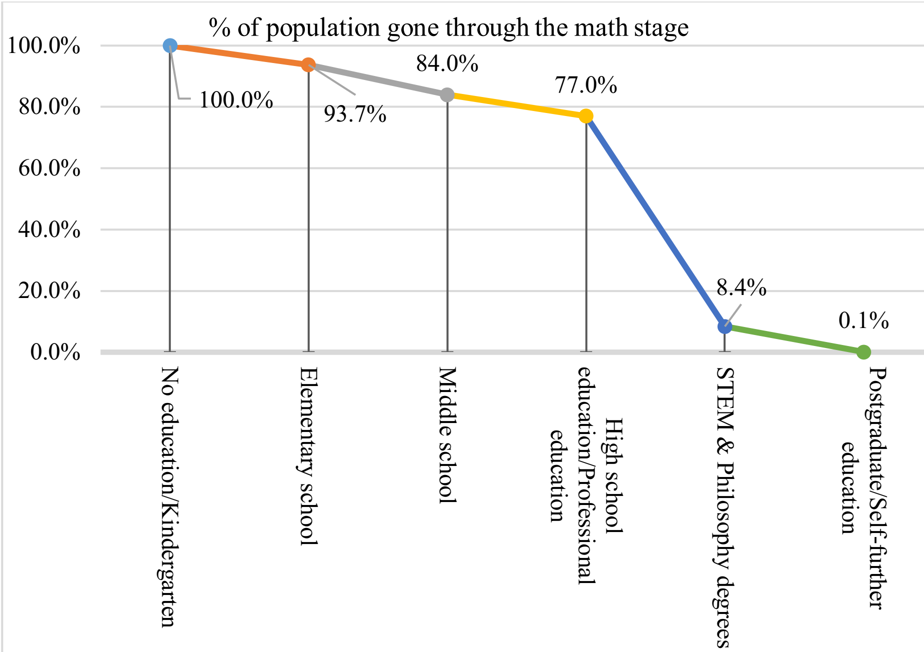

Fig. 1. % of population gone through the math stage.

4 Discussion

The first stage of mathematical understanding is, in essence, the question of extrinsic or cultural nature of mathematics. With the two sides of the argument of whether math is a property of this world or a cultural phenomenon. This debate has split the world of mathematics with the likes of David Hilbert and Henri Poincare taking opposing sides.
To add, in essence the difference between a mathematician and a person not familiar with the fundamentals is that a mathematician reflects on his faith in the basis of math while a person on the side of the argument is not willing to reach the fundamental concepts of faith at the core of math.

Some philosophical conceptions say that mathematics is the language of science. Werner Heisenberg considered maths to be the form in which we express our understanding of nature, not the content. To quote[8]: he thinks that modern physics has definitely decided in favor of Plato. In fact **the smallest units of matter are not physical objects in the ordinary sense; they are forms, ideas which can be expressed unambiguously only in mathematical language.** Here to master maths one has to master a language. In this way mathematical modelling is reduced to translation from a natural language to mathematical abstraction. When a person familiarizes oneself with the language to the point thoughts appear in the form of that language freedom of expression comes, thought and language become linked at the deepest level.

Mathematics is undoubtedly a singular language in the way it's being studied. It's commonly known that the most effective way to learn a language is immersion. Whereas in mathematics the learning starts on a tangent far away from the current state of affairs and few reach fluency. How can one learn a language the essence of which is not seen or known, only taken on faith that it exists?

It's possible that the approach to education has to be rethought. Children might be better off to be initiated into realization of the abstract, into the existence of objects that can be imagined but not felt. There is a certain underestimation of childrens' imagination, feeling that they won't be able to grasp abstract thoughts yet it's often said that childrens' imagination is more powerful than that of an adult. It is feasible to imagine that children must be told about the lack of universal forms and laws, but dogmas exist that are being followed. In this paradigm shift in the outlook on mathematics children can realize the fundamental problems that weren't tackled by their ancestors.

A new generation of mathematicians, those brought up with the outlook from the current state of math and looking into the past may fare better than those who followed the development of a historical narrative in the field of mathematics from the past into the current. This is one of the ways of eradicating dogmatic fundamentalism in mathematics.

Is the ture that simple to hard is equivalent to going from natural to abstract? Where is the confidence that children won't be able to grasp abstract concepts from the beginning of their education and be initiated into the true state of mathematical thought much sooner?

5 Conclusions

Analysis within this work points to mathematical understanding being in part a problem of the system of education that introduces abstraction in the later stages of education, ones inaccessible for the majority of the population. It mustn't be forgotten that value systems that maximize reflection and critique of the fundamentals are not in line with governmental policy. This approach however delays development in scientific and philosophical thought, and, as a consequence, development of humanity which tells that there is an urgent need to reconsider the educational and value systems currently in place.

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