

# Numerical calculation and experimental study on friction and lubrication of Stirling engine piston ring

Li Songjian<sup>1</sup>, Zheng Wei<sup>1</sup>, Xu Wei<sup>1</sup>, Ling Hong<sup>1</sup>

<sup>1</sup>Shanghai Micropowers Ltd, Shanghai 201203, China

**Abstract:** To solve the practical problem of Stirling engine piston ring, numerical model on friction and lubrication were established based on average Reynolds equation theory. Numerical calculation results show: piston rings are in the mixed lubrication state which gas lubrication film and micro-convex body contact existing simultaneously; contact pressure is 2 orders of magnitude less than gas film pressure; contact pressure is approximately linearly related to mean gas pressure. Simulated test rig was built and experimental results show: high-pressure working gas leak to intermediate chamber through piston rings, and the pressure of intermediate chamber drops sharply at lowest pressure of circulation; piston ring wear increases when rotational speed or working pressure increase. The accuracy of numerical model was proved by experimental phenomena and test data.

**Key words:** polymer piston ring, numerical calculation, contact model

## 1 Introduction

The Stirling engine is a piston engine with external heating. It uses gas as a working medium and works in a closed regenerative cycle. The Stirling engine is mainly composed of an external combustion system, a working fluid circulation system and a transmission system. Among them, the piston ring dynamic seal in the working fluid circulation system is one of the key technologies that affect the performance and reliability of the Stirling engine [1].

The structure and simplified force analysis of Stirling engine piston ring is shown in Figure 1. Stirling engine piston ring relies on the pressure of the working gas to produce a sealing effect, which is used to isolate the high-pressure gas from different Stirling cycles. It operates under high-speed and high-load working conditions. At the same time, in order to prevent lubricating oil from polluting the working fluid circulation system, the piston ring adopts an oil-free lubrication design and uses polymer materials to achieve efficient and reliable operation.

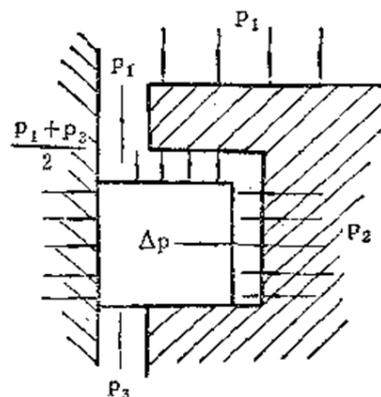


Fig.1 Structure and simplified force analysis of Stirling engine piston ring

The linear simplified force analysis of Stirling engine piston ring is as follows:  $P_1$  represents the pressure in the working chamber,  $P_2$  represents the pressure acting on the inner surface of the piston ring;  $P_2$  is approximately equal to  $P_1$ . And  $P_3$  represents the pressure in the intermediate chamber which is the lowest pressure of the Stirling cycle. The gas film pressure between piston ring and cylinder liner is linearly simplified as  $(P_1+P_3)/2$ , so the average contact pressure of the piston ring  $\Delta P$  is:

$$\Delta P = P_2 - \frac{P_1 + P_3}{2} \approx P_1 - \frac{P_1 + P_3}{2} = \frac{P_1 - P_3}{2} \quad (1)$$

Stirling engine piston ring of polymer material slides against cylinder liner of metal material, and its main wear mechanism is adhesive wear. According to the Holm-Achard adhesive wear formula<sup>[2]</sup>, Equation 2 can be derived as follow.

$$h = k \cdot P_N \cdot v \cdot t \quad (2)$$

In Equation 2,  $h$  ----the radial wear of the piston ring,  $k$ ---- wear rate,  $P_N$ ----the average contact pressure,  $v$  ----the movement speed of the piston ring,  $t$  ----wear time. According to Equation 1 and Equation 2, the wear rate of Stirling engine piston ring can be calculated.

Wear rates were calculated using the actual wear of the piston ring, average contact pressure and wear time. However, all wear rates calculated are less than  $5 \times 10^{-8} \text{mm}^3/(\text{N} \cdot \text{m})$ , which is much lower than the dry friction wear rate of conventional PTFE materials ( $10^{-6} \sim 10^{-5} \text{mm}^3/(\text{N} \cdot \text{m})$ )<sup>[3]</sup>. The product piston ring and cylinder liner samples were entrusted to a third party for dry friction testing. Under the working conditions of temperature 150°C, load 1MPa, and rotation speed 600rpm, the test coefficient of wear was  $1.2 \times 10^{-5} \text{mm}^3/(\text{N} \cdot \text{m})$ . It shows that the piston ring material used in the product does not have outstanding wear resistance characteristics. Therefore, it is inferred that the piston ring-cylinder liner of the Stirling engine is a special form of friction, and it is necessary to carry out detailed research on friction and lubrication of Stirling engine piston ring.

## 2 Numerical calculation method

Since the structure and motion characteristics of Stirling engine piston ring are similar to internal combustion engine, mature numerical calculation method of internal combustion engine is used for reference. The core calculation framework is built on the solution of average Reynolds equation. Aiming at the special characteristics of Stirling engine piston ring, such as working gas as lubricating medium, contact between polymer and metal, etc., this paper adopts innovative methods to solve related problems.

### 2.1 Average Reynolds equation

When the fluid film thickness reaches an order of magnitude close to that of the solid surface roughness, the influence of surface roughness on the lubrication performance needs to be considered. Patir and Cheng introduced the concepts of pressure flow factor and shear flow factor, and derived the average Reynolds equation to calculate the roughness Surface fluid lubrication<sup>[4]</sup>. The lubricating medium between Stirling engine piston ring and cylinder liner is gas, and the thickness of the gas film reaches the order of magnitude of the surface roughness, so the average Reynolds equation theory was adopted. In addition, the axial height of piston ring is much smaller than the radius of cylinder liner, which can be simplified to a one-dimensional problem according to Haddad's research results<sup>[5]</sup>. The

average Reynolds equation of the Stirling engine piston ring can be expressed as:

$$\frac{\partial}{\partial x} \left( \phi_x \frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) = 6U\rho\phi_c \frac{\partial h}{\partial x} + 6U\sigma \frac{\partial(\rho\phi_s)}{\partial x} + 12\rho\phi_c \frac{\partial h}{\partial t} \quad (3)$$

In equation 3,  $p$ -----gas film pressure;  $h$ -----nominal gas film thickness;  $\sigma$  -----comprehensive surface roughness;  $U$ -----velocity of piston ring;  $\eta$  -----lubricating hydrodynamic viscosity;  $\phi_x$  -----pressure flow factor;  $\phi_s$  -----shear flow factor;  $\phi_c$  -----contact factor;  $t$  -----time .

$\phi_x$  express the ratio of lubricating fluid flow through rough surface to smooth surface. The value is related to the film thickness ratio and aspect ratio of surface asperities. When the surface direction parameter is 1,  $\phi_x$  can be calculated by equation 4:

$$\phi_x = 1.0 - 0.9 \exp(-0.56H) \quad (4)$$

In equation 4:  $H$  ----- film thickness ratio,  $H = h / \sigma$  .

$\phi_s$  express the influence of flow change on lubrication when two rough surfaces move relatively. It is a function of film thickness ratio, surface direction parameter and surface roughness. When surface direction parameters of the friction pair are both 1,  $\phi_s$  can be calculated by equation 5~ 7 :

$$\phi_s = v_{r1}\Phi_s(H) - v_{r2}\Phi_s(H) \quad (5)$$

$$\Phi_s(H) = \begin{cases} 1.899H^{0.98} \exp(-0.92H + 0.05H^2) & H \leq 5 \\ 1.126 \exp(-0.25H) & H > 5 \end{cases} \quad (6)$$

$$v_{r1} = \left( \frac{\sigma_1}{\sigma} \right)^2, \quad v_{r2} = \left( \frac{\sigma_2}{\sigma} \right)^2 = 1 - v_{r1} \quad (7)$$

Where:  $\sigma_1$  ----- roughness of the piston ring;  $\sigma_2$  ----- roughness of the cylinder liner;  $\sigma$  ----- comprehensive surface roughness,  $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$  .

$\phi_c$  is related to film thickness ratio and asperity distribution of contact surface. When surface asperity distribution conforms to the normal distribution,  $\phi_c$  can be calculated by equation 8

$$\phi_c = \begin{cases} \exp(-0.6912 + 0.782H - 0.304H^2 + 0.0401H^3) & 0 \leq H \leq 3 \\ 1 & H > 3 \end{cases} \quad (8)$$

The lubricating medium of Stirling engine piston ring is gas, which is compressible, and its density, temperature and pressure have a strong coupling relationship. The test results show that the actual piston ring working temperature is about 60°C~90°C, so it can be considered that the piston ring works under an isothermal condition of 80°C. According to working fluid gas (helium) thermal

parameter table, a polynomial can be used to accurately fit the helium density under different pressure conditions. Moreover, the difference of helium viscosity is extremely small in working pressure range, so it can be assumed that viscosity of helium was not changed. In summary, the density and viscosity of working gas were calculated using working pressure, and then substituted into the average Reynolds equation.

## 2.2 Average Reynolds equation solution

Due to simple geometric structure, calculation accuracy of the difference method is close to the finite element method. The difference method is simple-programming and fast-calculating. Therefore, the difference method was used to solve average Reynolds equation in this paper. Average Reynolds equation is second-order partial differential equation that takes the derivative of time and space. When using the difference method to solve, center difference format was used in space direction, and backward difference format was used in time direction. Equation 3 was discretized as equation 9.

$$\begin{aligned} & \left[ \frac{(\phi_x \rho h^3 / \eta)_{i+1} - (\phi_x \rho h^3 / \eta)_{i-1}}{2\Delta x} \right] \left( \frac{\bar{p}_{i+1} - \bar{p}_{i-1}}{2\Delta x} \right) + \left( \frac{\phi_x \rho h^3}{\eta} \right)_i \left( \frac{\bar{p}_{i+1} - 2\bar{p}_i + \bar{p}_{i-1}}{\Delta x^2} \right) \\ & = 6U(\rho\phi_c)_i \frac{h_{i+1} - h_{i-1}}{2\Delta x} + 6U\sigma \frac{(\rho\phi_s)_{i+1} - (\rho\phi_s)_{i-1}}{2\Delta x} + 12(\rho\phi_c)_i \frac{h_i^j - h_i^{j-1}}{\Delta t} \end{aligned} \quad (9)$$

After sorting, average Reynolds equation was transformed into a set of nonlinear equations, and ultra-relaxation iterative method was used to calculate the minimum gas film thickness and gas film pressure distribution.

Reynolds boundary condition was used to ensure that the lubricating fluid flow was continuous. The inlet boundary pressure was circulating pressure of the working gas and the outlet boundary pressure was intermediate cavity pressure. The gas film geometric model was set to a wedge shape. Since the outer processing surface of the piston ring is a straight line, the ring side surface deforms due to pressure distribution. Therefore, gas film geometric model was a wedge shape with small height difference, specific calculating equation as follow.

$$h = h_{\min} + \frac{h_{\text{diff}}}{Th} x \quad (10)$$

In equation 10,  $h_{\min}$  -----minimum gas film thickness between piston ring and cylinder liner;  
 $h_{\text{diff}}$ -----the wedge height difference of the gas film due to deformation ;  $Th$ -----thickness of piston ring.

## 2.3 Balance condition

Radial force balance of piston ring was set as balance condition of the numerical calculation, and the radial force analysis of the Stirling engine piston ring is shown in Figure 2.

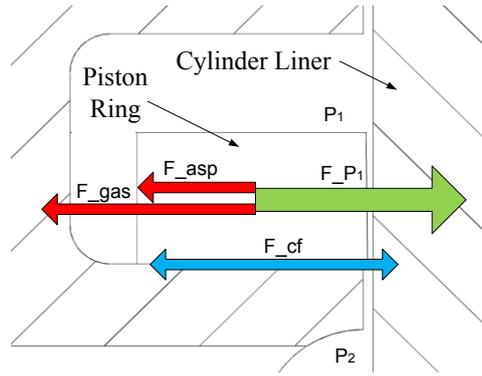


Fig.2 Radial force analysis of piston ring

The force balance equation of the piston ring per unit circumference is:

$$F_{asp} + F_{gas} + F_{cf} = F_{P1} \quad (11)$$

In equation 11,  $F_{P1}$  -----gas force on the back side of piston ring, which was calculated by circulating pressure  $P_1$ ;  $F_{gas}$  -----gas film bearing force, calculated using the integral of gas film pressure in space direction;  $F_{cf}$ -----friction force generated by the contact, calculated by the product of positive axial pressure and friction coefficient;  $F_{asp}$ -----contact pressure, calculated by the established rough surface contact model.

The contact of Stirling engine piston ring is between polymer and metal. On one hand, the actual contact area of micro convex is much larger than two metal surfaces. On the other hand, the adhesion of the polymer material significantly affects the contact force. Based on the Greenwood-Williamson rough surface contact model, this paper established a Stirling engine piston ring contact model through contact surface observation, actual contact area correction, adhesion correction and test data comparison. The G-W model assumes that all asperities have the same radius of curvature and the peak heights are randomly distributed around the mean, and then the following equations were derived to calculate contact pressure of the asperities.

$$p_{asp} = \frac{16\sqrt{2}}{15} \pi (\eta_1 \beta_1 \sigma)^2 E' \sqrt{\frac{\sigma}{\beta_1}} F_{\frac{3}{2}}(H) \quad (12)$$

$$A_c = \pi^2 (\eta_1 \beta_1 \sigma)^2 A F_2(H) \quad (13)$$

In equation 12 and 13,  $p_{asp}$  ----- Hertz contact pressure of actual contact per unit length, calculated based on the Hertz contact theory of elastomer and rigid body;  $\eta_1$  ----- number of asperities per unit area;  $\beta_1$  ----- radius of curvature of the asperities;  $A_c$  -----actual contact area;  $A$  -----nominal contact Area;  $E'$  ----- comprehensive elastic modulus of piston ring and cylinder liner;  $F_{\frac{3}{2}}(H)$  and  $F_2(H)$  -----correction term obtained by integrating the Gaussian distribution function [4].

Internal combustion engine piston rings contact calculation parameters are often chosen by experience, but lack of experience on Stirling engine piston rings. In this paper, the surface morphology of the piston ring (as shown in Figure 3) was observed through microscope, and the actual observation statistics were used as calculation parameters of the asperity.

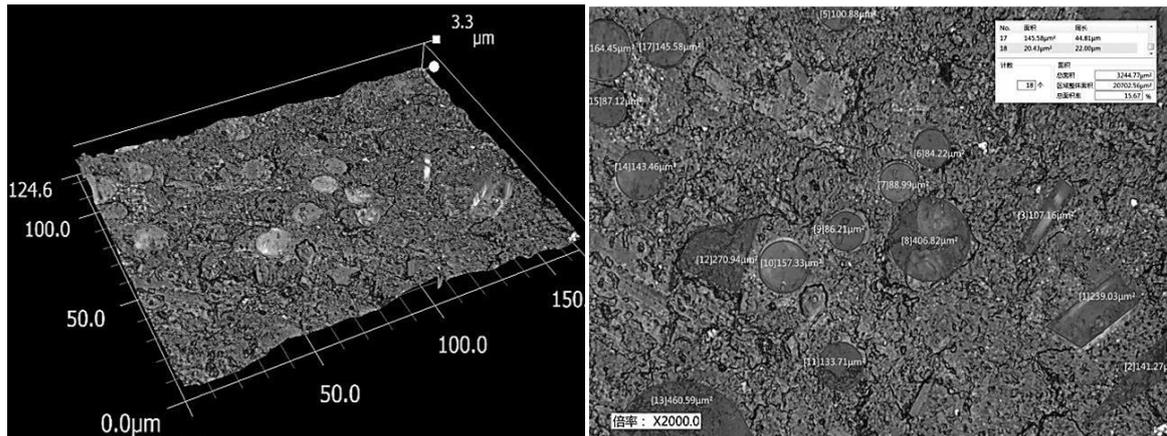


Fig.3 Surface morphology and test data of piston ring

For the problem that actual contact area is much larger than contact area between metal surfaces, the contact area was enlarged by setting correction factor. This article assumes that when the minimum gas film thickness is less than or equal to  $0.5\mu\text{m}$ , the actual contact area is equal to the nominal contact area. As the thickness increases, the actual contact area gradually decreases.

Contact adhesion problem was solved based on JKR adhesion theory, The asperities contact between piston ring and cylinder liner can be approximated as contact between an elastic sphere and a rigid plane. Adhesive contact force between single elastic sphere and rigid plane can be calculated according to Equation 14<sup>[2]</sup>.

$$F_A = -\frac{3}{2}\gamma_{12}\pi R \quad (14)$$

In equation 14,  $F_A$ —adhesive contact force;  $R$ —radius of the elastic sphere;  $\gamma_{12}$ —relative surface energy. After calculating the adhesive contact force of single asperity, adhesive contact force of entire piston ring surface was calculated using the actual contact number of asperities .

Based on aforementioned theory, numerical calculation programs were compiled to realize the transient simulation of Stirling engine piston ring’s lubrication and contact.

### 3 Calculating results analysis

Design parameters, material properties and gas pressure curves as input, transient lubrication results of Stirling engine piston ring were calculated using the established numerical model. Through analyzing calculating results, friction and lubrication mechanism of the Stirling engine piston ring was researched comprehensively.

Calculating results of minimum gas film thickness and contact pressure were shown in Figure 4, which shows that: 1) When gas pressure close to minimum pressure of Stirling cycle, minimum gas film thickness increases significantly. Resulting from enlarged leakage channel, pressure of intermediate cavity approach to working gas cavity; 2) Lubrication state often distinguished by film thickness ratio according to classical fluid lubrication theory. When film thickness ratio greater than 2~3, lubrication state is considered as full-film hydrodynamic lubrication. Otherwise lubrication state is considered as mixed state is where lubricating film and body contact existing at the same time<sup>[6]</sup>. Film thickness ratio of Stirling engine piston ring ranges from 0.2 to 1.6, indicating that it keeps on mixed state; 3) Contact pressure is much lower than gas film pressure, indicating gas lubrication plays

main load-bearing effect, and the linear estimation method will seriously overestimate contact pressure;  
4) Calculated average contacting Pv value was 0.78MPa•m/s, and the contacting Pv value estimated by actual piston ring wear was about 0.3MPa•m/s, which indicate that calculated results was accurate.in order of magnitude.

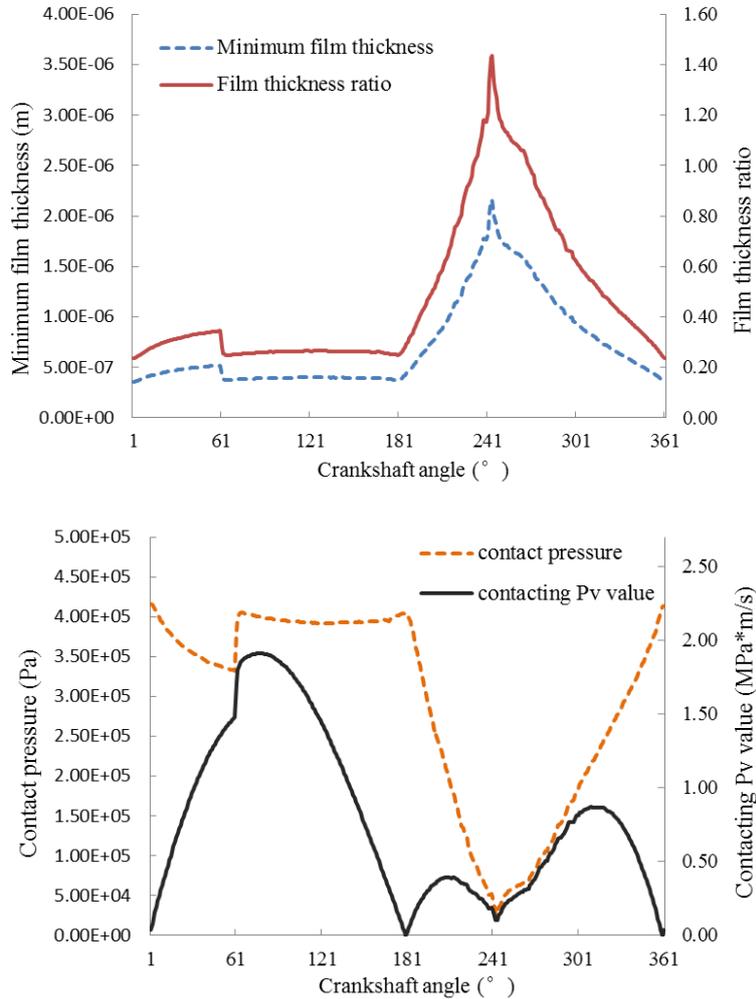


Fig.4 Calculating results of minimum film thickness (up) and contact pressure (down)

Relation curve between contact pressure and mean gas pressure was shown in Figure 5, which shows: Minimum film thickness and mean gas pressure approximately were linearly negative correlated, and contact pressure and mean gas pressure were linearly positive correlated, indicating that when mean gas pressure increases, gas film becomes thinner and contact pressure increases.

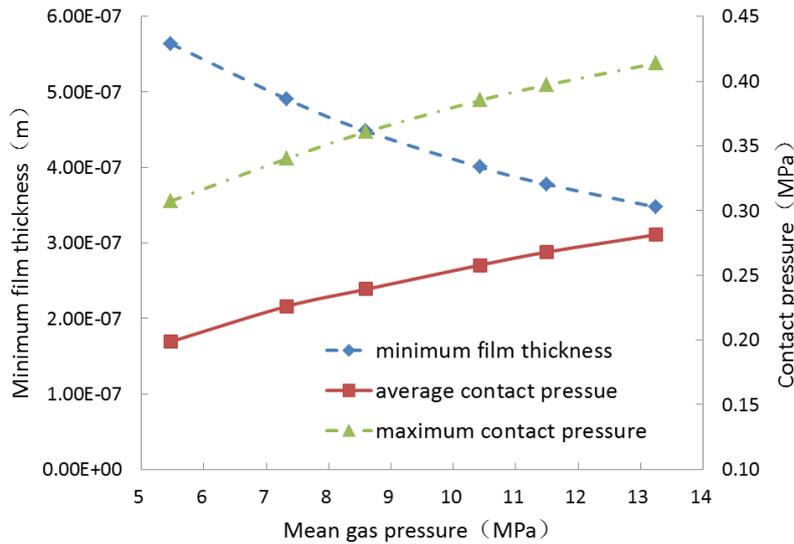


Fig.5 Relation curve between contact pressure and mean gas pressure

#### 4 Test research

According to principle of Stirling engine piston ring, a simulation test bench was established. The basic structure of the test bench was shown in Figure 6 (left). Actual piston rings can be experimented using the test bench, and working conditions as follow: working gas was helium, test speed was 330r/min, and stroke was 50mm.

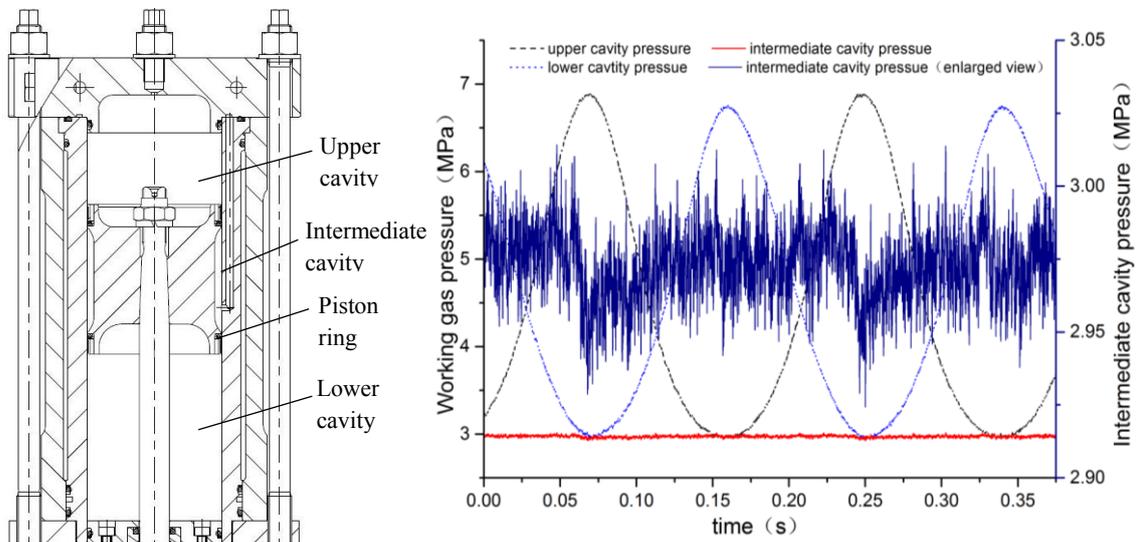


Fig.6 Simulated test rig structure (left) and cylinder pressure test curve (right)

To explore seal mechanism of piston ring, pressure data of three cavities was tested and shown in Figure 6 (right). Pressure test curve shows that intermediate cavity pressure was basically maintained at minimum pressure during operation; further enlarging intermediate cavity pressure, it was found that when approaching to minimum pressure, intermediate cavity pressure suddenly drops, which means piston ring leakage channel expanded at this moment; as lower cavity pressure rises, intermediate cavity pressure continues to improve slowly, which shows higher-pressure gas in compression cavities leaks to intermediate cavity continuously. This phenomenon accords with calculated gas film thickness curve, indicating that established simulation model can accurately explain seal mechanism of Stirling

engine piston ring.

Wear tests on different condition were carried out using the test bench. After each test, piston rings were destroyed to measure wear amount. Test condition and measurement results were shown in Table 1, which shows that: 1) as speed or pressure increases, wear of piston ring gradually increases; 2) some wear of piston rings were irregular or contrary to basic law; 3) test parameters as input, wear rates of piston rings were calculated using established simulation model. It was found that wear rates of different conditions were close to wear rate of conventional filled PTFE material, which verifies established calculating model is accurate.

**Table1 Wear test condition and results**

Test condition (minimum pressure)	Test time h		Wear mm	Wear rate $\times 10^{-7} \text{mm}^3 / (\text{N}\cdot\text{m})$
345r/min 2MPa	101.5	Upper ring	0.028	5.99
		Lower ring	0.020	5.30
345r/min 3.3MPa	102.5	Upper ring	0.054	9.22
		Lower ring	0.026	5.41
900r/min 2.2MPa	103	Upper ring	0.097	10.40
		Lower ring	0.087	9.23
900r/min 3.3MPa	104	Upper ring	0.075	6.29
		Lower ring	0.113	9.47

## 5 Conclusions

- (1) Based on average Reynolds equation theory, this paper established a calculating model to research seal and lubrication mechanism of Stirling engine piston ring.
- (2) Calculating results show: Stirling engine piston rings maintain in mixed lubrication state where lubricating film and body contact existing at the same time. Contact pressure is much lower than gas film pressure. When mean gas pressure increases, gas film becomes thinner and contact pressure increases.
- (3) Pressure test curve shows that higher-pressure gas in compression cavities leaks to intermediate cavity continuously, and when approaching to minimum pressure, intermediate cavity pressure drops suddenly. This phenomenon accords with calculated gas film thickness curve, indicating that established simulation model can accurately explain seal mechanism of Stirling engine piston ring
- (4) Wear tests on different condition show that as speed or pressure increases, wear of piston ring gradually increases. Wear rates calculated using established model were close to wear rate of conventional filled PTFE material, which verifies established calculating model is accurate.

## Reference:

- [1]Jin Donghan. Stirling engine technology[M], Harbin: Harbin Institute of Technology Press, 2009.
- [2]Valentin L. Popov. Principles and Applications of Contact Mechanics and Tribology [M], Beijing: Tsinghua University Press, 2011.
- [3]Pan Bingli. Tribology of Advanced Polymer Materials[M], Beijing: National Defense Industry Press, 2016.
- [4] Patir N, Cheng H S. Application of average flow model to lubrication between rough sliding

- surfaces. *Journal of Lubrication Technology*. 1979, 101: 220-230P.
- [5] Haddad S D, Tian K T. An analytical study of offset piston and crankshaft designs and the effect of oil film on piston slap excitation in a diesel engine. *Mechanism and Machine Theory*. 1995, 30: 271-284.
- [6] Wen Shizhu, Huang Ping. *Mechanism of Tribology[M]*, Beijing: Tsinghua University Press, 2012.