

# Damage Function and Its Application

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**Abstract.** The types of damage functions for objects in the event of various emergency situations - fires, accidents, emergencies, etc. - are considered. The methods of constructing damage functions using expert methods and taking into account the actions of emergency services to eliminate emergency situations and their consequences are presented. The possibility of comparing the effectiveness of various methods of emergency response using the damage function is shown. An example of assessing damage reduction while increasing the efficiency of fire extinguishing by automatic installations is provided.

## 1 Introduction

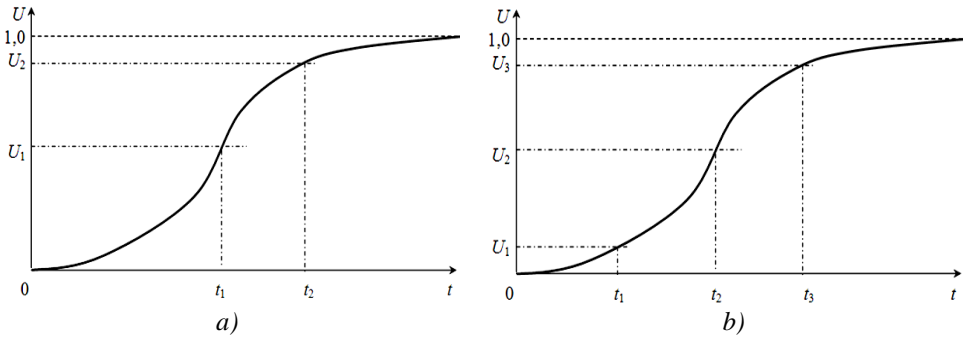
In the event of emergency situations (fires, accidents, emergencies, etc.) at industrial [1] and other facilities, they are usually damaged. At the same time, the amount of damage depends both on the object itself and on the type of emergency situation and measures to minimize it - on the effectiveness of fire extinguishing, emergency rescue operations (ERO) [2], etc. When developing planning documents [3-5] and risk assessment [6, 7] is of interest to determine the dynamics of possible damage, for which it was proposed [8, 9] to use a special curve  $U(t)$ , called the «damage function».

## 2 Materials and methods

The initial damage function  $U(t)$  is a curve that monotonically increases from  $U=0$  (at time  $t=0$  of the onset of an emergency situation, there is still no damage) to  $U=I$ , when the damage reaches its greatest value (Fig. 1).

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**Fig. 1.** Initial damage function: a) when represented by 2 points, b) when represented by 3 points

Mathematical expressions for the function  $U(t)$  that meet the requirements for it can have different forms (for example, [8, 9]):

$$U(t) = (1 - e^{-kt})^n \tag{1}$$

$$U(t) = 1 - (1 + At)^{-m} \tag{2}$$

$$U(t) = 1 - B^{-\lambda t} \tag{3}$$

$$U(t) = 0,5 \left[ (1 - e^{-\mu t})^a + (1 - e^{-\mu t})^b \right] \tag{4}$$

$$U(t) = c(1 - e^{-vt}) + (1 - c)(1 - e^{-\zeta t}) \tag{5}$$

where  $k, n, A, m, B, \lambda, \mu, a, b, c, v, \zeta$  – coefficients that are to be determined.

The representation of the damage function is carried out using expert methods [10] by 2 (fig. 1, a) or 3 (fig. 1, b) points. It's easy to notice that by 2 points:  $U(t_1)=U_1$  и  $U(t_2)=U_2$  the following coefficients may be obtained:  $(k, n)$ ,  $(A, m)$  и  $(B, \lambda)$  of the equations (1)-(3) respectively. Coefficients  $(\mu, a, b)$  and  $(c, v, \zeta)$  of equations (4) and (5) – by 3 points:  $U(t_1)=U_1$ ,  $U(t_2)=U_2$  и  $U(t_3)=U_3$  (fig. 1, b).

Coefficients  $(k, n)$ ,  $(A, m)$  и  $(B, \lambda)$  can be determined from solutions of systems of two equations:

$$\begin{cases} U_1 = [1 - \exp(-kt_1)]^n, \\ U_2 = [1 - \exp(-kt_2)]^n, \end{cases} \tag{6}$$

$$\begin{cases} U_1 = 1 - (1 + At_1)^{-m}, \\ U_2 = 1 - (1 + At_2)^{-m}, \end{cases} \tag{7}$$

$$\begin{cases} U_1 = 1 - B^{-\lambda t_1}, \\ U_2 = 1 - B^{-\lambda t_2}. \end{cases} \tag{8}$$

Coefficients  $(\mu, a, b)$  can be determined from solutions of systems of three equations:

$$\begin{cases} U_1 = 0,5[1 - \exp(-\mu t_1)]^a + 0,5[1 - \exp(-\mu t_1)]^b, \\ U_2 = 0,5[1 - \exp(-\mu t_2)]^a + 0,5[1 - \exp(-\mu t_2)]^b, \\ U_3 = 0,5[1 - \exp(-\mu t_3)]^a + 0,5[1 - \exp(-\mu t_3)]^b. \end{cases} \tag{9}$$

Coefficients  $c, v, \zeta$  can be determined similarly.

Systems of equations (7)-(9) can be solved numerically [11], and in [9] a method for determining the coefficients  $k$  and  $n$  of equation (1) by two stages is presented. First, the relation  $t_2/t_1=f_1(n)$  function is found as follows:

$$\frac{t_2}{t_1} = \frac{\ln(1-U_2^{1/n})}{\ln(1-U_1^{1/n})} \tag{10}$$

whence, by the known relation  $t_2/t_1$  the value  $n$  is selected Then  $k$  coefficient is being determined:

$$k = -t_1^{-1} \ln(1-U_1^{1/n}) = -t_2^{-1} \ln(1-U_2^{1/n}) \tag{11}$$

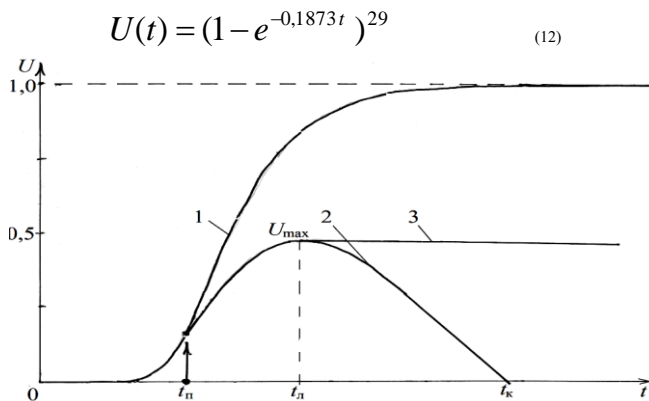
For the case when  $U_1=0,1$ ,  $U_1=0,5$  and  $U_2=0,9$  the functions  $f_1(n)$  are obtained, and they are presented in table 1.

**Table 1.** Function  $f(n)$  at  $U_2=0,9$

$n$	0,1	0,2	0,3	0,4	0,5	0,6	0,8	1	$U_1$
$\frac{t_2}{t_1}$	$4,3 \cdot 10^9$	89279	2621	469,1	165,24	59,84	36,152	21,85	0,1
$\frac{t_2}{t_1}$	438,8	28,12	11,646	7,520	5,773	4,827	3,836	3,322	0,5
$n$	1,5	2	3	5	10	20	50	100	$U_1$
$\frac{t_2}{t_1}$	13,163	7,812	5,396	3,863	2,882	2,366	1,988	1,812	0,1
$\frac{t_2}{t_1}$	2,707	2,419	2,133	1,893	1,686	1,553	1,438	1,378	0,5

Note: at  $U_1=0,5$ :  $f(29)=1,5$ ;  $f(3,88)=2$ ;  $f(1,204)=3$ ;  $f(0,577)=5$ ;  $f(0,3286)=10$ .

For example, appearance of the damage fuction (1), presented at  $U(t_1=20)=0,5$  and  $U(t_2=30)=0,9$  using the equations (9), (10) and table 1, is the following (curve 1 on fig.2):



**Fig. 2.** The initial damage function (1) and its appearance in the elimination of the consequences, for example, of the release of hazardous chemicals (2) and when extinguishing a fire (3)

A similar approach is applicable to model (7), when the value of  $m$  is determined by the relation  $t_2/t_1=f_2(m)$  (table2):

$$\frac{t_2}{t_1} = \frac{(1-U_2)^{-1/m} - 1}{(1-U_1)^{-1/m} - 1} \tag{13}$$

**Table 2.** Function  $f_2(m)$  at  $U_1=0,5$  and  $U_2=0,9$

$m$	1	2	3	5	10	100	1000	$\infty$
$t_2/t_1$	9	5,20	4,441	3,934	3,607	3,348	3,324	3,322

However, there is a limitation corresponding to the condition  $m \rightarrow \infty$ :

$$\min \frac{t_2}{t_1} = \frac{\lg(1-U_2)}{\lg(1-U_1)} \quad (14)$$

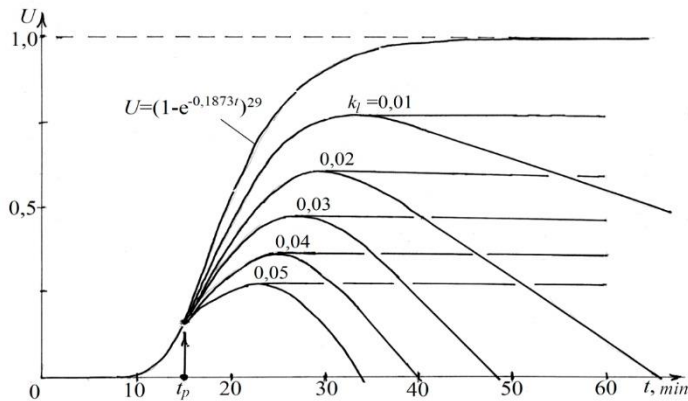
Applied to equation (4) using numerical methods [11] from solving the equations system (9) for the case  $U(10)=0,1$ ,  $U(20)=0,5$  and  $U(30)=0,9$  the following values of coefficients are determined  $\mu=0,2228$ ,  $a=14,12$  and  $b=161,1$ , and as a result:

$$U(t) = 0,5(1 - e^{-0,2228t})^{14,12} + 0,5(1 - e^{-0,2228t})^{161,1} \quad (15)$$

### 3 Results and discussion

Using the damage function, it is possible to solve the problem of assessing the effectiveness of eliminating an emergency situation. In this case, two situations should be distinguished - when at the moment  $t_l$  the damage reaches its maximum  $U_{max}$  and remains at this level for some time (for example, a fire) and when after reaching  $U_{max}$  the damage is completely eliminated after the time  $t_k$ , i.e.  $U(t_k) = 0$ , which may correspond to the neutralization of the release of an emergency chemical hazardous substance. Graphically similar situations are shown in Fig. 2.

If we assume that the elimination of an abnormal situation will begin in time  $t_p$  after its occurrence and will occur at a rate of  $k_l$  (can be determined by expert methods [10]), then the dynamics of damage is shown in Fig. 3, provided that equation (12) is valid.



**Fig. 3.** An example of a damage function during actions to localize and eliminate an emergency situation

Taking into account the actions to localize and eliminate the contingency situation, the dynamics of damage  $U(t)$  can be described in equation:

$$U(t) = \begin{cases} U_0(t) & \text{when } t < t_p, \\ U_0(t) - k_l(t - t_p) & \text{when } t \in [t_p, T), \\ U_k & \text{when } t > T, \end{cases} \quad (16)$$

where  $U_0$  – is an initial damage function determined by (1)-(5);  $T=t_l$  and  $U_k=U_{max}$ , when, for example, fire extinguishing takes place;  $T=t_k$  and  $U_k=0$ , when, for example, there is a neutralization of the release of hazardous chemicals.

Time  $t_l$  can be found from the solution of the transcendental equation:

$$\frac{dU_0}{dt} = k_l \tag{17}$$

If the damage function is described by expression (1), then equation (17) takes the form [9]:

$$k_l = knE(1 - E)^{n-1} \tag{18}$$

where  $E = \exp(-k(t_l - t_p))$ .

Maximal damage  $U_{\max} = U(t_l)$  at the initial damage function (1) can be determined from the equation (16):

$$U_{\max} = (1 - e^{-kt_l})^n - k(t_l - t_p) \tag{19}$$

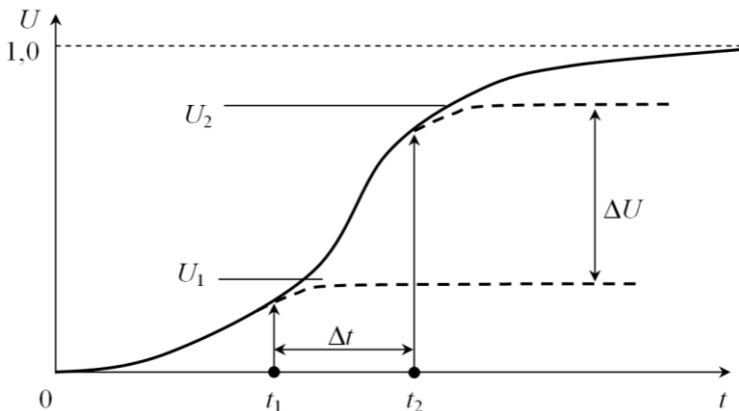
Equations (18) and (19) can be solved numerically [11].

Time  $t_k$  can be found from the solution of the transcendental equation:

$$U_0(t_k) = k_l(t_k - t_p) \tag{20}$$

As an example, Fig. 3 shows function (12) and damage reduction functions at different speeds  $k_l$  from 0.01 to 0.05 min<sup>-1</sup>.

With the help of the damage function, it becomes possible to solve the problem of comparing various methods of neutralizing an abnormal situation at an object, for example, a fire in a room when using automatic fire extinguishing installations (AFEI) [12]. In Fig. 4, using the damage function, it is clearly shown that with an increase in the operational efficiency of the AFEI by the value of  $\Delta t$ , the direct damage from the fire can be reduced by the value of  $\Delta U$ . Specific values of  $\Delta t$  and  $\Delta U$  can be obtained using the damage function for specific objects and specific types of AFEI.



**Fig. 4.** An example of reducing damage at an object while increasing the response time of the localization system and eliminating an emergency situation

## 4 Conclusion

Thus, if the facility has a risk of various emergency situations (for example, fires), the above methods for constructing the damage function make it possible to objectively assess the

characteristic time parameters of damage localization, taking into account the onset and speed of action of the corresponding systems, for example, AFEI.

Also, the problem of comparing two or more ways to minimize damage at a specific object can be solved, which will allow to objectively choose the best of them.

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