Analytical and numerical study of influence of the shape of the section on lateral buckling of doubly symmetrical sections

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Abstract. This paper presents an analytical and numerical study of the lateral buckling of beams with double symmetrical I and H cross sections having substantially the same plastic modulus of resistance around the strong axis subjected by a uniformly distributed load in order to understand the influence of the one of the forms during lateral buckling. For this, a critical elastic moment analysis is carried out using ANSYS software using the element SHELL181 and analytical formulas from Eurocode3. Finally, there is a presentation of the non-linear behavior of these two cross sections.

1 Introduction

Modern construction is characterized in all areas of civil engineering by the usage of large span structures and the progressive reduction of massive sections. This reduction is due to economic and assembly concerns, thus to overcome all these difficulties, thin structures are used. These thin structures are characterized by lightness and good flexural strength.

In the case of slender elements, their resistance is greatly affected by instabilities, such as lateral buckling of beams. The lateral buckling of the beams is characterized by a bending-torsion coupling, that is to say the lateral torsional buckling simultaneously generates a lateral bending strain and a longitudinal torsional rotation along the unsupported portion of the beam. In addition, twisting is often accompanied by warping. We then speak of non-uniform torsion. In this case, the classical theory based on the Saint-Venant model in uniform torsion is no longer valid. In general, for the study of these problems, the Vlassov model [1] is the most used.

Several factors like the shape of the cross section, the boundary conditions, the point of application of the load, etc. whether it is symmetrical or not can directly influence the stability of the beams under any load.

Many analytical, numerical and experimental studies have been carried out in order to better elucidate this phenomenon of instability. The first studies date from the 20th century, like the work of Timoshenko [2] on a symmetrical bi-supported beam subjected to a pure bending moment. This work made it possible to establish the formula for the critical moment of lateral buckling in specific cases. Throughout the 20th century other authors have also worked on this topic. In 2001 based on the method of Galarkin, Mohri and Al [3] , who presented a simplified analytical solution to calculate the elastic critical moment of I-beams under lateral loading In 2011 Yoo and Lee [4] presented in their book "structural stability" an analytical solution of the critical load and the critical moment for doubly symmetrical cross sections on fork supports subjected to a concentrated load at mid-span and a uniformly distributed load. The formulas used in several standards such as Eurocode [5] are the results of experimental and theoretical research by certain researchers

The most recent studies are very strongly marked by numerical methods in particular the finite element method. The contribution of numerical computation softwares like ANSYS and Abaqus has indeed changed the way of approaching these problems. Numerical resolution methods have thus opened up new horizons in understanding the postcritical behavior of structures. In 2015 Fourdi and Al [6] developed a finite element approach that can describe the non-linear behavior of certain sections under lateral buckling. Arash and Al in 2019 [7] developed a finite element model on the resistance of a planar frame to lateral buckling. Carlos and Al [8] performed a numerical study to understand the influence of certain imperfections on the lateral buckling of slender I-beams. The work of Rossi and Al [9] have detailed the influence of imperfections on the resistance of I-beams on lateral buckling.The combination of numerical and experimental methods has enabled certain authors [10-15] to understand the failure mode of I-beams under lateral buckling.

The objective of this work is focused on studying the influence of the shape of the doubly symmetrical I and H section having substantially the same plastic modulus of resistance around the strong axis and thus analyzing their non-linear behavior. The study will be done by means of finite element computation using the element SHELL181 of the ANSYS code then by analytical normative formulas.

2 Materials and methods

The fundamental case for the study of lateral buckling is a simple beam , see Fig.1, subjected in pure bending. Starting from the deformed state of the beam, one can calculate the value of the critical load for which the system is in metastable equilibrium. This fundamental case of the simple beam was solved by Timoshenko.

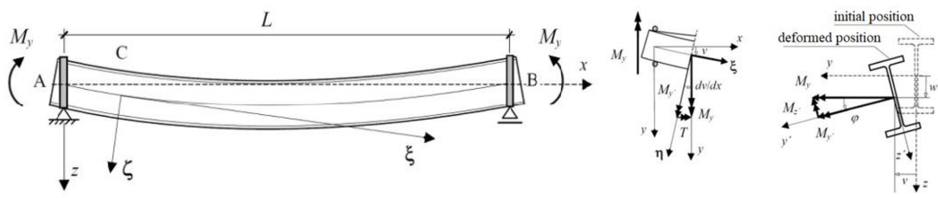


Fig. 1. Lateral buckling of a doubly symmetrical I-beam, subjected in pure bending.

The differential equation describing this phenomenon of instability of this beam is given by the following relation:

$$EI_w \frac{d^4\varphi(x)}{dx^4} - GI_t \frac{d^2\varphi(x)}{dx^2} - \frac{M_y^2}{EI_z}\varphi(x) = 0 \qquad (1)$$

Solving the differential equation (1) based on the boundary conditions allows us to determine the critical moment of the elastic buckling Mcr, for which the system is in metastable equilibrium. This critical moment is formulated as follows:

$$M_{cr} = \frac{\pi}{L}\sqrt{GI_t EI_z\left(1+\frac{\pi^2 EI_w}{L^2 GI_t}\right)} \qquad (2)$$

E – Young's modulus

G – Shear modulus

I_z – Second moment of inertia about weak axis

I_t – Torsion constant

I_w – Warping constant

L – Length between lateral restraints

2.1 Elastic critical moment of buckling according to Eurocode 3

The formula adopted by Eurocode3 [5] to calculate the elastic critical moment in doubly symmetrical sections I and H is given by:

$$M_{cr} = C_1\frac{\pi^2 EI_z}{(k_z L)^2}\left[\sqrt{\left(\frac{k_z}{k_w}\right)^2\frac{I_w}{I_z}+\frac{(kL)^2 GI_t}{\pi^2 EI_z}+(C_2 z_p)^2}-C_2 z_p\right] \qquad (3)$$

k_z – effective length factor which is related to the restrain against lateral bending at the boundaries

k_w – effective length factor which is related to the restrain against warping at the boundaries

C_1 and C_2 are coefficients depending on the loading and end restraint conditions. These coefficients are given in Eurocode3 for different types of loadings.

2.2 Numerical analysis method in ANSYS

The numerical work proposed here on the lateral buckling of I and H beams having substantially the same plastic modulus of resistance is carried out by means of a finite element analysis using the element SHELL181 of the ANSYS code.

SHELL181 is a quadrilateral element has four nodes and six degrees of freedom per node (displacements UX, UY, UZ and rotations of the normal vector ROTX, ROTY, ROTZ), being able to work in membrane (compression) and in bending. This element is capable of being used for large deformations, and can also be used for isotropic and orthotropic materials.

The geometry, the locations of the nodes and the coordinates of the system are represented in the next figure:

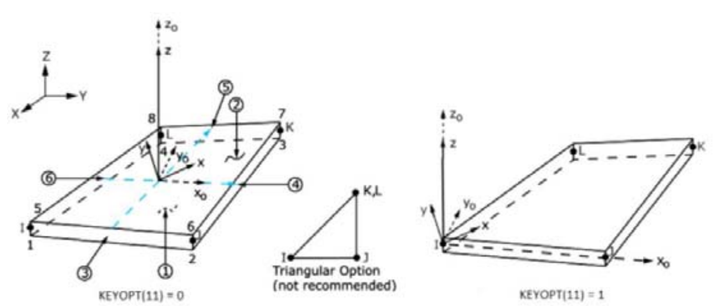


Fig. 2. The geometry of the element SHELL181.

The determination of the critical multiplier of the bending moment distribution M in the beam is carried out by resolution of the problem with the associated eigenvalues.

$$det\left(\left[K_L\right]+\mu_{cr}\left[K_G\right]\right)=0 \quad (4)$$

If M_{max} is the maximum moment in the beam under the applied loads, then the critical moment is obtained by:

$$M_{cr}=\mu_{cr}.M_{max} \quad (5)$$

$\left[K_L\right]$ is the linear stiffness matrix corresponding to the basic state of the structure
$[K_G]$ is the stress stiffness matrix

3 Results

In general, it emerges from Table 2 of comparison between the values calculated analytically according to the formula of Eurocode 3 and those calculated numerically (ANSYS) that the different methods give practically the same values of the elastic critical moment. The study was done on I and H beams having substantially the same plastic modulus of resistance around the strong axis with spans ranging from 3 to 5 m under a distributed uniform load.

By observing the image of the first form of buckling of H-beams, we notice a certain nuance on the strain of the upper flanges compared to the strains observed on the I-beams which all deform in the same way whatever the length of beam. Thus for the H-beam with a length of 3 m, its upper flange takes a wavy shape. It is from the span of 4 m that the deformation becomes more or less similar to that of I-beams.

By carrying out an analysis of the non-linear buckling ,see Fig. 8 and Fig. 9 of the two beams I and H in steel S235 with a span of 3 m, we noticed that the first plastic deformation happens practically at the same time as the difference in the loss of stability.

3.1 Numerical investigation

In this part, we present the results obtained by our analytical and numerical study for the symmetrical sections I and H having approximately the same plastic modulus of resistance wpl.y. Analytical calculations are established by the Eurode 3 formula and numerical calculations by ANSYS. Calculations are made for beams with a span of 3 to 5 m. These beams are subjected by a uniformly distributed load. The geometric properties of these cross-sections are given in Table 1.

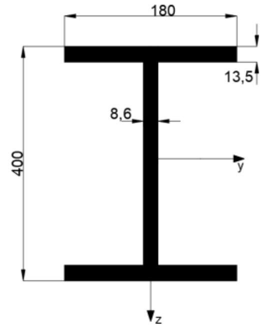
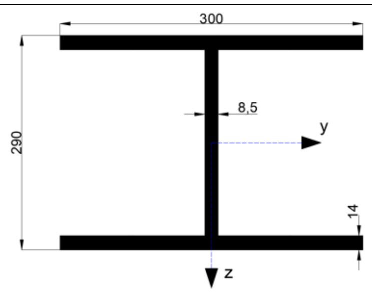
In the case of a simple beam with fork supports, the k_z and k_w coefficients of equation (3) are equal to 1.

Since the point of load application is on the upper flange of the section, $z_p=\frac{h}{2}$

The coefficients C1 and C2 in Eurocode3 for this loading case are respectively 1.13 and 0.45. The critical moment formula becomes:

$$M_{cr}=1{,}13\frac{\pi^2 EI_z}{(L)^2}\left[\sqrt{\frac{I_w}{I_z}+\frac{L^2 GI_t}{\pi^2 EI_z}+\left(0{,}45z_p\right)^2}-0{,}45z_p\right] \quad (6)$$

Table 1. The geometric properties of these cross-sections.

Cross-section	**Geometrical properties**
 I cross-section	$w_{pl.y} = 1,24.10^6\ mm^3$ $A = 8067,8\ mm^2$ $I_y = 2,1876.10^8\ mm^4$ $I_z = 1,3142.10^7\ mm^4$ $I_w = 4,8939.10^{11}\ mm^6$ $I_t = 3,7715.10^5\ mm^4$
 H cross-section	$w_{pl.y} = 1,3.10^6\ mm^3$ $A = 10627\ mm^2$ $I_y = 1,7285.10^8\ mm^4$ $I_z = 6,3013.10^7\ mm^4$ $I_w = 1,199.10^{12}\ mm^6$ $I_t = 6,0048.10^5\ mm^4$
Steel S235 $E = 210000\ N/mm^2$ $G = \dfrac{E}{2(1+\nu)} = 80769\ N/mm^2$ $\nu = 0,3\quad Poisson's\ ratio$	

The analysis is carried out on a beam with fork supports subjected to a uniformly distributed load. The load is applied to the upper flange, see Fig. 3.

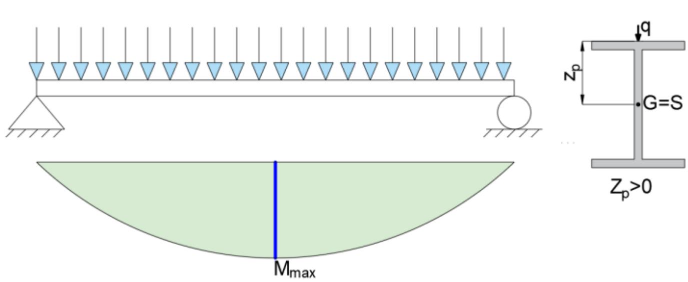


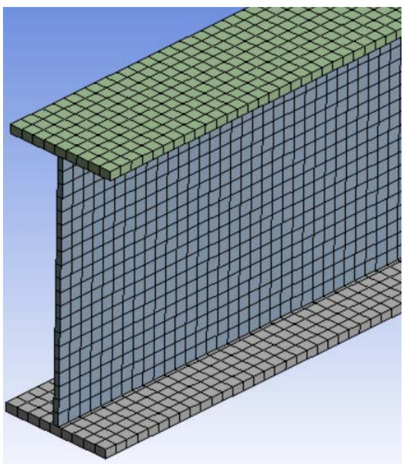
Fig. 3. The beam under a uniformly distributed load.

To perform the numerical calculation in ANSYS we used a uniformly distributed load q = 50 kN / m. The maximum moment for a beam at the fork supports under a uniformly distributed load is given by the following formula:

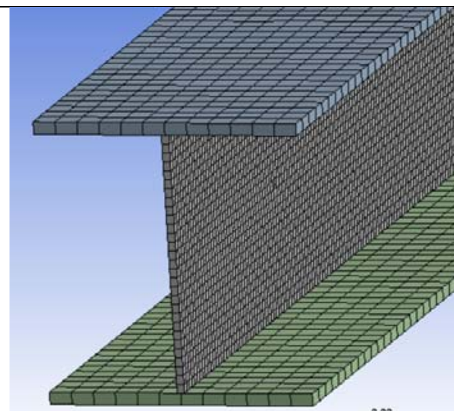
$$M_{max} = \frac{qL^2}{8} \quad (7)$$

The numerical elastic critical moment is calculated by the relation (5), the values μ_{cr} of each beam are given in Figure 6.

The mesh in ANSYS was done with linear elements 20x20 for the I beams, for the H beam 20x20 for the flanges and 10x10 for the web, see fig. 4



Mesh size I-cross section (20x20)



Mesh size H-cross section (20x20) for Upper flange
and (10x10) for Lower flange

Fig.4. Mesh size for I and H beams.

Table 2. Numerical and normative critical moments of the beam I and H simply supported subjected to a distributed load.

L(m)	**Cross-setion**	**The elastic critical moment, KN.m**		**Δ% (ANSYS/ Eurocode)**	$\frac{M_{cr}(H-\text{beam})}{M_{cr}(I-\text{beam})}$
		FEM (ANSYS)	Eurocode 3		
3	I	495	497.18	0.438	2.79
	H	1381.725	1386.54	0.347	2.79
4	I	308.2	310.6	0.77	2.74
	H	844.62	847.205	0.3	2.73
5	I	221.54	222.52	0.44	2.66
	H	589.84	595.15	0.89	2.67

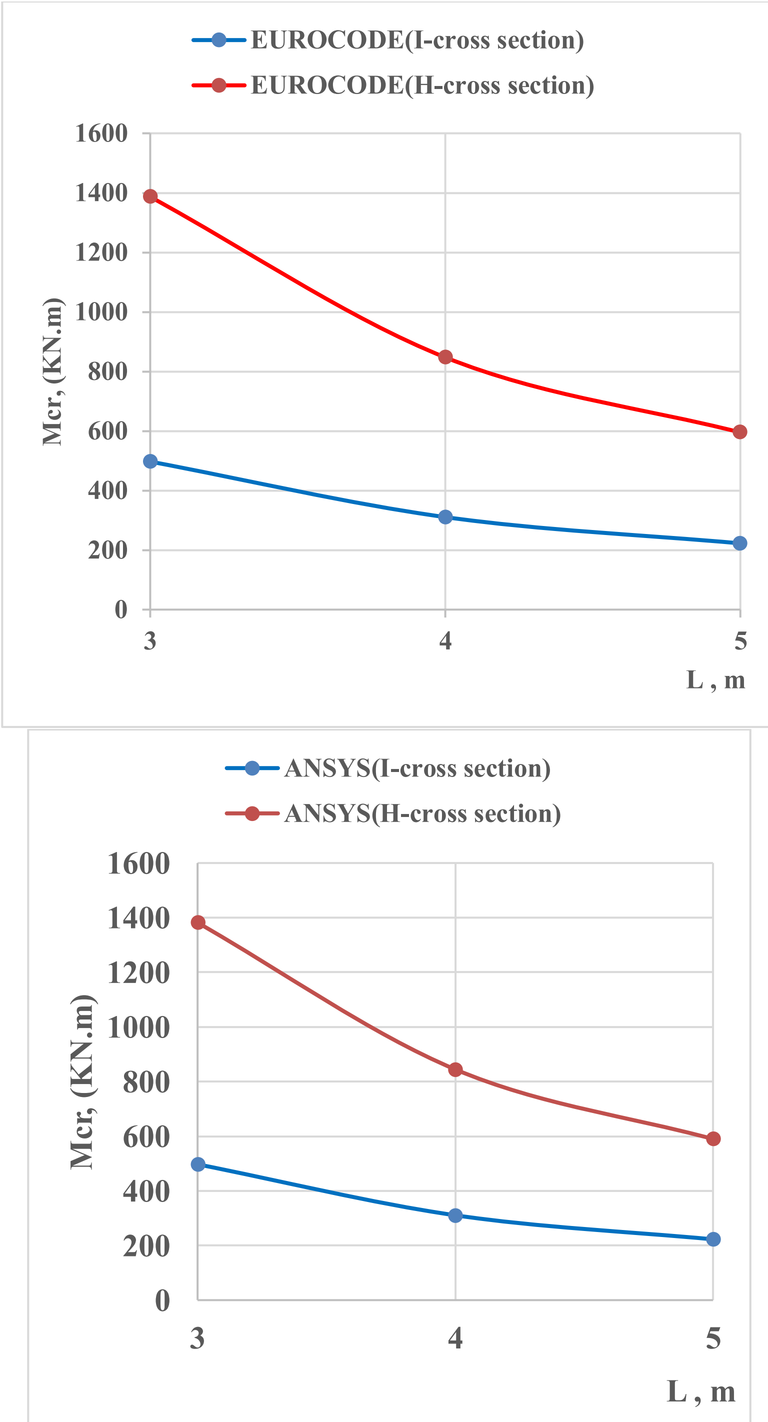


Fig. 5. Variation of numerical (ANSYS) and normative critical moments.

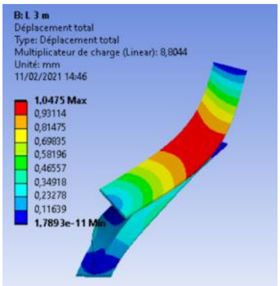
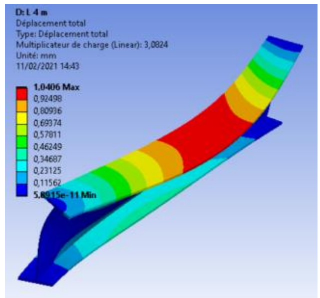
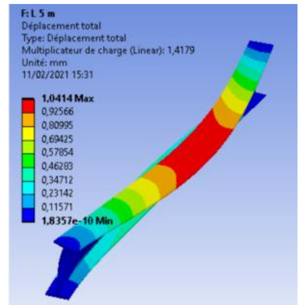
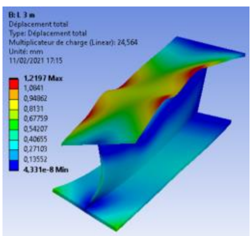
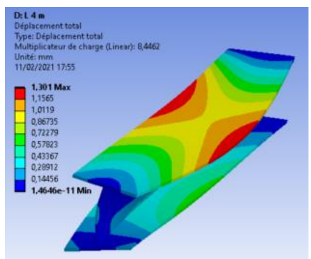
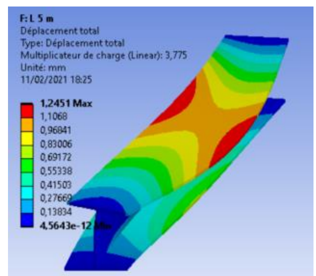
 I-cross section, L=3 m, $\mu_{cr} = 8,8$	 I-cross section, L=4 m, $\mu_{cr} = 3,08$	 I-cross section, L=5 m, $\mu_{cr} = 1,417$
 H-cross section, L=3 m, $\mu_{cr} = 24,56$	 H-cross section, L=4 m, $\mu_{cr} = 8,44$	 H-cross section, L=5 m, $\mu_{cr} = 3,775$

Fig.6. The first mode buckled shape of the different beams.

To perform the non-linear analysis we took only the beams with the span of 3 m for the two cases of cross section (I and H).

S235 steel with a yield strength f_y = 235 N/mm^2 was used. The bilinear material law is given in Figure 7.

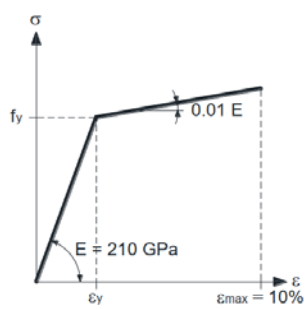


Fig. 7. The bilinear law used for this analysis.

The plastic modulus of resistance for cross sections I and H with respect to the strong axis is calculated as follows:

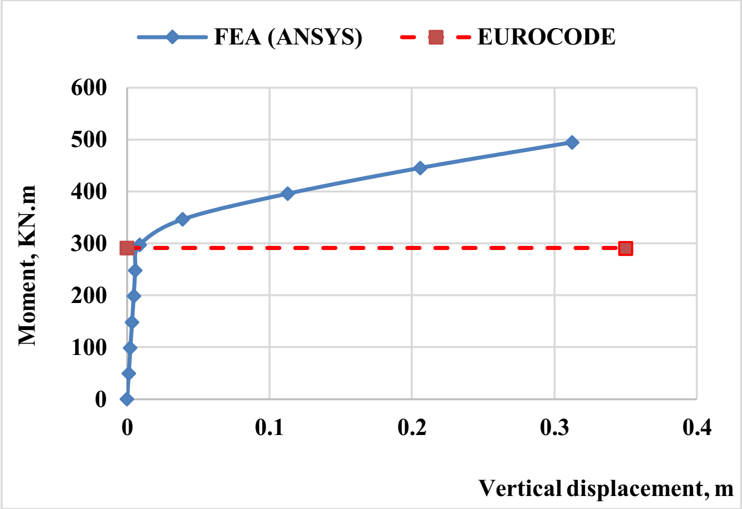
$$M_{pl.y} = \left[B.t_f \left(H - t_f \right) + \frac{t_w \left(H - 2t_f \right)^2}{4} \right] f_y \qquad (8)$$

Where:

$$W_{pl.y} = \left[B.t_f \left(H - t_f \right) + \frac{t_w \left(H - 2t_f \right)^2}{4} \right] \quad (9)$$

$$M_{pl.y} = W_{pl.y}.f_y \quad (10)$$

For I-cross-section $M_{pl.y} = W_{pl.y}.f_y = 1{,}24x235 = 291{,}4\ KN.m$
For H-cross-section $M_{pl.y} = W_{pl.y}.f_y = 1{,}3x235 = 305{,}5\ KN.m$
Figures 7 and 8 show the non-linear analysis of the lateral buckling of I and H beams under a uniformly distributed load.



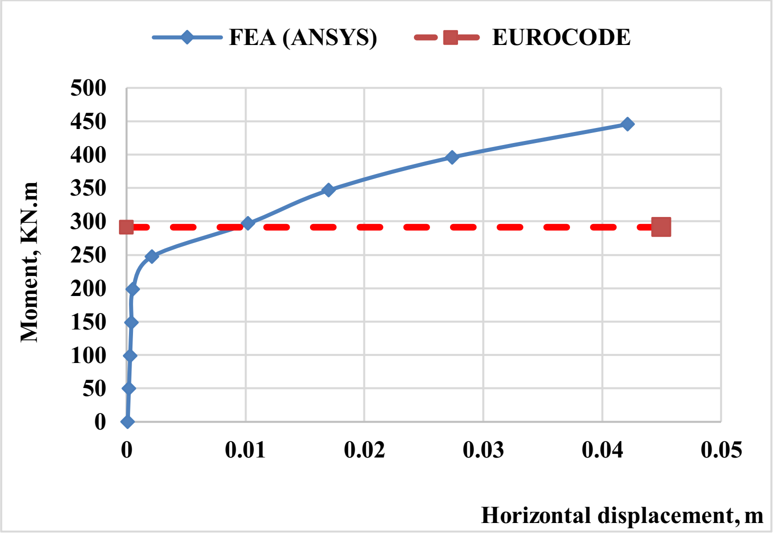


Fig. 8. Behavior of non-linear buckling of I-beam under uniform load.

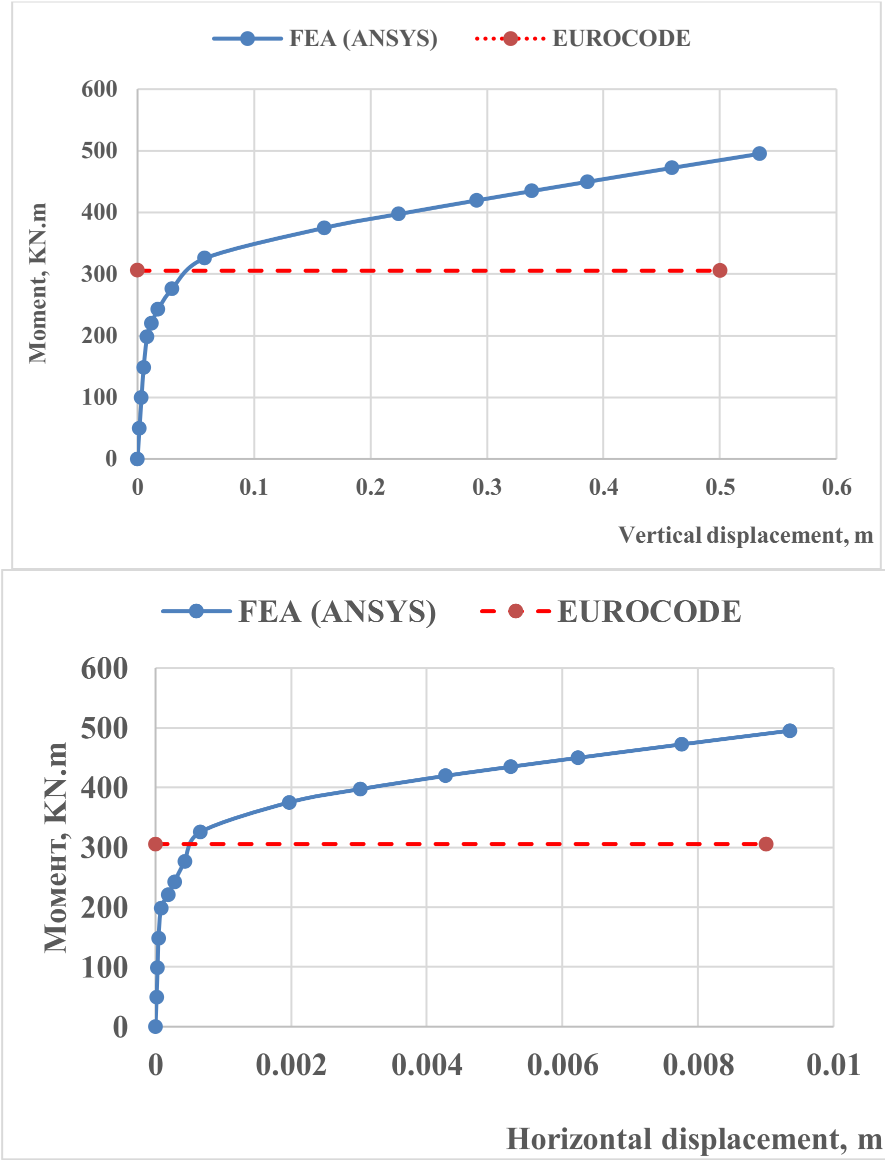


Fig. 9. Behavior of non-linear buckling of H-beam under uniform load.

4 Conclusion

This study has highlighted the following points:

1. the influence of the geometry of the cross sections is very significant on the resistance to lateral buckling;

2.For doubly symmetrical I and H sections, the distance from the point of application of the load to the center of gravity or of shear hardly influences the resistance of a beam to lateral buckling;

3.When flexural rigidity of the section in the small plane of inertia is low, the lateral buckling is more worrying, and the length of the beam plays a key role regardless of the cross section;

4.For the case of doubly symmetrical sections I and H having substantially the same plastic modulus of resistance around the strong axis considered in this study, the elastic critical moment which can cause the lateral buckling of the H beams under the distributed uniform load is almost three times that of I-beams;

5.The numerical and analytical results show a strong convergence if the element mesh was well done in the numerical calculation. Numerical finite element computation remains one of the most efficient methods to describe the non-linear behavior of a structure.

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