Reliability analysis of concrete beams reinforced with FRP bars

Firass Mohamad[1*], *Moussa* Leblouba[1], *Samer* Barakat[1], *Zaid A.* Al-Sadoon[1], and *Mohamad* Taklas[1]

[1]Department of Civil and Environmental Engineering, University of Sharjah, P.O. Box 27272, Sharjah, UAE

Abstract. The strength reduction factors (ϕ) adopted by ACI-440.1R-15 standard code for the flexure design of Fiber Reinforced Polymers Reinforced Concrete (FRP-RC) beams are calibrated in this study. The code calibration process aims to achieve a particular target reliability index (β_T) for the selected FRP-RC rectangular beams. An extensive experimental database is collected and used in the calibration process of the resistance factors. It was found that the calibrated ϕ led to a more economical and less conservative design while satisfying the safety requirements imposed by the design code. To meet the β_T, a recommended value of ϕ, is found to be 0.70 for cast-in-situ beams failing in compression and 0.65 for tension-controlled cast-in-situ beams. Thus, the proposed values are slightly greater than comparable factors adopted by the ACI code of 0.65 for compression-controlled and 0.55 for tension-controlled cast-in-situ beams.

1 Introduction

The conventional steel reinforcement's deterioration in the concrete is one of the major issues in the failure of (RC) structures, especially when the structural elements get exposed to harsh environments such as inland water crossings and marine locations where the water is acidic. In such situations, the cracks begin to form and create paths for the agents of the aggressive environments until it reaches the steel reinforcement bars. At that time, the corrosive oxidation process takes place [1]. To overcome this issue, an innovative approach in which the conventional steel reinforcement bars are replaced with Fiber-Reinforced Polymer (FRP) reinforcing bars. As per ACI-440.1R-15 [2], FRP materials are anisotropic characterized by high tensile strength only in the direction of the reinforcing fibers. Therefore, FRP materials are not yielding; instead, they are elastic until failure.

The flexural behavior of FRP-RC members and the material's modes of failure, according to the study done by Fesser and Brown [3], can only be brittle. Therefore, in concrete internally reinforced by FRP bars, the concrete may be over-reinforced by making sure that the provided amount of reinforcement will lead to concrete crushing (compression-controlled) before FRP rupture, and in this case, the section will be compression-controlled.

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*Corresponding Author: U19104411@sharjah.ac.ae

Under-reinforced (tension-controlled) sections are also permitted, but the failure mode is much different from steel-reinforced concrete. Tension-controlled failures result in FRP rupture before concrete crushing, and the failure may be catastrophic due to the linear to elastic failure of the FRP material. Also, if the FRP reinforcement ruptures, sudden and catastrophic failure can occur. Therefore, concrete crushing has typically been considered to be the more desirable failure mode.

Abed and Alhafiz [4] found that adding the Basalt Fiber-Reinforced Polymer (BFRP) reinforcement ratio leads to improving the flexural capacity of BFRP beams, regardless of the type of concrete used, whether it is Fiber-Reinforced Concrete (FRC) or ordinary Portland Cement Concrete (PCC). Also, it was observed that for the same reinforcement ratio, the capacity and stiffness of the reinforced concrete (RC) members were approximately the same when adding more longitudinal BFRP bars.

Several studies conducted reliability studies on concrete members reinforced with fiber-reinforced polymers (FRP). One of the studies was done by Carol Shield [5], and it investigated the concrete flexural member reinforced with fiber-reinforced polymers (FRP) structural reliability. Also, equations from ACI 440.1R-06 [6] of sections subjected to flexure were used to obtain the reliability indices that apply specific load factors. Similarly, resistance factors for flexure were obtained based on ACI 440.1R-06 [6]. The reliability indices used were between 3.5 and 4.8, and it was evident that previous versions of the studied codes lead to higher reliability. It was also concluded that curvature analysis of beams at failure demonstrated that flexural members fail by concrete crushing (Compression-Controlled) have almost the same amount of ductility as those members that fail by FRP reinforcement rapture (Tension-Controlled). Behnam and Eamon [7] performed a reliability-based design optimization (RBDO) to find the lowest cost Ductile Hybrid Fiber-Reinforced Polymers (DHFRP) bar configurations by strengthening the basic constraints. Different applications are considered in the study as bridge decks and building beams with different material bars. It was concluded that optimal bar configuration changes based on the application were conducted, but overall, as the number of bar materials increases, the optimized bar cost decreases. It is also found that DHFRP-reinforced flexural members' reliability is $\beta = 3.8 - 3.9$, which is higher than the conventional RC with steel bars which was considered in the study to be $\beta_T = 3.5$ which shows that the ϕ adopted by ACI-440.1R-15 [2], which is 0.55, is so conservative.

2 FRP-reinforced concrete ACI 440.1R provisions

As per ACI-440.1R-15 [2], the design of RC sections usually ensures tension-controlled behavior controlled by yielding steel before the concrete's crushing. The steel yielding gives a warning before the failure of the beam. The compression-controlled mode of failure is preferable for FRP-reinforced flexural sections.

Comparing the balanced reinforcement ratio with the FRP reinforcement ratio is considered to be the controlling limit state. The balanced ratio of FRP reinforcement is calculated using its design tensile strength since FRP does not yield. The FRP reinforcement ratio and the balanced FRP reinforcement ratio can be calculated from Eqs. 7.2.1a and 7.2.1b were obtained from the ACI-440.1R-15 [2]. If the reinforcement ratio is more than the balance ratio ($\rho_f > \rho_{fb}$), the concrete crushing limit state controls. Otherwise, if ($\rho_f < \rho_{fb}$), the FRP rupture limit state controls. When $\rho_f > \rho_{fb}$, the controlling limit state is the crushing of concrete, and the stress distribution in the concrete can be approximated with the ACI rectangular stress block. The equilibrium of forces and strain compatibility Eqs. 7.2.2a, 7.2.2b, and 7.2.2c which are obtained from the ACI-440.1R-15 [2] can be derived. Substituting (a) from Eq. 7.2.2b into Eq. 7.2.2c and solving for f_f gives Eq. 7.2.2d which is obtained from the ACI-440.1R-15 [2]. The nominal flexural strength can be computed from Eqs. 7.2.2a, 7.2.2b, and 7.2.2c. At the compression-controlled failure mode, the FRP reinforcement is linearly elastic. Therefore, the stress level in the FRP can be determined from Eq. 7.2.2c since it is lower than f_{fu}. In another approach, it can be computed as shown in Eq. 7.2.2e which is obtained from the ACI-440.1R-15 [2] to replace Eq. 7.2.2a.

For a section controlled by tension, the ultimate value for this product equals $\beta_1 c_b$ and is attained when the ultimate concrete strain value 0.003 is achieved. Hence, a conservative lower bound and a simplified computation of the nominal moment of the section shall be determined as shown in Eqs. 7.2.2g and 7.2.2h which are obtained from the ACI-440.1R-15 [2]. Whereas a limit state for a concrete crushing can be estimated as per calculations, the section as constructed might not collapse accordingly. The factor of safety (ϕ) for flexure can be calculated by Eq. 7.2.3 which is obtained from the ACI-440.1R-15. This formula provides ϕ of 0.55 for the tension-controlled members and ϕ of 0.65 for the compression-controlled ones, and a linear transition is provided between the two failure modes.

3 Reliability analysis

A data of 155 FRP-reinforced rectangular beams subjected to flexure has been recorded from the literature; 43 specimens are set to be tension-controlled, and 112 ones are considered compression-controlled as per the ACI-440.1R-15 model [2]. Most of the data obtained from literature are listed as shown in Table 1.

From the experimental data gathered, a reliability analysis is conducted on the data to calibrate the ϕ provided by the ACI code, taking into account the modes of failure considered in the code.

Table 1. Recorded Data obtained from the literature

Author(s)	**Number of chosen specimens**	**Author(s)**	**Number of chosen specimens**
Lau.D., & Pam. H. J. (2010) [8]	4	Pecce et al. (2000) [21]	2
Toutanji. H. A., & Saafi. M. (2000) [9]	4	Goldston et al. (2016) [22]	6
Thériault, M., & Benmokrane, B. (1998) [10]	8	Liu et al. (2019) [23]	6
Kassem et al. (2011) [11]	12	Habeeb, M. N. (2013) [24]	4
Benmokrane et al. (1995) [12]	9	Yost et al. (2001) [25]	12
Ashour, A. F. (2006) [13]	6	Ashour, A. F., & Family, M. (2006) [26]	2
Masmoudi et al. (1998) [14]	8	Ovitigala, T. (2012) [27]	12
Barris et al. (2009) [15]	6	Mahmoud, K. A. A. (2015) [28]	17
Ashour et al. (2008) [16]	5	Alsayed et al. (2000) [29]	4
El-Mogy et al. (2010) [17]	4	Alsayed, S. H. (1998) [30]	3
Thiagaraian. G. (2003) [18]	6	Aiello, M. A., & Ombres, L. (2000) [31]	3
Benmokrane, B., & Masmoudi, R. (1996) [19]	3	Duranovic et al. (1997) [32]	4
Brown et al. (1993) [20]	5		

3.1 Statistical parameters of the professional factor (P)

The model accuracy was evaluated by computing the professional factor bias (λ_P) used in the calibration analysis. It is calculated as the ratio of the experimental moment (M_{exp}) to the predicted moment (M_{pred}) capacities computed using the ACI-440.1R-15 model, as shown in Eq. 1.

$$\lambda_P = \frac{M_{exp}}{M_{pred}} \qquad (1)$$

Table 2 demonstrates a summary of the statistical properties of λ_P. For the compression and tension controlled collected data. The ultimate limit state design approach obtained using ACI-440.1R-15 provides conservative results compared with the empirical data. In addition, rectangular beams under compression-controlled give less conservative results since the mean ($\mu = 1.115$) compared to the tensioned-controlled ones ($\mu = 1.116$). The rectangular beams Coefficient of Variation ($COV = \sigma/\mu$) for the compression-controlled data is higher than the tension-controlled comparable values.

Table 2. Statistical Properties (P)

Statistical Properties of P	Compression-Controlled	Tension-Controlled
Bias (λ_P)	1.115	1.116
Standard Deviation (σ_P)	0.208	0.201
Coefficient of Variation (COV_P)	0.187	0.180

3.2 Statistical parameters of the material and fabrication factors (MF)

The combined effect of the material and fabrication parameters used in the ACI-440.1R-15 [2] design model equations is considered a term of (MF). The flexural moment capacity equations of the design model are shown in Eq. 7.2.2e for the compression-controlled members and Eq. 7.2.2f for the tension-controlled members. The statistical parameters, λ, COV, and the Probability Distribution Type for these parameters are illustrated in Table 3, mentioning the reference for each considered parameter.

Table 3. Statistical Properties of parameters considered in the (MF)

Parameter	Bias Factor (λ)	Coefficient of Variation (COV)	Probability Distribution Type	Reference
Effective beam depth (d)	0.99	0.040	Normal	Szerszen and Nowak [33]
Beam width (b)	1.01	0.040	Normal	
Concrete compressive strength (f_c')	1.14	0.100	Normal	
Ultimate strain of concrete (ε_{cu})	1.13	0.150	Lognormal	Baji and Ronagh [34]
β_1 parameter	1.11	0.080	Lognormal	
Area of FRP bars (A_f)	1.00	0.015	Normal	Kulkarni [35]
Ultimate tensile strength of FRP bars (f_{fu})	1.10	0.130	Weibull	
Modulus of elasticity of FRP bars (E_f)	1.04	0.080	Normal	Carol Shield [5]

Using MATLAB software and the statistical parameters mentioned above, the material and fabrication factor bias (λ_{MF}) and the coefficient of variation of the material and fabrication factor (COV_{MF}) of each mode of failure for rectangular beams subjected to flexure are obtained. The summary of the results is shown in Table 4.

Table 4. Statistical Properties (MF)

Statistical Properties of MF	Compression-Controlled	Tension-Controlled
Bias (λ_{MF})	1.197	1.072
Standard Deviation (σ_{MF})	0.145	0.065
Coefficient of Variation (COV_{MF})	0.121	0.060

3.3 Statistical parameters of the resistance models (R)

Once statistical parameters of P, MF, the statistical parameters of the Resistance model of the ACI-440.1R-15 are estimated, the design model can be computed for both crushing of concrete and FRP rapture modes of failure. The Resistance bias (λ_R) follows the same lognormal distribution followed by λ_P. Using Eq. 7.2.1b and Eq. 7.2.2a, λ_R and the resistance coefficient of variation (COV_R) can be computed. The results are shown in Table 5.

$$\lambda_R = \lambda_{MF}\, \lambda_P \tag{2}$$

$$V_R = \sqrt{V_{MF}^2 + V_P^2} \tag{3}$$

Table 5. Statistical Properties (R)

Statistical Properties of R	Compression-Controlled	Tension-Controlled
Bias (λ_R)	1.335	1.196
Coefficient of Variation (COV_R)	0.223	0.190

3.4 Calibration process

Reliability analysis for the rectangular beams subjected to flexure has been performed using Monte-Carlo Simulation (MCS) via UQLAB software, following the Load and Resistance Factor Design (LRFD) approach to assessing the ϕ adopted by the ACI-440.1R-15 design model. Both crushing of concrete and FRP rapture failure modes are considered, assuming many samples were obtained from each random variable based on their probabilistic distributions. The total number of simulations needed for each run was ($N = 1 \times 10^6$). The reliability index (β) was generated for each live-to-total load ratio starting from 0 and ending with 1 with a 0.05 increment, as shown in Eq. 4.

$$0 \leq \frac{L}{L+D} \leq 1 \tag{4}$$

The β_T adopted for cast-in-place rectangular beams is 3.5. Therefore, for the rectangular beams subjected to flexure under both crushing of concrete and FRP rapture failure modes, if the ϕ adopted by the ACI-440.1R-15 design model satisfies this β_T, then their model is reliable. Otherwise, these factors need to be adjusted by performing a calibration process using reliability analysis.

For the FRP-reinforced rectangular beams having crushing of concrete failure modes (compression-controlled beams) and using reduction strength factor (ϕ) adopted by the ACI-440.1R-15 [2] provisions, the reliability indices reveal a design model of 0.65 using a range of live to total load ratios from 0 to 1. As shown in Figure 1, the reliability indices computed, using Monte-Carlo Simulation are evident. ACI-440.1R-15 adopted ϕ (0.65 – Compression-Controlled) are reaching a value of 4.0, while the β_T needed for the rectangular beams is 3.5. Therefore, the reliability analysis demonstrated that the adopted ϕ is conservative and needs to be reevaluated to provide a less conservative and more economical design. The reliability indices correspond to different ϕ were obtained to reevaluate the ϕ. **Error! Reference source not found.** illustrates the values of the β for each ϕ attempted. It can be concluded that a ϕ of 0.7 will give a β value more than the β_T of 3.5 at the practical range that falls between 0.2 and 0.5, which means that using a ϕ of 0.7, a safe and cost-effective value can be obtained.

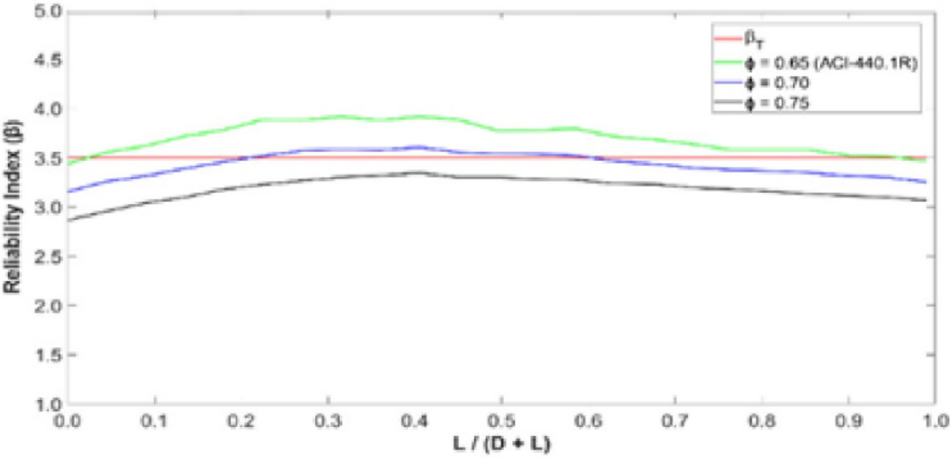

Fig. 1. Reliability indices (β) (compression-controlled).

A conclusion can be drawn using the same procedure of reliability analysis. ACI-440.1R-15 adopted ϕ (0.55 – Tension-Controlled) reach a value of 4.7, as illustrated in Figure 2. Hence, the reliability analysis demonstrated that the adopted ϕ is low and needs to be reevaluated.

Figure 2 illustrates the values of the β for each ϕ attempted. It can be concluded that a ϕ of 0.65 will give a β value more than the target reliability of 3.5 at the practical range that falls between 0.2 and 0.5, which means that using a ϕ of 0.65 results are safe and cost-effective values of β can be achieved.

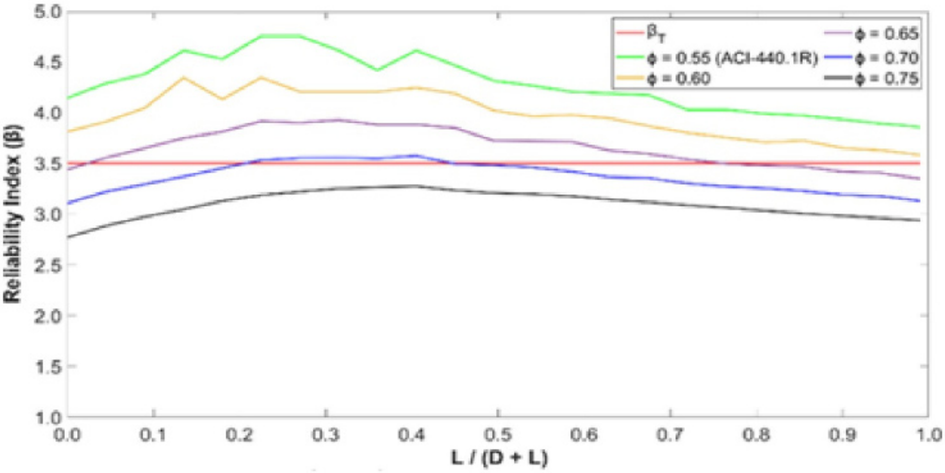

Fig. 2. Reliability indices (β) (tension-controlled).

4 Conclusions

In this study, reliability analysis for the flexure design of FRP-RC beams was performed to calibrate the ϕ adopted by the ACI-440.1R-15 code. The main outcomes of the study are summarized below. It should be noted that the proposed resistance factors are preliminary and limited to the studied data. A more data is required for better accurate estimation.

- A recommended value of ϕ for compression-controlled cast-in-situ beams is $\phi = 0.70$, which is slightly greater than the one adopted by the ACI code, i.e., $\phi = 0.65$.
- For tension-controlled cast-in-situ beams, a $\phi = 0.65$ is recommended to meet the β_T imposed by the ACI code instead of the currently used, $\phi = 0.55$.

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