terms like heat transfer losses etc. The simple analysis is an enhancement of adiabatic analysis. The solution procedure for simple analysis is same as for adiabatic analysis and spaces of expansion and compression are considered as adiabatic. In a simple analysis, losses (heat transfer losses, load losses, imperfect regeneration losses) terms are included along with the adiabatic model. By adding these losses, indicated cycle work is reduced from ideal cycle work. In this way, non-ideal analysis is being developed in simple analysis and that predicts the real performance of the Stirling engine. These three methods are developed in MATLAB and the RK-4 method is employed.
Mass of gas within the Stirling engine is a significant parameter for adiabatic analysis. Therefore, Schmidt's analysis is used to calculate the mass of gas inside the Stirling Engine. It required mean pressure inside the Stirling engine as input MATLAB program. The Schmidt analysis required seven, hot and cold gas wall temperature, the phase angle of displacement piston overpower piston, clearance volume, expansion, and compression swept volume, and mean pressure [6].

To get a set of the equation for this engine, the engine is divided into six compartments, expansion chamber, heater tubes, regenerator, cooler, cooler connecting pipes, compression chamber. Heat transfer in heat exchanger is unsteady and temperature in cooler and heater varies concerning crank angle. For formulating a set differential equation for governing equations, the derivation of each parameter with respect to crank angle (θ) is denoted by d and defined as d=d/dθ hereunder. By applying the law of conservation of energy and equation of state on each cell.

$$dQ + c_p\left(T_i m_i - T_0 m_0\right) = dW + dE_{loss} + c_v d\left(mT\right) \tag{7}$$

Whereas, dE_{loss} is a loss term that included all losses occurring in the Stirling engine. The losses occurring in different volume of cell in Stirling engine is given below. In adiabatic analysis, the total mass of gas in the engine is constant.

$$M = m_c + m_{k1} + m_{k2} + m_r + m_h + m_e \tag{8}$$

The derivative of the mass equation is given by

$$dm_c + dm_{k1} + dm_{k2} + dm_r + dm_h + dm_e = 0 \tag{9}$$

Using the equation of state, the mass in each cell of the equation is given by

$$m_i = \frac{P_i V_i}{RT_i} \tag{10}$$

where i= k1,k2, r, h (cooler tube, cooler fin, regenerator, and heater). The gas pressure equation is given by

$$P = \frac{mR}{\dfrac{V_c}{T_c} + \dfrac{V_{k1}}{T_k} + \dfrac{V_{k2}}{T_k} + \dfrac{V_r}{T_r} + \dfrac{V_h}{T_h} + \dfrac{V_e}{T_e}} \tag{11}$$

Temperature and volume of a heater, cooler tube, cooler fin, the regenerator is constant, so differential form of the equation of state is

$$\frac{dm}{m}=\frac{dP}{P} \tag{12}$$

$$dm=\frac{dP}{R}\frac{V_i}{T_i} \tag{13}$$

where i= k, r, h (cooler fin, cooler tube, regenerator, and heater)

The mass accumulation in the compression and expansion chamber is

$$dm_c=\frac{PdV_c+\frac{V_c dP}{\gamma}}{RT_{ck1}} \tag{14}$$

$$dm_e=\frac{PdV_e+\frac{V_e dP}{\gamma}}{RT_{he}} \tag{15}$$

The mass flow rate between two continuous cells is

$$m_{ck1}=-dm_c \tag{16}$$

$$m_{k1k2}=m_{ck1}-dm_{k1} \tag{17}$$

$$m_{k2r}=m_{k1k2}-dm_{k2} \tag{18}$$

$$m_{rh}=m_{k2r}-dm_r \tag{19}$$

$$m_{he}=m_{rh}-dm_h \tag{20}$$

The pressure variation in the equation is given by

$$dP=\frac{\gamma P\left(\frac{dV_c}{T_{ck1}}+\frac{dV_e}{T_{he}}\right)}{\frac{V_c}{T_{ck1}}+\gamma\left(\frac{V_{k1}}{T_k}+\frac{V_{k2}}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}\right)+\frac{V_e}{T_{he}}} \tag{21}$$

The temperature Tck and The are conditional temperatures in direction of flow given
If mass $m_{he}>0$ then $T_{he}=T_h$ else if $T_{he}=T_e$
If mass $m_{ck}>0$ then $T_{ck}=T_c$ else if $T_{ck}=T_k$
The temperature in compression and expansion space in differential form is

$$dT_e=T_e\left(\frac{dP}{P}+\frac{dV_e}{V_e}-\frac{dm_e}{m_e}\right) \tag{22}$$

$$dT_c=T_c\left(\frac{dP}{P}+\frac{dV_c}{V_c}-\frac{dm_c}{m_c}\right) \tag{23}$$

The temperature of the regenerator is determining by

$$T_r=\frac{T_h-T_k}{\ln\left(\frac{T_h}{T_k}\right)} \tag{24}$$

The heat transfers in a heater, regenerator, and cooler (combine as the cooler and cooler fin) are by

$$dQ_h = \frac{V_h dPc_v}{R} - c_p\left(T_{rh}m_{rh} - T_{he}m_{he}\right) \tag{25}$$

$$dQ_r = \frac{V_r dPc_v}{R} - c_p\left(T_{kr}m_{kr} - T_{rh}m_{rh}\right) \tag{26}$$

$$dQ_k = \frac{\left(V_{k1} + V_{k2}\right) dPc_v}{R} - c_p\left(T_{ck1}m_{ck1} - T_k m_{k2r}\right) \tag{27}$$

The work done in compression and expansion space is

$$dW_e = PdV_e \tag{28}$$

$$dW_c = PdV_c \tag{29}$$

The total work done is presented

$$dW = dW_e + dW_c \tag{30}$$

The efficiency is outlined as

$$\eta = \frac{W}{Q_h} \tag{31}$$

3.3 Non-ideal analysis

The regenerator stores energy from the gas oscillating in the Stirling engine. Different types of materials are used in the regenerator to store energy. Due to this advantage, it greatly improves the performance of the Stirling engine. The thermal efficiency for the non-ideal model is

$$\eta_i = \frac{W'}{Q_K} = \frac{Q'_H + Q'_K}{Q'_H} \tag{32}$$

The heat exchange in the heater and cooler for the non-ideal case is given by

$$Q_H = Q'_H + Q'^{(1-\varepsilon)}_R \tag{33}$$

$$Q_K = Q'_K - Q'^{(1-\varepsilon)}_R \tag{34}$$

Substituting the value of Q_h, Q_k, and ideal efficiency

$$\eta = \frac{\eta_i}{\left[1 + \left(\frac{Q'_R}{Q'_H}\right)^{(1-\varepsilon)}\right]} \tag{35}$$

In a simple analysis, the number of heat transfer units are given as

$$NTU = N_{ST}\frac{A_{WG}}{2A} \tag{36}$$

The Stanton number is obtained by

$$N_{ST} = h/\rho u c_p \tag{37}$$

The regenerator effectiveness is obtained as

$$\varepsilon_{reg} = \frac{NTU}{\left(1 + NTU\right)} \tag{38}$$

NTU is also used to estimate the effectiveness of heater and cooler

$$\varepsilon_{he} = 1 - e^{NTU} \tag{39}$$

It is also important to mention here that the wall temperature of the cooler and heater are lower than the respective gas temperature inside the Stirling engine, which reduces the performance of the engine. Using the convective heat transfer equation, it is easy to determine inside the gas temperature of the heater and cooler. For this purpose, the value of Q_k and Q_h are found from the adiabatic cycle. The basic heat transfer equation in adiabatic is

$$Q = hA_{wg}\left(T_w - T\right) \tag{40}$$

By using the above equation, the individual equation for cooler and heater temperature can be written as

$$T_k = T_{wk1} - fQ_k \Big/ \left(h_k A_{wgk1}\right) \tag{41}$$

$$T_h = T_{wh} - fQ_h \Big/ \left(h_h A_{wgh}\right) \tag{42}$$

In a practical Stirling engine, fluid friction is also known as load losses, an important factor in calculating the actual performance of the Stirling engine. Due to fluid motion pressure drops around the heat exchanger resulting in reduces the output power and efficiency of the Stirling engine. It is also important to note here that due to the oscillating flow of nature in the Stirling engine. The pressure drop across three heat exchangers is calculated by the following equation.

$$W = \oint p\left(dV_E + dV_C\right) - \oint \sum \Delta p dV_E = W_i + \Delta W \tag{43}$$

$$\Delta W = \int_0^{2\pi} \left(\sum_{i=1}^{4} \Delta p_i \frac{dV_E}{d\theta} \right) d\theta \tag{44}$$

The pressure drops, friction factor, and Reynold number is given by

$$\Delta p = \frac{-2C_f \,\mathrm{Re}\, \mu V}{d_h^2 A} \tag{45}$$

$$C_f = \frac{\tau}{0.5 \rho \mu^2} \tag{46}$$

$$\mathrm{Re} = \frac{\rho u d_h}{\mu} \tag{47}$$

It was first time describe by Petrescu et al. [20] that pressure on the surface of a piston will differ from pressure inside a closed system of a Stirling engine. The pressure during the compression process on the piston surface will greater than the pressure inside the engine and similarly, during the expansion, it will be lower. This study leads to fact that an increase of expansion and decrease of compression work reduce the output power of the Stirling engine. So the pressure correction must be concluded. The pressure loss due to finite speed thermodynamics and mechanical friction is calculated as follows.

$$P_{cylinder\_corrected} = P_{cylinder}\left[1 \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}}\right] \tag{48}$$

$$P_{finite\_speed} = \pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \tag{49}$$

$$a = \sqrt{3\gamma} \tag{50}$$

$$c = \sqrt{3RT} \tag{51}$$

$$\Delta P_f = 0.97 + 0.15 \frac{N}{1000} \tag{52}$$

$$W_{loss\_finite\_speed} = \int \left(\pm \frac{au_p}{c} \pm \frac{\Delta P_f}{P_{cylinder}} \right) dV \qquad (53)$$

In the above equations, delta P_f is piston-cylinder friction loss. (+) a sign denotes the compression space, (-) denotes expansion space and N is the speed of the engine. These are set of differential equations with 16 containing derivative and 24 variables in which 7 derivatives are solved by using the Rk-4 order method

4 Results and discussions

4.1 Model Validation

Figure 3 depicts indicated PV diagram related to experimental, ideal, adiabatic, and simple analysis. It can be seen that the simulated PV diagram closely fits with the experimental PV diagram in the figure. It is important to mention here that the simple analysis of the Stirling engine is obtained by excluding losses of Stirling (i.e., regenerator wall heat leakage losses, heat transfer losses, pressure drop losses, and piston finite speed losses) engine which is also known here as non-ideal simulated curve. The evaluated mean pressure of the non-ideal simulated curve is 3.59 bar pressure while the experimental mean pressure is 3.58 bar pressure. However, maximum pressure is the difference for the experimental and non-ideal curves. This is due to consideration of non-ideal heat exchanger causes large change non-ideal curve with an experimental curve. Reader et al. (1983) [22] mentioned that the phenomenon is caused by the fact that cylindrical walls of heat exchangers do not provide sufficiently high conduction heat transfer medium to provide a constant gas temperature within the cylinders, generating substantial curve deviations. This phenomenon is clear from figure 4 which shows Texp and Tcom temperature fluctuation in expansion and compression chamber. Figure 4 also shows the temperature gradient of wall and gas temperature within the heater and cooler heat exchangers. The T_h and T_k are the gas temperatures within heater and cooler respectively which is much higher than the user-defined wall temperature of the heater and cooler, respectively. The wall temperature of the heater and cooler are defined as 424 K and 294 K respectively while the gas temperature is found as 389.6 K and 308.6 K, respectively. There is a big difference between the wall and gas temperature which means heat exchanger walls are not providing enough conduction for constant gas temperature. This proves the fact mentioned by Reader et al. (1983) [22] in his research work. The indicated PV diagram of the experimental curve is obtained from actual pressure drops in all spaces of the Stirling engine. The speed and pressure of the engine are 882 rpm and 3.58 bar, respectively. The indicated work of isothermal, adiabatic, simple, and experimental analysis is 133.82 W, 143.75 W, 93.2W, 111.43W, respectively. The thermal efficiency of Schmidt, adiabatic, and simple analysis is calculating as 30.7 %, 30.9%, 21.20% respectively.

Figure 5 and figure 6 depict the mass and energy flow in each compartment of the Stirling engine. In figure 5, the values of heat energy added to a heater, rejected from a cooler, and stored in a regenerator are shown below. It is important to note that there is a difference in magnitude of energy flow and total energy flow for regenerator is a sum to zero. The heat transfer to a heater and cooler is 30.33 J and -24.08 J respectively. The total work done is calculated as 6.35 J. Figure 6 shows instantaneous mass flow in each compartment of the engine. At the crank angle of 145-degree, the large amount of mass flow takes from

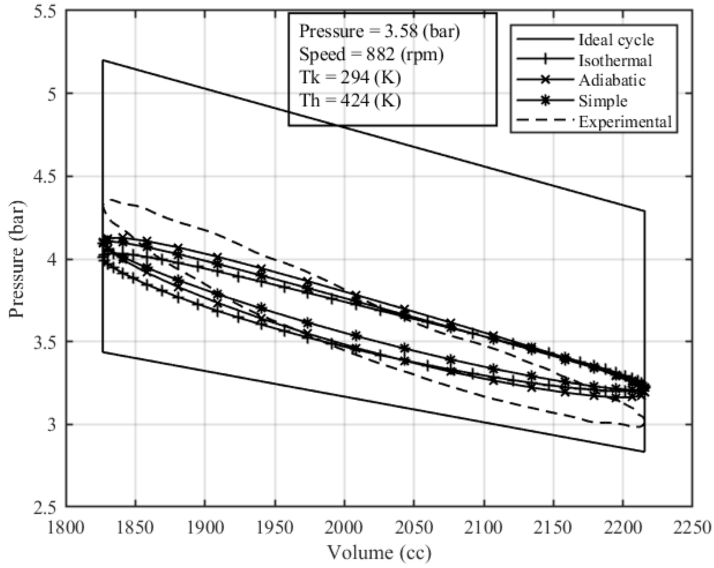

Fig.3. Pressure volume diagram of experimental, adiabatic, isothermal, and simple analysis

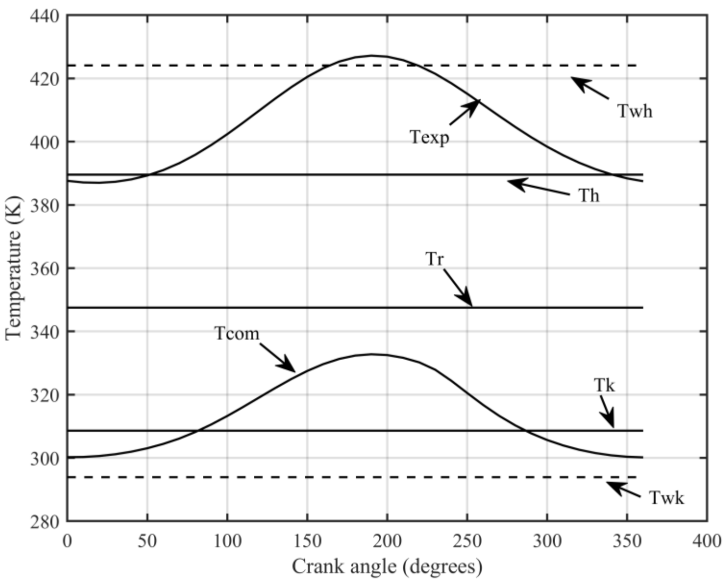

Fig.4. Temperature fluctuation for simple analysis.

compression to cooler (CK1) and less amount of mass flow from heater to expansion space. In a reverse manner, mass flow takes place after crank angle of 150 degree.

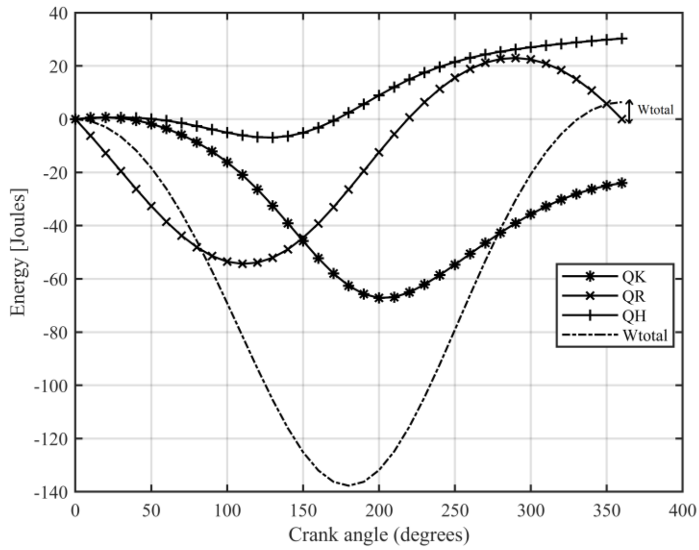

Fig. 5. Energy flow in a heater, cooler, and regenerator

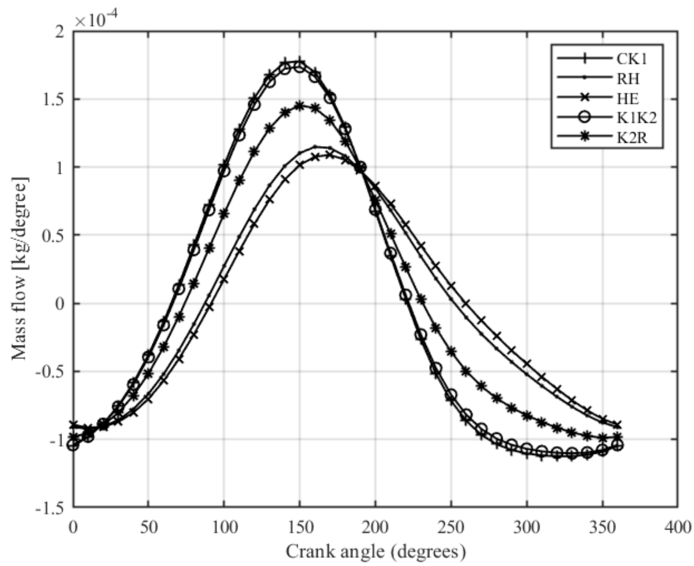

Fig. 6. Mass flow in each compartment of Stirling engine.

Figure 7 represents the work loss due to piston finite speed. Throughout the cycle, work loss fluctuates. At the angles 100- and 260-degree, maximum finite speed loss occurred 1.79 W. At an angle of 10, 170, and 360 degrees, the finite speed loss is calculated as -1.9 W. The sum of loss to the whole of a cycle is calculated as -4.3 W.

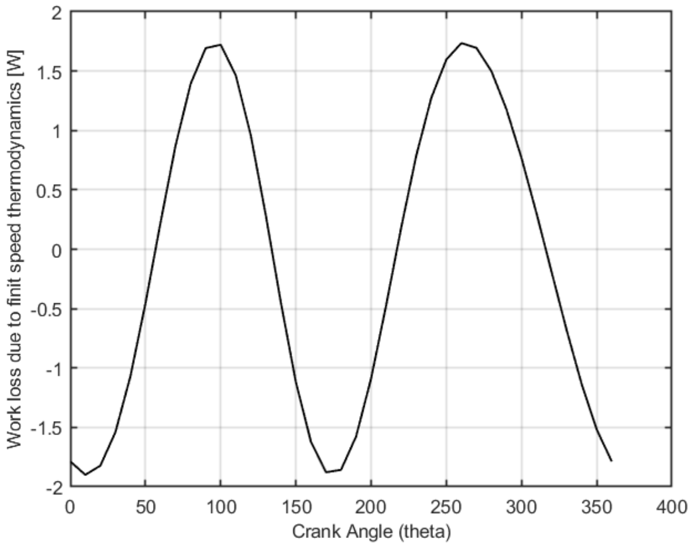

Fig. 7. Work loss due to piston finite speed

The model shows the sum of finite speed piston losses is -4.3 W. Moreover, the summary of the results is shown in table 2.

Table 2. Summary of thermal model results

	Isothermal	**Adiabatic**	**Simple**	**Experimental**
Work $_{in}$ (J)	9.08	9.70	6.30	7.50
Power (W)	133.8	143.75	93.2	111.43
Thermal efficiency (%)	30.70	30.90	21.2	24.70

In the above table, indicated work of the isothermal model is 9.08 J. In this model, the effect of friction losses, heat transfer losses, load losses are not included and therefore heat exchangers are insignificant. Therefore, power and efficiency estimated due to the isothermal model are 133.8 W and 30.70% respectively. The adiabatic model is much accurate because it provides true information on each compartment of the Stirling engine. It is limited because it does not contain any losses of the Stirling engine. The indicated work, power, and efficiency calculated due to this model is 9.7 J, 143.75 W, and 30.90 % respectively. In adiabatic including all the losses and seeing the non-ideal regenerative case, simple analysis is found which depicts the real performance of Stirling engine. The indicated work, power, and efficiency evaluated due to the simple model is 6.30 J, 93.2 W, 21.2%, and experimentally, these are found 7.50 J, 111.43 W, and 24.70 % respectively. The percentage error between the simple model and the experimental result is found -16.7 %, -14.1% for power and efficiency respectively.

5 Conclusion

Throughout this work, it is tried to simulate the design methodology of gamma type double piston cylinder engine for waste heat recovery of exhaust gas of diesel engine. The engine is divided into six compartments (i.e., expansion chamber, heater tubes, regenerator, cooler, cooler connecting pipe, compression chamber). Connecting pipe of cooler is also included for evaluation of real pressure variation in the chamber. Moreover, real piston motion variations are also included in our model for performance of Stirling engine. This research work is purely based on an enhanced adiabatic model. In this work, isothermal, adiabatic, and enhanced simple analysis of double piston cylinder Stirling engine is done. The exhaust gas of the diesel engine is used as a heat source for the Stirling engine. The heater, cooler, mean pressure, and frequency of the engine are 424 K, 294 K, 3.5 bar, and 14.67 Hz with helium as a working fluid inside the Stirling engine respectively. Simulation results show indicated work done by isothermal, adiabatic simple, and experimentally calculated as 9.08 J, 9.7 J, 6.3 J, and 7.50 J respectively and power estimated by these models are 133.82 W, 143.75 W, 93.2 W, and 111.43 W respectively. Moreover, the simulation shows that heater and cooler walls are not conducting very well. The temperature of the wall heater wall is 424 K while the temperature of the gas inside heater tubes is 384 K. In addition, the power losses due to piston finite speed are calculated as -4.3 W in our Stirling engine. The percentage error between experimental and simulation results are found -16.7 %, -14.1% for power and efficiency respectively. The above-mentioned results show a close agreement with simulation and experimental results.

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