

NEW APPROACH TO COMPUTE ACCURATELY THE ENTROPY GENERATION DUE TO NATURAL CONVECTION IN A SQUARE CAVITY

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Abstract. The idea to carry out an exercise to compare the calculation of entropy generation for unsteady natural convection in a square cavity with vertical sides that are maintained at different temperatures was motivated by the observation, in the literature, of inaccurate or often erroneous results concerning the values of this significant physical entity. It then appeared necessary to reconsider this problem in order to ensure its consistent assessment. The new approach that we propose allows a direct access to the value of the entropy generation by considering the exact values of the thermophysical properties of the working fluid, which depends on the Prandtl and the Rayleigh numbers.

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1 Introduction

It is well established that convective heat transfer processes are always accompanied by a loss of energy commonly referred to as "Entropy Generation" in the scientific literature [1]. Accurately determining the entropy generation in applied thermal engineering is the indispensable prerequisite for the design of different types of thermal exchanging devices [2]. Entropy generation has been and continues to be the subject of many research activities over the past decades, as shown by the significant review articles by Oztop and Al-Salem [3], Sciacovelli et al. [4], and more recently by Kumar et al. [5], and Cai et al. [6]. Solving the entropy generation equation during fluid flows in a natural convection regime, which will be developed later, requires the calculation of a coefficient called "the irreversibility distribution ratio", oftentimes symbolized by the Greek letter ϕ .

One real problem lies in an accurate determination of the irreversibility distribution ratio value. Until now, the computing of the relevant value of this ratio has not yet been well established and the gap between the values proposed by different authors can be quite significant. Hussein et al. [7], and Oztop et al. [8], present and discuss the results for entropy generation without specifying any value of the irreversibility ratio. Shavik et al. [9], Yejjer et al. [10], Jassim et al. [11] and Seyyedi et al. [12], impose values of the same ratio ranging from 10^{-3} to 10^{-5} with no explanation. In the same way, Erbay et al. [13, 14], specify values which increase linearly with the Rayleigh number (Ra), starting at 10^{-13} for $Ra=10^2$ to reach 10^{-9} for $Ra=10^6$, with constant steps of 10. Rathnam et al. [15], use an order of magnitude analysis of parameters to evaluate the same ratio at 10^{-3} . Magherbi et al. [16], Ilis et al. [17], De C. Oliveski et al. [18], and Bouabid et al. [19], consider this coefficient as an investigative parameter in the same way as the Rayleigh, or Prandtl numbers for example in [16], ϕ varies from 10^{-4} to 10^{-1} and in [17-18-19], ϕ varies from 10^{-4} to 10^{-2} . However, we would also point out that, until very recently (05 February 2021) Karki et al. [20] investigates the exergy analysis of Rayleigh–Benard natural convection by varying the irreversibility ratio ϕ between 10^{-2} and 10^{-5} . Finally, it must be stressed that we identified a numerical study of natural convection and entropy generation reported by Khorasanizadeh and Nikfar [21], in which they have doubts about the fact that the irreversibility distribution ratio ϕ remains constant while at the same time Rayleigh number varies considerably. The values suggested by these authors vary according to the Rayleigh number and are certainly more important than such as the ones already proposed but they are still far from reality, and above all, there is no propose that set forth how these values were concretely obtained.

Based on the aforementioned brief review of the literature, which revealed many shortcomings in the determination of the exact value of the irreversibility distribution ratio ϕ , that we propose a new pragmatic approach in order to estimate the fair value of the entropy generation in confined spaces.

2 Problem description

The benchmark problem considered here is the differentially heated square cavity problem depicted in Figure 1. It is similar to those referenced by Magherbi et al. [16], Ilis et al. [17], De C. Oliveski et al. [18], and Bouabid et al. [19]

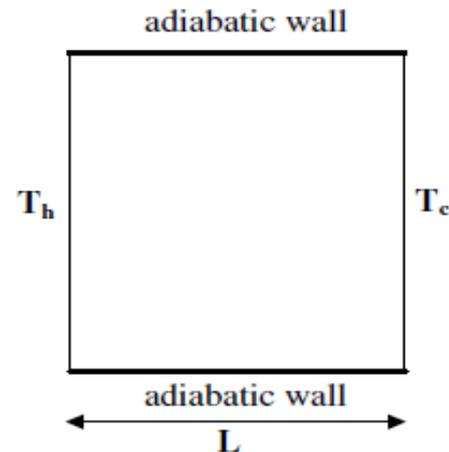


Fig. 1. A schematic view of the benchmark problem [16].

3 Mathematical formulation

The stream function-vorticity (ψ, ω) formulation is used to express the dimensional governing equations for the laminar and unsteady state natural convection in Cartesian coordinates x and y .

$$\left. \begin{aligned} \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -\omega \\ \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} &= \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + g\beta \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \end{aligned} \right\} \quad (I)$$

here $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

The numerical resolution of the previous system of equations requires the following initial and boundary conditions:

At $\tau=0$ and for whole space.

$$\psi = \omega = 0, \quad T = T_0 + \Delta T \left(0.5 - \frac{x}{L} \right) \quad 0 \leq x \leq L$$

At $\tau > 0$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2} \text{ and } T = T_h \text{ at left wall}$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2} \text{ and } T = T_c \text{ at right wall}$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial y^2} \text{ and } \frac{\partial T}{\partial y} = 0 \text{ at bottom and top walls.}$$

The average Nusselt number \overline{Nu} can be expressed, on the basis of the dimensional variables, as follows:

$$\overline{Nu} = -\frac{1}{\Delta T} \int_0^L \left. \frac{\partial T}{\partial x} \right|_{x=0} dy \quad (1)$$

The local entropy generation is given by [21]:

$$s_{gen} = \underbrace{\frac{k}{T_0^2} \left\{ \left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right\}}_{s_{th}} + \underbrace{\frac{\mu}{T_0} \left\{ 2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\}}_{s_{ff}} \quad (2)$$

The total entropy generation is the integral over the system volume of the local entropy generation:

$$\overline{s_{gen}} = \frac{1}{\mathcal{G}} \int_{\mathcal{G}} s_{gen} d\mathcal{G} \quad (3)$$

Another parameters that characterizes the irreversibilities distribution are the local and the average Bejan number defined respectively as [16]

$$be = \frac{s_{th}}{s_{gen}} \quad \overline{be} = \frac{1}{\mathcal{G}} \int_{\mathcal{G}} be d\mathcal{G} \quad (4)$$

The dimensionless form of the governing equations may be written with following dimensionless variables:

$$X, Y = \frac{1}{L}(x, y), \quad \Theta = \frac{T - T_0}{T_h - T_c}, \quad U, V = \frac{L}{\alpha}(u, v), \quad \tau = \frac{\alpha}{L^2}t$$

$$\left. \begin{aligned} \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} &= -\Omega \\ \frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} &= Pr \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + RaPr \frac{\partial \Theta}{\partial X} \\ \frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} &= \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \end{aligned} \right\} \quad (II)$$

Where $U = \frac{\partial \Psi}{\partial Y}, V = -\frac{\partial \Psi}{\partial X}, \Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}$

$$Pr = \frac{\nu}{\alpha}, \quad Ra = \frac{g\beta\Delta T Pr}{\nu^2} L^3$$

The average Nusselt number \overline{Nu} can be expressed, on the basis of the dimensionless variables, as follows:

$$\overline{Nu} = -\int_0^1 \left. \frac{\partial \Theta}{\partial X} \right|_{X=0} dY \quad (5)$$

And the local entropy generation, with the same dimensionless variables is given by:

$$S_{gen} = \underbrace{c_1 \left(\frac{\partial \Theta}{\partial X} \right)^2 + \left(\frac{\partial \Theta}{\partial Y} \right)^2}_{s_{th}} + \underbrace{c_2 \left\{ 2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right\}}_{s_{ff}} \quad (6)$$

The total entropy generation is given by:

$$\overline{S_{gen}} = \int_0^1 \int_0^1 S_{gen} dXdY \quad (7)$$

The two irreversibilities coefficient are:

$$c_1 = k \left(\frac{\Delta T}{T_0 L} \right)^2 \quad c_2 = \frac{\mu}{T_0} \left(\frac{\alpha}{L^2} \right)^2 \quad (8)$$

The first term in equations (2) and (6) is the local entropy generation due to heat transfer and the second term is the local entropy generation due to fluid friction.

The dimensionless form of Eq. (4) takes the form

$$Be = \frac{S_{th}}{S_{gen}} \quad \overline{Be} = \int_0^1 \int_0^1 Be dXdY \quad (9)$$

At this point it must be underlined that the lowercase letters correspond to the dimensional variables and the uppercase letters correspond to the dimensionless variables.

It should be noted that to appraise the total volumetric entropy generation all the authors previously cited use the local entropy generation number defined by:

$$N_s = \frac{S_{gen}}{c_1} = \left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 + \varphi \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right] \quad (10)$$

Where φ is the irreversibility distribution ratio defined

$$\text{as: } \varphi = \frac{c_2}{c_1} = \frac{\mu T_0}{k} \left(\frac{\alpha}{\Delta T} \right)^2 \frac{1}{L^2} \quad (11)$$

The total dimensionless entropy generation number is written:

$$\overline{N_s} = \int_0^1 \int_0^1 N_s dXdY \quad (12)$$

The primary objective of this work is to suggest a method that is mathematically and physically correct for the determination of the irreversibility distribution ratios thus providing the true value of the entropy generation.

4 Numerical procedure

4.1 Discretisation method

The above-mentioned systems of equations (I) and (II) subject to their respective initial and boundary conditions have been discretized by the finite difference method. Temporal discretization has been achieved using the Runge-Kutta fourth-order method (R.K.4). The convective terms have been discretized with a third-order upwind scheme as proposed by Kawamura et al. [23]. The diffusive terms, as well as the terms including the first derivatives, have been discretized by a fourth-order accurate scheme. An iterative procedure based on the successive Non Linear Over Relaxation method (NLOR) was used to solve the discretized stream function equation. Once the velocities and temperatures have been determined, the average Nusselt number (Eqs.1,5) and the characteristics of the total entropy generation in dimensional and dimensionless variables are computed

using equations (3), (4), (7), (9), and (12). An in-house FORTRAN code, with a double precision accuracy, has been developed for solving the systems of the discretized equations.

4.2 Grid independency test

Numerical tests have been made to ensure the accuracy of results for the grid used in this study. Five grid sizes (41x41; 81x81; 101x101; 161x161 and 201x201) have been considered. Table 1 shows the convergence of the average Nusselt number and the total entropy generation for Pr=0.7 and Ra=103. It should be noted that the maximum relative error does not exceed 2% between the grid sizes of 81x81 and 101x101 compared to the grid size of 201x201. Therefore, it was decided to use a non-uniform grid with 101x101 grid points for all calculations allowing a balance between accuracy and CPU time.

Table 1. Grid independence test: average Nusselt number and total entropy generation for Ra=10³, Pr = 0.

Grid	\overline{Nu}	$\overline{S_{gen}}$
41x41	1.1196	0.16057
81x81	1.1182	0.16034
101x101	1.1181	0.16034
161x161	1.1179	0.16031
201x201	1.1178	0.16030

4.3 Code Validation

In order to ensure the effectiveness of the developed code, validation was carried out to compare the values of the average Nusselt number obtained in the classic case of natural convection occurring in a square cavity with differentially heated vertical sides and insulated horizontal walls. Table 2 shows this comparison for different Rayleigh numbers when the Prandtl number is set at 0.7.

Table 2. Comparison of the present average Nusselt number with available literature

Ra	Ref.[24]	Ref.[25]	Ref.[26]	Present
10 ³	1.116	1.127	1.117	1.118
10 ⁴	2.238	2.245	2.246	2.249
10 ⁵	4.509	4.521	4.530	4.537
10 ⁶	8.817	8.800	8.822	8.825

In view of this validation, it can be concluded that the computation code developed for this study gives results in accordance with those cited in the literature.

5 Results and discussion

5.1 Details data associated with the benchmark

Numerical simulations have been performed for different Rayleigh numbers (Ra). The thermo-physical properties of the working fluid, for Prandtl number equal to 0.7, have been taken from the book by the authors Incropera et al.[27] at the bulk temperature T₀=350 K and are listed in Table 3. Furthermore, the temperature difference ΔT are kept constant at 10 K.

Table 3. Thermo-physical properties of air (Pr=0.7) at T₀=350 K expressed in the International System (SI) of units. [27]

ρ	k	μ	α	β
0.995	3.10-2	2.082 10-5	2.9910-5	2.85710-5

5.2 The irreversibility ratios calculation method

It can be seen that the irreversibility ratios c₁ and c₂ in equation (8) are directly proportional to the thermo-physical properties of the fluid, to the temperature difference ΔT, and (perhaps more importantly) to the cavity length L. All these parameters with the exception of the cavity length L are known for each Prandtl number. This effective length could be evaluated numerically from the Rayleigh number as follow:

$$Ra = \frac{g\beta\Delta TPr}{\nu^2} L^3 \Rightarrow L = \sqrt[3]{C^m Ra} \quad (13)$$

Where $C^m = \frac{\nu^2}{g\beta\Delta TPr}$ is a constant, and $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity.

This new approach does take into account all these parameters and thus provides a better reflection of the real value of the irreversibility ratios (which are given in table 4), thus providing the true value of the entropy generation.

Table 4. Real values of the irreversibility ratios for different Rayleigh numbers (Ra) and Pr=0.7

Ra	c ₁	c ₂	φ = c ₂ /c ₁
10 ³	1.4340 10 ⁻¹	1.8235 10 ⁻⁹	1.2716 10 ⁻⁸
10 ⁴	3.0896 10 ⁻²	8.4640 10 ⁻¹¹	2.7396 10 ⁻⁹
10 ⁵	6.6563 10 ⁻³	3.9287 10 ⁻¹²	5.9022 10 ⁻¹⁰
10 ⁶	1.4340 10 ⁻³	1.8235 10 ⁻¹³	1.2716 10 ⁻¹⁰

5.3 Comparison with Benchmark results

5.3.1 Method validation

The validity of the proposed method has been verified by solving the systems of equations (I) and (II) expressed with the dimensional, and the dimensionless variables respectively.

The comparisons of average Nusselt number, total entropy generation, total entropy generation due to fluid friction and average Bejan number for a Rayleigh number equal to 104 are shown in table 5. As seen, the obtained results are quite similar, because we have taken into account the thermo-physical properties of the fluid (see Table 3) and the cavity length L calculated from equation (13).

Table 5. Different parameters obtained from the equations solved with the dimensional and dimensionless variables (Ra=10⁴)

Dimensional variables	Dimensionless variables
\overline{nu} 2.249050	\overline{Nu} 2.249070
$\overline{S_{sen}}$ 6.949419 10 ⁻²	$\overline{S_{sen}}$ 6.949482 10 ⁻²
$\overline{S_{ff}}$ 8.819882 10 ⁻⁷	$\overline{S_{ff}}$ 8.820095 10 ⁻⁷
\overline{be} 0.999987308	\overline{Be} 0.999987308

5.3.2 Total entropy generation calculation

The total entropy generation equations (6) and (7) subject to the exact values of the two irreversibilities coefficient c1 and c2 given by equation (8) have been numerically solved for different Rayleigh numbers. The obtained results, given in figure 2, provide a direct measurement of the total entropy generation in the physical domain expressed in W/m³K. It should be noted that they are most helpful to the engineers and designers in order to design and optimize different types of equipment involving heat transfer in fluid flows.

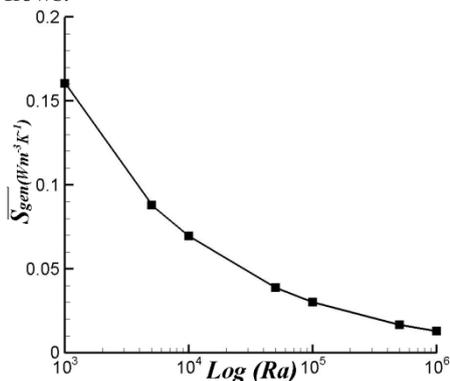


Fig. 2. Total entropy generation $\overline{S_{gen}} (Wm^{-3}K^{-1})$ versus the Rayleigh numbers (Ra).

5.3.3 Total dimensionless entropy generation number

Total dimensionless entropy generation number versus Rayleigh numbers is shown graphically in Figures 3 and 4. In figure 3 the total dimensionless entropy generation number is computed with an the irreversibility distribution ratio ϕ , by taking account of the thermo-physical properties of the fluid (see Table 3) and the cavity length L calculated from equation (13), while, in figure 4 the same parameter is computed with an the irreversibility distribution ratio ϕ , which is set arbitrarily at 10⁻⁴, according to the approach adopted by Magherbi et al. [16], Ilis et al. [17], De C. Oliveski et al. [18], and Bouabid et al. [19].

We clearly observe some of the numerical differences in solutions between the two approaches for the total dimensionless entropy generation number (\overline{Ns}). In figure 3 \overline{Ns} varies on a scale of 1 to 9 with a slightly increasing slope, while in figure 4 \overline{Ns} varies from 1 to

400. This variation is moderately progressive for Ra less than 10⁵ and becomes exponential afterwards.

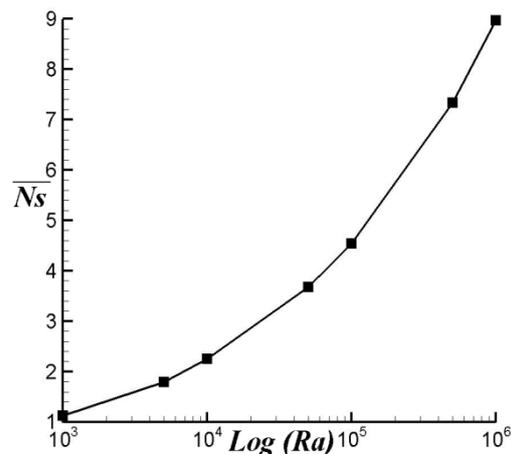


Fig. 3. Total dimensionless entropy generation number versus the Rayleigh numbers (Ra) according to our approach.

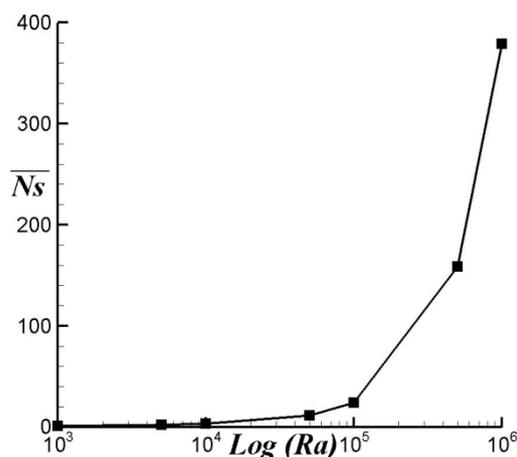


Fig. 4. Total dimensionless entropy generation number versus the Rayleigh numbers (Ra) according to the approach referenced in [16-19].

5.3.4 Formatting the title

The evolution of the average Bejan number is shown in Figure 5 and, 6 respectively, for different Rayleigh numbers. The same qualitative trend in the evolution of the average Bejan number appear in the bench mark solution (figures 5-6).The obvious difference between the two solutions lies in the fact that the average Bejan number is close to one, according to our approach, while the same parameter varies significantly from 0.02 to 1, according to the approach referenced in [16-19], when the irreversibility distribution ratio value is set at $\phi=10^{-4}$. This could be explained by the fact that irrespective of the Rayleigh number the irreversibility distribution ratio ϕ is very small (see table 4) compared to that chosen for the benchmark test ($\phi=10^{-4}$). As a result, the total entropy generation due to fluid friction is negligible, which implies an average Bejan number close to one. This result confirms the commonly adopted hypothesis, which states that the viscous dissipation function is neglected in the energy transport equation.

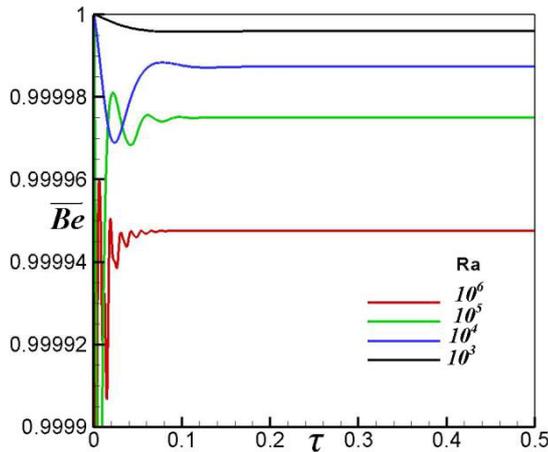


Fig. 5. Evolution of average Bejan number versus the Rayleigh numbers (Ra) according to our approach.

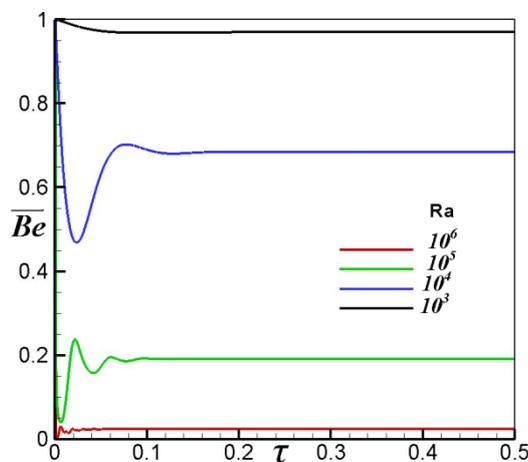


Fig. 6. Evolution of average Bejan number versus the Rayleigh numbers (Ra) according to the approach referenced in [16-19], ($\phi=10^{-4}$)

6 Conclusion

In the present work, details are given of a new approach used to obtain an accurate determination of the entropy generation for unsteady natural convection in a square cavity with differentially heated sidewalls. The conclusions are summarized as follows:

- 1 From the mathematical point of view, it is clear that the characteristic length L is proportional to the Rayleigh number and then cannot be assumed as constant, especially if this number varies quite considerably.
- 2 The irreversibility distribution ratio number varies significantly according to the Rayleigh number and hence it can, in no case, be considered as a constant.

- 3 One and only one value of the irreversibility distribution ratio ϕ corresponds to each fluid characterized by its Prandtl and Rayleigh numbers.
- 4 The irreversibility distribution ratio ϕ is proportional to the inverse of the characteristic length L squared, and thus its value decreases when the characteristic length L (or Rayleigh number) increases.
- 5 The values of the irreversibility distribution ratio ϕ proposed by different authors are being overestimated resulting often lead to an over-sizing of the heat transfer processes equipment.
- 6 Though the present results are restricted to natural convection in confined spaces of limited Prandtl number and Rayleigh numbers, the procedure is general and can be equally applied to various problems including mixed convection, nanofluids flows, turbulent flows, and so on.

ACKNOWLEDGMENTS

The authors like to express their thankfulness to the Computational Fluid Dynamics Laboratory of the Mechanical Engineering Department, of Istanbul Medeniyet University, Turkey for having provided computer facilities during this work.

NOMENCLATURE

Be	Bejan number
g	Gravitational acceleration, ms^{-2}
Gr	Grashof number ($= g\beta\Delta TL^3 / \nu^2$)
k	Thermal conductivity, $\text{Wm}^{-1} \text{K}^{-1}$
L	Length of the square cavity, m
Pr	Prandtl number, ($= \nu/\alpha$)
Ra	Rayleigh number, ($= \text{Gr Pr}$)
S_{gen}	Entropy generation, $\text{Wm}^{-3}\text{K}^{-1}$
t	Time, s
T	Absolute temperature, K
T_0	Bulk temperature $(T_h + T_c)/2$, K
u, v	Velocity components in x, y directions, m s^{-1}
U, V	Dimensionless velocities in X and Y direction
\mathcal{V}	System volume
x, y	Dimensional Cartesian coordinates, m
X, Y	Dimensionless Cartesian coordinates

Greek symbols

α	Thermal diffusivity, $\text{m}^2 \text{s}^{-1}$
β	Thermal expansion coefficient, K^{-1}
ΔT	Temperature difference $((T_h - T_c)$, K
Θ	Dimensionless temperature
ν	Kinematic viscosity, $\text{m}^2 \text{s}^{-1}$
μ	Dynamic viscosity, $\text{kg m}^{-1}\text{s}^{-1}$
ϕ	Irreversibility distribution ratio

ρ	Density, kg m^{-3}
τ	Dimensionless time
ψ	Stream function, $\text{m}^2.\text{s}^{-1}$
Ψ	Dimensionless stream function
ω	Vorticity, s^{-1}
Ω	Dimensionless vorticity

Subscripts

ff	Fluid friction
th	Heat transfer

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