Rock slope kinematic analysis for planar failure: A probabilistic approach

Ibnu Rusydy[1,7*], *Mulkal* Mulkal[2], *Haqul* Baramsyah[2], *Hendri* Ahmadian[3], *Hamzah* Hussin[4], *Nafisah Al*-Huda[1,5], *Marwan* Marwan[6]

[1]Geological Engineering Department, Engineering Faculty, Universitas Syiah Kuala, Indonesia
[2]Mining Engineering Department, Engineering Faculty, Universitas Syiah Kuala, Indonesia
[3]Information Technology Department, Faculty of Science and Technology, UIN Ar-Raniry, Indonesia
[4]Department of Geoscience, Faculty of Earth Science, Universiti Malaysia Kelantan, 17600 Jeli, Kelantan, Malaysia
[5]Civil Engineering Department, Engineering Faculty, Universitas Syiah Kuala, Indonesia
[6]Geophysical Engineering Department, Engineering Faculty, Universitas Syiah Kuala, Indonesia
[7]Tsunami and Disaster Mitigation Research Center (TDMRC), Universitas Syiah Kuala, Indonesia

Abstract. The probabilistic kinematic analysis for rock slope has been conducted on one slope along USAID road in Aceh Province, Indonesia. This research aims to develop the modelled probability density function (PDF) and determine the probabilistic of planar failure occurrence (P_{op}). The geometry of discontinuity planes (dip and dip directions) and slope geometry (slope angle and slope face) were collected from our previous study. One slope with planar failure criteria was selected. The Monte Carlo simulations were performed in generating 100 new random values in 100 time iterations to produce modelled PDFs for the geometry of discontinuity plans based on statistical parameters of field observed data. The probabilistic of each experimental PDFs were computed to produce the probabilistic of planar failure occurrence. The result reveals that the distribution of dip and dip directions for experimental PDFs are considered Beta and Normal distributions. The statistical parameters produced in the model are almost likely similar to observed data. It means the model that was developed are reliable and conscientious. The rule of $\Phi < \beta j < \beta s$ and dip directions (α_j) within ±20° to slope face (α_s) are utilised as the boundaries to calculate the probabilistic of planar failure occurrence (P_{op}) which revealing 0.26.

1 Background

Rock slope failure analysis for mining and civil projects has attracted rock engineers to develop numerous methods to analyse the rock slope stability, including the rock slope kinematic analysis, limit equilibrium, numerical modelling, empirical approach, and rock mass classification [1–5]. In a conventional approach, those methods merely concern a single input parameter value known as deterministic, and it previously applies in deterministic slope kinematic analysis. Nevertheless, the rocks' physical, mechanical, and structural properties vary from point to point and naturally in rock [6]. Thus, the conventional approach is required to be improved obviously to the probability approach considering the uncertainty of geological structural data collected from the surface of rock slope to increase the reliable result in slope design and prevention measure.

The rock slope kinematic analysis approach studies the typology of slope failure without any consideration of the force working on the slope [2,5,7]. The type of potential failures determined in the rock slope kinematic approach was based on the stereography interpretation, which has been successfully conducted by numerous researchers [2,5,7]. However, their approaches are assumed as a deterministic approach picking the mean values from stereography as a single value. The probabilistic rock slope kinematic analysis has been introduced by numerous researchers [8–10], considering the effect of variability of joint orientations. Furthermore, the probability of planar failure due to variability of structural features have been conducted by Rahim et al. [11] and Farhan and Rai [12] and give reliable results. The comprehensive study on how the variability of discontinuity plane drives the planar failure of rock slope is elaborated and scrutinised in this paper. Hence, this study aims to develop the modelled probability density function (PDF) and determine the probabilistic of planar failure occurrence (P_{op}) at USAID highways rock slope in Aceh province, Indonesia.

Overall, this paper discusses the deterministic kinematic analysis, which previously has been conducted by Rusydy et al. [5] for Slope 1, revealing the planar failure potential at the joint set (J2). The data were then re-analysed by performing Monte Carlo simulation to generate a large number of random values based on statistical parameters from the joint set (J2). The probabilistic kinematic analysis was conducted to determine the probabilistic of planar failure occurrence

* Corresponding author: ibnu@unsyiah.ac.id

(P_{sp}) and how the variability of joint and slope geometries influence the failure.

2 Methodology

This research was conducted at the rock slopes along USAID highways side, build after Indian Ocean Tsunami hit Aceh province. During construction, a multitude of rock slopes have been exposed; hence, some slopes are vulnerable and need to be investigated and analysed. The dip and dip direction of discontinuity planes (e.g. joints, bedding) on the rock slopes were recording directly employing geological compass by Rusydy et al. [2,5,7,13]. The dip and dip direction discontinuity planes data were plotted in stereographic projection to determine the number of planes most likely having a similar direction with the slope face, and the dips of planes are higher than friction angle (Φ) but lower than slope angle. The Alejano et al. [14] suggested method was utilised in this study to determine the basic friction angle.

2.1 Rock slope kinematic analysis

Rock slope kinematic analysis is associated with the rock slope movement process without considering the forces working the rock slope. This approach is introduced by Hoek & Bray [15] and Goodman [16]. The analysis and calculation are stands on the stereography projection, which are projected in the dip (β_j) and dip direction of discontinuity planes (α_j), which is 3D into the 2D model [2,5,7]. Other input data required in the kinematic analysis is friction angle (Φ) determined by employing the tilt testing suggested by Alejano et al. [14].

The Orient software developed by Vollmer [17] was utilised in this research to produce a stereography

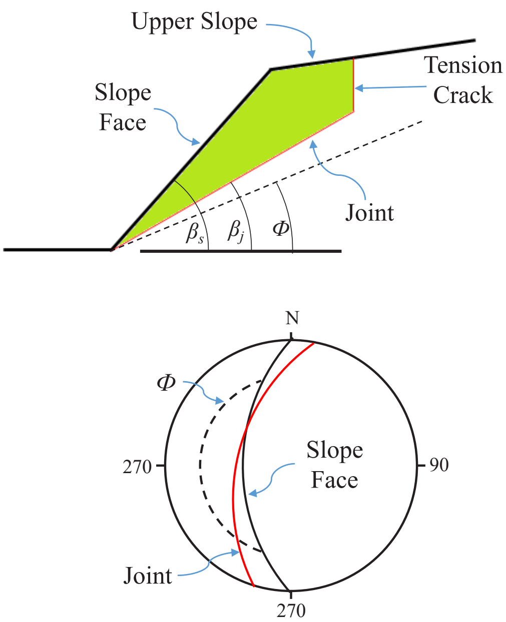

Fig. 1. Geometric circumstances for Plane failure (a) The graphic view of planar failure, (b) stereography projection, after modified from Wyllie & Mah [18].

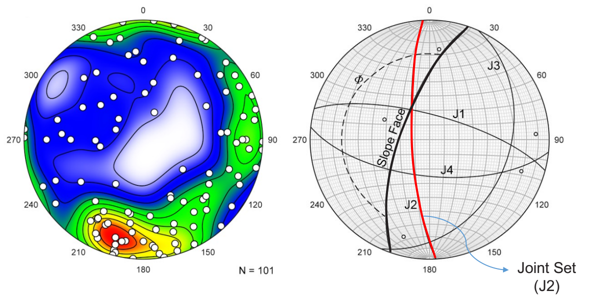

Fig. 2. Stereography projection plot for Slope 1 from Rusydy et al. [5], Joint set (J2) in red line formed the planar failure mode.

projection model (see Fig. 2). The mean of plane orientations (β_j *and* α_j) are determined from Orient software as the joint sets. Those joint sets combine with the orientation of slope (β_s *and* α_s) and friction angle (Φ) are deployed to determine the typology of failures (e.q. planar, wedge, toppling). Using the mean of plane orientation without considering the variability of those orientations is known as the deterministic approach.

Ten rock slopes have been investigated by Rusydy et al. [2] and Rusydy et al. [5], employing the scan-line method. Four from ten slopes are potentially had planar failure refer to deterministic kinematic analysis. Nonetheless, this paper merely discusses one slope as an example to introduce the probabilistic approach in analysing the planar failure probabilities.

Planar failure is a common failure typology in rock slope stability study; nevertheless, it requires numerous geometric circumstances to occur. Wyllie and Mah [18] noted that the planar failure occurs when the dip direction of joint planes (α_j) are within ±20° to the slope face (α_s). The following condition to planar failure to occur is when the dip of joint (βj) is lower than slope angle (βs), yet it must be higher than friction angle (Φ), or in another way it can be written as $\Phi < \beta j < \beta s$. The last circumstances is a present of tension crack in the upper part of the slope, as denoted in Fig. 1. In this study, those circumstances play a crucial role as the boundaries in determining probabilistic of planar failure occurrence except the presence of tension crack due to it is undefined in the field.

2.2 Probabilistic kinematic analysis

Probabilistic analysis is performed when the field data are insufficient or burdensome to assign a single value to calculate the model. Probabilistic describe the degree of belief in the truth of circumstances [19]. Hence, this study revealed probabilistic values between 0 and 1 to express the degree of belief for planar failure to occur. Obregon and Mitri [20] argued that the probability of failure for slope stability could be computed by encountering two models (series and parallel) influenced by numerous parameters. The series model is described as a single line from starting point to the end. If one parameter fails, it could trigger the rock slope failure. While in the parallel system, it requires all failure components to trigger the rock slope failure. The

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planar failure in rock slope is more likely similar to the parallel system requiring at least two geometric circumstances to slope to have planar failure as mentioned previously ($\Phi < \beta j < \beta s$) as the first component and the second component is dip direction (α_j) within ±20° to the slope face (α_s), please see Fig. 3. According to Obregon and Mitri [20], the probability of all components are expressed as $P[p_1 \cap p_2 \dots \cap p_N]$. It means the failure could happen in a parallel system when all components fail, and in probability analysis, they have an intersection relationship.

The probabilistic of failure (*Pf*) for parallel system failure in rock slope can be simplified as denoted in Equation 1. This rule assumes all the parameters (*N*) governing the rock failure are independent and has their probabilities (*Pi*).

$$Pf = \prod_{i=i}^{N}(Pi) \quad (1)$$

Naturally, the values of dip direction (α_j), dip (βj), and friction angle (Φ) vary from site to site, from point to point while the slope angle (βs) and slope face (α_s) have a single value. The variability α_j, βj, and Φ lead to uncertainty; accordingly, the probabilistic kinematic analysis approach is necessary to cope with this variability. The probabilistic density function (PDF) of α_j and βj were generated at the first stages utilising the R programming software, while the Φ is assuming as a single value. The example of PDFs and histograms in this study is denoted in Fig. 4.

After the dip and dip direction PDFs are developed, the kinematic instability is quantified as the probability of pole or plane triggering the failure mechanism. The probabilistic of planar failure occurrence (P_{op}) is written in Equation 2 as suggested by Obregon and Mitri [20].

$$P_{op} = Pr[\Phi \le \beta_j \le \beta_s].Pr[\alpha_s - 20^o \le \alpha_j \le \alpha_s + 20^o] \quad (2)$$

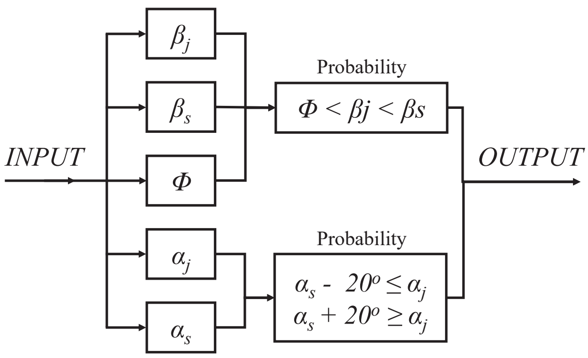

Fig. 3. Parallel system for planar failure in this study

2.3 Monte carlo simulation

The Monte Carlo simulation was run in this study to generate large numbers of dips and dip directions. This simulation is the most common approach in stochastic analysis in that the random values to the model is generated from sampled statistical distribution or frequency distributions like histograms and probability density function (PDF) [21,22].

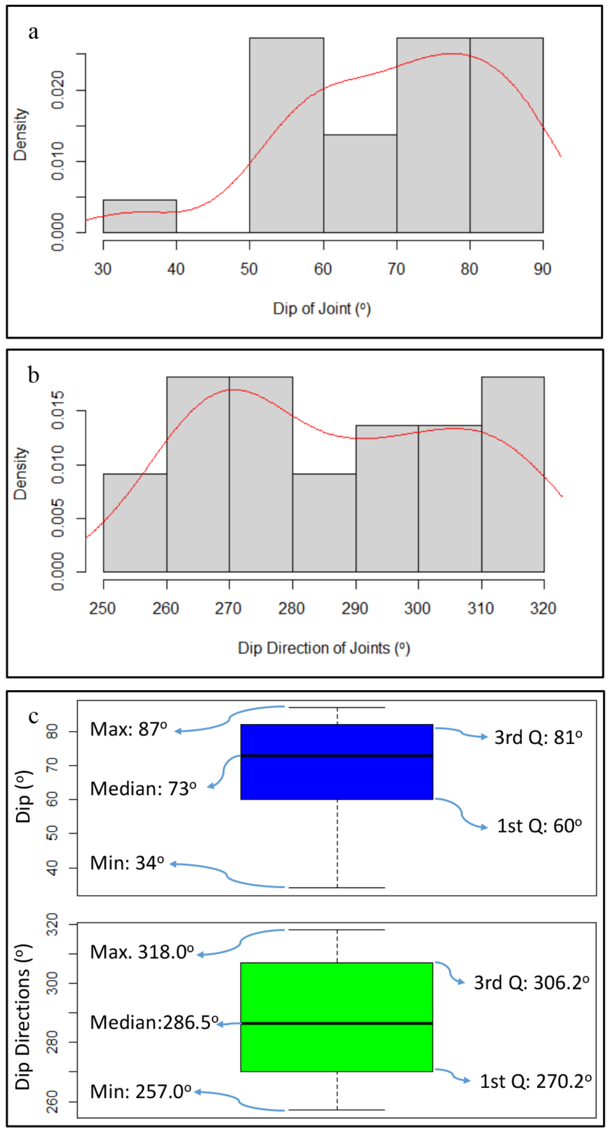

Fig. 4. (a) The Histogram, (b) PDF, and (c) Boxplot of joint geometries for Slope 1 joint set 2 (J2).

The number of observations for the joint set (J2) is merely 22 data, and it is insufficient to perform the probabilistic analysis directly from the PDF of field observed data. Thus, the Monte Carlo simulation was performed to generate large new random values based on statistic characterisation from field observed data (βj, α_j) distributions (see Fig. 3). 100 new data have been generated in 100x iteration for dips and dip directions using R data analysis software. As denoted in Fig. 4, the dip of joints has a Beta distribution typology while the dip direction is recognised as Normal distribution typology. In performing the simulation for Beta distribution, it requires the *Shape1*, and *Shape2* from observed data. The EnvStats package in R programming software developed by Millard [23] was employed to determine the *Shape1* and *Shape2* values from field observed data. Due to the dip direction data distribution is assumed as a normal distribution, it merely requires the mean (μ) and standard deviation (σ) values.

3 Results

The observed data distributions are the key in developing the experimental model utilising the Monte Carlo simulation; accordingly, the result of experimental models varies from 100x of iteration, and probabilistic kinematic analysis for J2 discusses.

3.1 The experimental models

The EnvStats R package was employed for Beta distribution and revealed the *Shape1* value for a dip of joints is 7.753, and *Shape2* is 3.324 from observed data (*J2*). Furthermore, the dip direction was recognised as Normal distribution, having 287.136 of μ and 20.185 of σ. The experimental distribution was developed by employing those statistical parameters.

In developing experimental distribution, 100 times iteration was run for 100 data, and all those simulations are plotted in one histogram and PDF as denoted in Fig. 5. Even though the range of the dip from the observed data is 34 to 87, the Monte Carlo simulation reveals bit higher data than expected. Yet, this simulation result is reliable, and the first quarter (*Q1*) and the third quarter (*Q3*) are similar to observed data. The distribution of dip direction experimental histogram and PDF is considered as normal distribution where the median and mean value are almost likely similar.

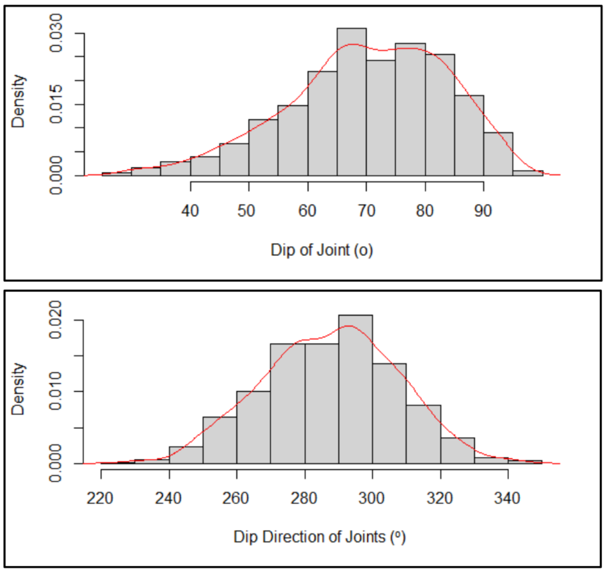

Fig. 5. The Experimental Histogram and PDF joint geometries for Slope 1 joint set 2 (J2) with N=100 and iteration 100 times. The Dip has Beta distribution typology, whilst the Dip direction has Normal distribution.

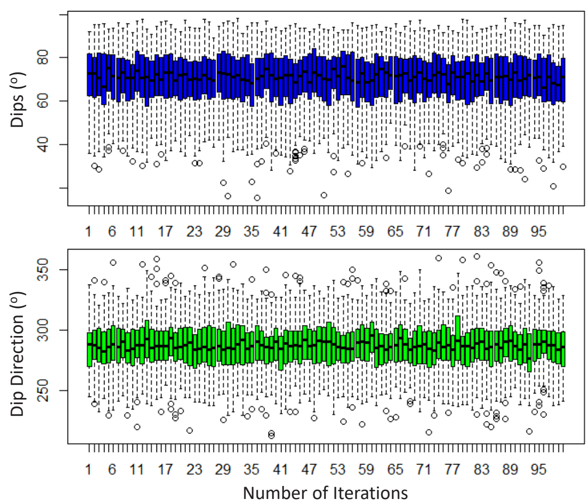

Fig. 6. Box plot of experimental data distributions from 100x iterations for dip and dip direction simulations

Fig. 6 denotes the boxplots of experimental data in a different iteration, the number of iteration as shown in the x-axis. The boxplot reveals the minimum, *Q1*, median, *Q3*, and maximum values sequentially from the bottom to the top. From 100 iteration, the median values of dip vary from 66.41° to 75.51°, while the actual median value from field data is 73°. The field observed median value for dip direction is N 286.5°E, whilst the modelled values fluctuate in 100 times iteration from N 276.9°E to N 295.3°E as shown in Fig. 6.

3.2 Probabilistic kinematic

After all simulated models reveal reliable results, the next stage is computing the probabilistic planar failure occurrence (P_{pf}) employing Equation 2. The geometric circumstances for planar failure are played an essential role as the boundaries in calculating the probabilistic to dip and dip direction in the PDF model.

According to Rusydy et al. [5], the investigated slope has 27° of friction angle (Φ) and the slope angle (βs) is 70°; accordingly, those data are utilised as boundaries in a dip of joint PDF curve to estimate the area between them. The area of 0.47 represents the probability of the first component as denoted in Fig. 7a; obviously, this area is calculated in R programming software. The second component as shown in Fig. 7b, 0.56 probability, act as the probability of dip direction (α_j) toward ±20° to the slope face (α_s). In this research, the slope facing to N 289° E; means any joints having

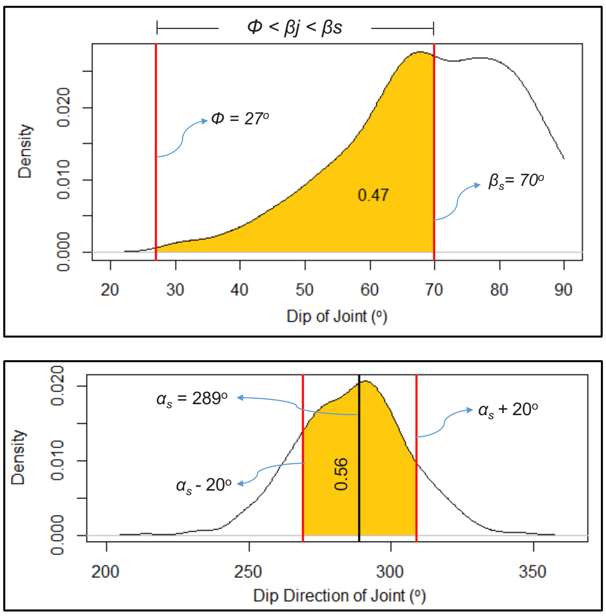

Fig. 7. The Experimental PDFs for a). Dips, b). Dip directions for Slope 1 joint set 2 (J2).

dip direction ≥ N 268° E to ≤ N 309° N are categorised as planar failure potential.

Table 1. Summary of fitted parameters for joint orientations

Data	Geometry	Distributions	Parameters*
Observed Data	Dip	Beta	Shape1 = 7.753 Shape2 = 3.324
	Dip Direction	Normal	Mean = 287.136 Sd = 20.185
Experimental Data	Dip	Beta	Shape1 = 7.500 Shape1 = 3.193
	Dip Direction	Normal	Mean = 286.56 Sd = 19.787

Note: Sd: Standard Deviation*

The connection between the first component and second component is recognised as an intersection probability relationship. The total probability of two components (0.47 and 0.56) are computed by multiplying those components; hence, the probabilistic of planar failure occurrence (P_{op}) in this study is 0.26. All parameters are employing in simulation as denoted in Table 1.

4 Conclusion

The slope in this study area has structural geology features from bedding, joints, and minor fault crossing the rock faces. The planar failure mainly occurs due to the dip direction of joints higher or lower than 20° from the slope face, and the dips are higher than the internal friction angle yet lower than the slope angle. Those geometric rules for planar failure obviously can be utilised as the boundaries in calculating the probability in PDF curves either for observed data or experimental/simulated data.

The experimental PDFs were developed based on the statistical parameters from observed data, and they must be scrutinised carefully to produce reliable results. This study identifies two typologies of data distributions, beta and normal which are required *shape1*, *shape2* for beta and mean, the standard deviation for normal distribution.

This merely conducted probabilistic kinematic analysis for planar failure in one rock slope. A multitude of rock slopes needs to be investigated by applying probability analysis to deal with the uncertainty and variability of rock properties. It is suggested to conduct more comprehensive research and cover more rock slopes in the future to mitigate rock slope failure alongside roads or probably in mining areas.

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