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Study of unsteady thermal conductivity in a multilayer rubber-metal product during post-vulcanization cooling and after-vulcanization

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Abstract. The article considers analytical solutions for a non-stationary problem of heat and mass transfer in a multilayer elastomeric material. Determined are the criteria, influence the process of temperature change in the treated material. A functional relationship between the main criteria of heat transfer and the temperature of the processed material was established, which is the basic relation during the development of an engineering method for calculating an industrial installation.

1 Introduction

Various industries pose high demands for the anti-corrosion protection of machine parts operating in strong aggressive working environments at high temperatures and pressures, with periodic changes in the composition of these media, under their effective mixing.

Rubber coating occupies a special position among the existing methods of protecting surfaces, due to the specific mechanical properties of rubber, i.e. high elasticity, shock absorption, good wear resistance, fatigue and strength characteristics, heat and frost resistance, resistance to aggressive media, heat, gas and water resistance. It is used to protect against aggressive media during the production of chemical equipment, in the automotive, aviation, and engineering industries, as well as for the production of various parts: gaskets, shock absorbers, buffers, bearings, tread rings to protect drill pipes from wear, etc.

Vulcanization of coatings is the final and most important process of the gumming cycle of any metal object, accompanied by high energy costs and it especially needs improvement.

The modes of heat treatment of rubber-metal objects are established experimentally from the laboratory tests or by measuring the product temperature and then determining the duration of vulcanization.

A.I. Lukomskaya [1], A.V. Lykov [2], S.S. Kutateladze, V.P. Isachenko, K. J. Baumeister [3], G.S. Emmerson [4], B.S. Gottfried [5], F.K. McGinnis [6], W.-J. Yang [7] and others conducted theoretical studies of heat and mass transfer processes during convective heat supply.

It is necessary to highlight not only the theoretical studies [1-4], but also the practical results of the Russian scientists [8-9], which were further used to develop the thermal regimes of heat treatment of rubber coatings for enterprises of the real sector of the economy.

A large number of works study heat transfer processes under film boiling conditions on a flat plate [3-7], which, in turn, influenced the mathematical model obtained in this work and became the basis for this research.

Mathematical modeling of heat and mass transfer processes in complex heat engineering systems was carried out by the authors in [10-11], as a result of which the dependences of heat transfer in non-stationary conditions were obtained.

The performed analysis of works allows us to conclude the importance and significance of the processes of heat and mass transfer during the heat treatment of gumming coatings and to state that the internal problems during heat treatment of elastomeric coatings require further studies.

2 Materials and methods

An internal heat release occurs at the initial moment of time during cooling of a rubber-metal product. The internal heat generation associated with the internal vulcanization reaction is proved by the fact that during vulcanization the middle part of the elastomeric coatings was heated above the temperature of the heat transfer medium. The increase in temperature is directly related to the content of bound sulfur in the elastomer. In ebonite mixtures, up to 920•103 J/kg of rubber is released during vulcanization [1, 2].

The rate of vulcanization and the degree of its course (degree of vulcanization) depend on temperature, and, consequently, the amount of heat energy supplied. In turn, the heat of reaction is a function of the degree of vulcanization. During vulcanization, its speed can be quantified. If a lot of heat is released in the system, and its reactivity is limited, the release of volatile products and the transformation and destruction of the polymer occur. Comparing the rates of heat generation and vulcanization, one can notice the conditions under which the thermal effects of vulcanization have a negative impact on the quality of the covers of rubber-metal objects. This is especially noticeable at high vulcanization temperatures (above 428 K).

The main task of thermal conductivity of a multilayer cover is to find the temperature field inside the elastomer based on its known characteristics [5, 6].

Consider a model of a cooled sample: a layer of rubber or ebonite is attached to a metal layer using a glue seam. Thermophysical properties of steel, determined by thermal conductivity and thermal diffusivity, differ significantly from the same properties of layers of glue and rubber. The thermal conductivity of steel is more than 2 times higher than the thermal conductivity of rubber.

In this regard, considered is a multilayer flat wall, which separates media, the temperatures of which arbitrarily change in time. Let the heat exchange on the outer surfaces of the wall with media occur according to Newton's law, and within the limits of each of its layers, an internal source of heat that changes in time acts. Then, assuming that an ideal thermal contact occurs between the layers of the wall, and the thermophysical properties of the layers and the intensity of internal heat sources do not depend on temperature, the problem of determining the unsteady temperature field in the considered multilayer flat wall can be reduced to integrating the following differential equation of unsteady heat conduction [3, 4, 10]:

$$C_\upsilon(x)\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left[\lambda(x)\frac{\partial T}{\partial x}\right]+q_\upsilon(x,\tau)$$
$$(0\le x\le x_n;\ \tau>0). \qquad (1)$$

With the initial condition

$$T(x,0)=f(x), \qquad (2)$$

and the boundary conditions

$$\left[-\lambda(x)\frac{\partial T}{\partial x}+\alpha_1 T(x,\tau)\right]_{x=0}=\alpha_1\varphi_1(\tau);$$
$$\left[\lambda(x)\frac{\partial T}{\partial x}+\alpha_2 T(x,\tau)\right]_{x=x_n}=\alpha_2\varphi_2(\tau), \qquad (3)$$

where x is the linear coordinate measured from one of the outer surfaces of the wall; $\varphi_1(\tau)$ and $\varphi_2(\tau)$ are the media temperatures; α_1 and α_2 are the heat exchange coefficients.

3 Results and discussion

The thermophysical characteristics of a multilayer wall as a whole and internal heat sources acting in it with intensity $q_\upsilon(x,\tau)$ as a function of x coordinate are presented in the form

$$\lambda(x)=\lambda_1+\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)S_-(x-x_i)$$

$$C_\upsilon(x)=C_{\upsilon_i}+\sum_{i=1}^{n-1}(C_{\upsilon_{i+1}}-C_{\upsilon_i})S_-(x-x_i)$$

$$q_\upsilon(x)=q_{\upsilon_1}+\sum_{i=1}^{n-1}[q_{\upsilon_{i+1}}(\tau)-q_{\upsilon_i}(\tau)]S_-(x-x_i)$$

where λ_i and C_{υ_i} are the thermal conductivity and the volumetric heat capacity of the *i*-th layer of the wall, respectively; q_{υ_i} is the intensity of internal heat sources in the *i*-th layer of the wall; x_i is the coordinate of conjugation of the *i*-th and *i*+1-th layers of the wall; *n* is the number of layers; $S_-(x-x_i)$ is the asymmetric unit function [7].

If we switch from from variable x to a new independent variable z according to the relation

$$z=\int_0^x\sqrt{\frac{C_\upsilon(x')}{\lambda(x')}}dx'=\sqrt{\frac{C_{\upsilon_1}}{\lambda_1}}x+$$
$$+\sum_{i=1}^{n-1}\left(\sqrt{\frac{C_{\upsilon_{i+1}}}{\lambda_i}}-\sqrt{\frac{C_{\upsilon_i}}{\lambda_i}}\right)(x-x_i)S_-(x-x_i) \qquad (4)$$

Then the boundary problem (1) - (3) will take the form

$$\frac{\partial T}{\partial \tau}=\frac{1}{\sqrt{C_\upsilon(z)\lambda(z)}}\frac{\partial}{\partial z}\left[\sqrt{C_\upsilon(z)\lambda(z)}\frac{\partial T}{\partial z}\right]+\frac{q_\upsilon(z,\tau)}{C_\upsilon(z)},$$
$$(0\le z\le z_n;\tau>0), \qquad (5)$$

$$T(z,0)=f(z) \qquad (6)$$

$$\sqrt{C_{\upsilon_1}\lambda_1}\frac{\partial T(0,\tau)}{\partial z}+\alpha_1 T(0,\tau)=\alpha_1\varphi_1(\tau);$$
$$\sqrt{C_{\upsilon_n}\lambda_n}\frac{\partial T(z_n,\tau)}{\partial z}+\alpha_2 T(z_n,\tau)=\alpha_2\varphi_2(\tau), \qquad (7)$$

We will find the solution to equation (5) in the form:

$$T(z,\tau)=\theta(z,\tau)+\sum_{m=1}^{\infty}\left[A_m+\int_0^{\tau}a_m(t)\cdot e^{k_m^2 t}dt\right]\Psi_m(z)e^{-k_m^2 t} \quad (8)$$

We require that the quasi-stationary component θ(z, τ) of the general solution (8) satisfy the differential equation

$$\frac{\partial}{\partial z}\left[\sqrt{C_\upsilon(z)\lambda(z)}\,\frac{\partial\theta(z,\tau)}{\partial\tau}\right]=0 \qquad (9)$$

with non-uniform boundary conditions

$$-\sqrt{C_{\upsilon_1}\lambda_1}\,\frac{\partial\theta(0,\tau)}{\partial z}+\alpha_1\theta(0,\tau)=\alpha_1\varphi_1(\tau);$$

$$\sqrt{C_{\upsilon_n}\lambda_n}\,\frac{\partial\theta(z_n,\tau)}{\partial z}+\alpha_2\theta(z_n,\tau)=\alpha_2\varphi_2(\tau), \quad (10)$$

and the eigenvalues k_m and the corresponding eigenfunctions $\Psi_m(z)$ were determined from the solution of the uniform boundary problem

$$\frac{d}{dz}\left[\sqrt{C_\upsilon(z)\lambda(z)}\,\frac{d\Psi_m(z)}{dz}\right]+k_m^2\sqrt{C_\upsilon(z)\lambda(z)}\Psi_m(z)=0, \quad (11)$$

$$-\sqrt{C_{\upsilon_1}\lambda_1}\,\frac{d\Psi_m(0)}{dz}+\alpha_1\Psi_m(0)=0;$$

$$\sqrt{C_{\upsilon_n}\lambda_n}\,\frac{d\Psi_m(z_n)}{dz}+\alpha_2\Psi_m(z_n)=0. \quad (12)$$

When, if to assume

$$\sum_{m=1}^{\infty}a_m(\tau)\Psi_m(z)=-\frac{\partial\theta(z,\tau)}{\partial\tau}+\frac{q_\upsilon(z,\tau)}{C_\upsilon(z)}, \quad (13)$$

then solution (8) will satisfy the differential equation (5) and boundary conditions (7). Substituting (8) into the initial condition (6), we obtain an expression for determining the coefficients A_m:

$$A_m=\frac{1}{N_m^2}\int_0^{z_n}[f(z)-\theta(z,0)]\sqrt{C_\upsilon(z)\lambda(z)}\Psi_m(z)dz, \quad (14)$$

where N_m^2 are the squared norm of eigenfunctions $\Psi_m(z)$,

$$N_m^2=\int_0^{z_n}\sqrt{C_\upsilon(z)\lambda(z)}[\Psi_m(z)]^2dz. \quad (15)$$

Further we find an expression for coefficients $a_m(\tau)$ from (13):

$$a_m(\tau)=\frac{1}{N_m^2}\int_0^{z_n}\left[\frac{q_\upsilon(z,\tau)}{C_\upsilon(z)}-\frac{\partial\theta(z,\tau)}{\partial\tau}\right]\sqrt{C_\upsilon(z)\lambda(z)}\Psi_m(z)dz. \quad (16)$$

By determining the quasi-stationary component of the sought temperature field (8) from the solution of (9) with the subsequent satisfaction of the non-uniform boundary conditions (10), we obtain

$$\theta(z,\tau)=\varphi_1(\tau)+\frac{\frac{1}{\alpha_1}+\frac{z}{\sqrt{C_1\lambda_1}}+\sum_{j=1}^{n-1}\left(\frac{1}{\sqrt{C_{j+1}\lambda_{j+1}}}-\frac{1}{\sqrt{C_j\lambda_j}}\right)(z-z_j)S_-(z-z_j)}{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{z}{\sqrt{C_1\lambda_1}}+\sum_{j=1}^{n-1}\left(\frac{1}{\sqrt{C_{j+1}\lambda_{j+1}}}-\frac{1}{\sqrt{C_j\lambda_j}}\right)(z_n-z_j)}[\varphi_2(\tau)-\varphi_1(\tau)] \quad (17)$$

Hereinafter, in order to simplify mathematical notation, it is assumed that $C_\upsilon(z)=C(z)$. To solve the special Sturm-Liouville problem (11), (12), we reduce the differential equation (11) to a partially degenerate form

$$\frac{d^2\Psi_m(z)}{dz^2}+k_m^2\Psi_m(z)+$$
$$+\sum_{j=1}^{n-1}\left(\sqrt{\frac{C_{j+1}\lambda_{j+1}}{C_j\lambda_j}}-1\right)\frac{d\Psi_m}{dz}\Bigg|_{z=z_j}\delta_-(z-z_j)=0, \quad (18)$$

where $\delta_-(z-z_j)=S_-(z-z_j)$ is the asymmetric impulse function [7].

The general solution to equation (18) can be represented as follows [9**Ошибка! Источник ссылки не найден.**]:

$$\Psi_m(z)=C_m\sin k_m z+D_m\cos k_m z-$$
$$-\frac{1}{k_m}\sum_{j=1}^{n-1}\left(\sqrt{\frac{C_{j+1}\lambda_{j+1}}{C_j\lambda_j}}-1\right)\frac{d\Psi_m}{dz}\Bigg|_{z=z_j}\sin k_m(z-z_j)\cdot S_-(z-z_j), \quad (19)$$

where C_m and D_m are the integration constants.

By solving successively the system of algebraic equations, we find the unknown quantities and obtain a solution to equation (11) in a closed form:

$$\Psi_m(z)=C_m\left[\sin k_m z-\sum_{j=1}^{n-1}\left(1-\sqrt{\frac{C_j\lambda_j}{C_{j+1}\lambda_{j+1}}}\right)\cdot R_j\sin k_m(z-z_j)\cdot S_-(z-z_j)\right]+$$
$$+D_m\left[\cos k_m z-\sum_{j=1}^{n-1}\left(1-\sqrt{\frac{C_j\lambda_j}{C_{j+1}\lambda_{j+1}}}\right)\cdot Q_j\sin k_m(z-z_j)\cdot S_-(z-z_j)\right] \quad (20)$$

Satisfying the general solution (20) to the boundary conditions (12), we find, up to an arbitrary constant, the form of the eigenfunctions

$$\Psi_m(z)=\sin k_m z+\frac{k_m\sqrt{C_1\lambda_1}}{\alpha_1}\cos k_m z+$$
$$+\sum_{j=1}^{n-1}\left(1-\sqrt{\frac{C_j\lambda_j}{C_{j+1}\lambda_{j+1}}}\right)\left(\frac{k_m\sqrt{C_1\lambda_1}}{\alpha_1}Q_j-R_j\right)\sin k_m(z-z_j)S_-(z-z_j) \quad (21)$$

and the characteristic equation for determining the eigenvalues k_m

$$\frac{\alpha_2 F(k_m) + k_m\sqrt{C_n\lambda_n}\,E(k_m)}{k_m\sqrt{C_n\lambda_n}\,L(k_m) - \alpha_2 M(k_m)} = \frac{k_m\sqrt{C_1\lambda_1}}{\alpha_1} \quad (22)$$

Now we calculate the coefficients A_m and $a_m(\tau)$ according to expressions (14) - (16) taking into account the eigenfunctions (21). Further we substitute them and expression (17) into the sought solution (8), and this completes the construction of a general solution to the problem posed.

Implementation of the mathematical model

Consider a rubber-to-metal plate. The initial temperature distribution over the wall thickness is constant and equal to 428 K. The heat transfer coefficients at the plate boundaries are α1=α2=200. The ambient temperature is 283 K. Figure 1 shows a graph of temperature distribution over the thickness of a rubber-metal sample (a layer of steel is covered by a layer of ebonite). The dashed lines show the interfaces between the layers. The temperature distribution was observed at different points in time. The numbers on the curves correspond to the following points in time: 1 - after 30 s, 2 - after 60 s, 3 - after 120 s, 4 - after 160 s, 5 - after 180 s, 6 - after 240 s, 7 - after 360 s, 8 - after 500 s, 9 - after 700 s, 10 - after 1530 s, 11 - after 1600 s, 12 - after 1650 s. Points correspond to the temperatures obtained experimentally. The solid line is the result of analytical solution.

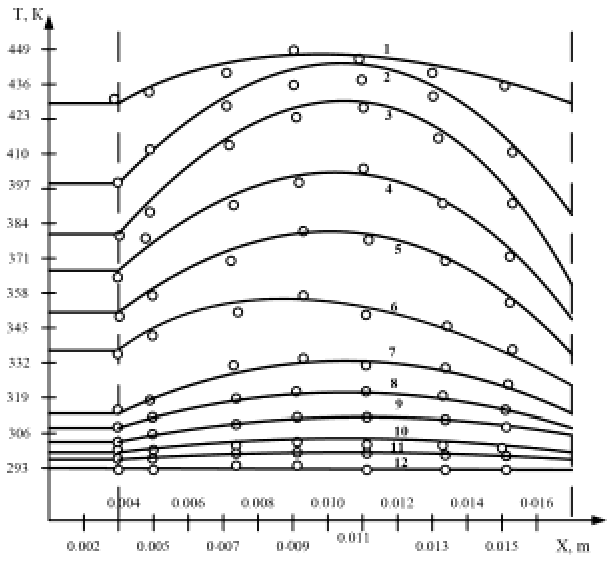

Fig. 1. Graph of temperature distribution over the thickness of a rubber-metal sample.

As one can see, the convergence of the results is quite high. With a decrease in cooling time, the discrepancy between the calculated and experimental data does not exceed 1 - 2%, gradually decreasing and tending to zero.

4 Conclusions

The development and implementation of calculation methods for determining the thermal modes of vulcanization will intensify and optimize the process while maintaining the high quality of rubberized products.

The developed calculation methods and the presented design solutions make it possible to create new and improve the existing industrial installations, which would reduce the duration of the cooling process without compromising the quality of the rubber-metal product. The practical value of the results obtained is the development and implementation of an engineering technique for calculating the cooling of coatings of gummed objects, designed to intensify the heat treatment process, improve the quality of rubber-metal products and the performance of gumming equipment in anticorrosive shops.

The results of the study were tested at the industrial enterprises of OJSC Severstal (Cherepovets), OJSC Ammofos (Cherepovets), OJSC Agrokhim (Sokol) during production of gummed objects and became the basis of their technological process.

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