Evaluating the methods for predicting permeability of unsaturated soils

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Abstract. The unsaturated permeability (kuns) is one of the key parameters in geotechnical and geo-environmental engineering involving unsaturated soils. To measured kuns in a matrix suction range of few Pa to thousands of kPa is time consuming and sometimes not practical. Thus, various methods for predicting the values of kuns were established, and their performances are quite different. The performance of two commonly used methods, i.e. Mualem's method adopting Van Genuchten's soil water characteristic curve (SWCC) (designated as MVG), and Fredlund et al.'s method (designated as FXM), have been evaluated comparing the predicted values with the measured data, which are test data from this study and the data from literature. The analysis results indicate that MVG method yielded a better prediction.

1 Introduction

Water movement in unsaturated zones is very important in geotechnical and geo-environmental engineering. Generally, the unsaturated permeability is measured experimentally or estimated from the soil water characteristic curve (SWCC). However, measuring unsaturated permeability requires a long time and special equipment [1], especially for the unsaturated permeability measurement at high suction range [2]. Therefore, estimating unsaturated permeability from SWCC have been developed, which can be divided into three categories: empirical equations [3,4], macroscopic models [5–9], and statistical models [10–12]. However, for a given soil, generally, each model results in a different estimated suction versus unsaturated permeability curve [13]. Among these models, the equations proposed by Mualem [12] and Fredlund [14] are two of the most widely used unsaturated permeability estimation models.

In this paper, performances of Mualem's [12] and Fredlund's [14] methods have been investigated by comparing the measured and predicted values of unsaturated permeability. Part of the measured data are from the test results of this study and part

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of them are from literature. Firstly, a brief description of the two prediction methods is given. Then the laboratory tests and the results, as well as comparative analysis results are presented.

2. Two prediction methods

Unsaturated permeability estimation models usually link a relative permeability (k_r) which is defined as unsaturated permeability (k_{uns}) divided by saturated permeability (ks) (kr = k_{uns}/k_s) with SWCC. Two widely used methods proposed by Mualem [12] and Fredlund [14] are employed to make a comparison. In this study, all the relative permeability equations are expressed in degree of saturation and suction since the volume change is neglected. Mualem [12] proposed a statistical model to predict relative permeability based on SWCC, it can be shown as follows:

$$k_r(S) = \left[\frac{(S - S_r)}{(S_{sat} - S_r)}\right]^{0.5} \left(\frac{\int_0^S \frac{dS}{\psi(S)}}{\int_0^{S_{sat}} \frac{dS}{\psi(S)}}\right)^2 \quad (1)$$

Where S is degree of saturation; S_r is residual degree of saturation; S_{sat} is saturated degree of saturation.

Equation (1) is usually combined with Van Genuchten's [15] SWCC equation:

$$S = S_r + \frac{1 - S_r}{[1 + (\alpha\psi)^b]^c} \quad (c = 1 - \frac{1}{b}) \quad (2)$$

Where a and b are fitting parameters, ψ is matric suction.

The combination of Mualem's [12] and Van Genuchten's [15] equation is designated as MVG method here, and the equations is expressed as:

$$k_r(\psi) = \frac{\{1 - (\alpha\psi)^{b-1}[1 + (\alpha\psi)^b]^{-c}\}^2}{[1 + (\alpha\psi)^b]^{c/2}} \quad (c = 1 - \frac{1}{b}) \quad (3)$$

Fredlund's [14] unsaturated permeability predicting equation (designated as FXM) is as:

$$k_r(\psi) = \frac{\int_{ln(\psi)}^{B} \frac{[S(e^y) - S(\psi)]}{e^y} S'(e^y)dy}{\int_{ln(\psi_{aev})}^{B} \frac{[S(e^y) - S_{sat}]}{e^y} S'(e^y)dy} \quad (4)$$

Where y a dummy variable of integration representing suction, ψ_{aev} is the air entry value, and B=ln(1000000). $S'(\psi)$ is the SWCC function proposed by Fredlund and Xing [14] with an expression of:

$$S(\psi) = \{1 - \frac{ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{1000000}{C_r}\right)}\} \frac{S_{Sat}}{\left\{ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (5)$$

Where a, n, and m are fitting parameters. Cr is a correction factor related to the suction corresponding to the residual degree of saturation of a soil, and a value of 3000 kPa was proposed by Fredlund and Xing [16].

$S'(\psi)$ is the derivative of equation (5) and can be expressed as [14]:

$$S'(\psi) = C'(\psi)\frac{S_{sat}}{\left\{ln\left[e + (\psi/a)^n\right]\right\}^m} - C(t)\frac{S_{sat}}{\left\{ln\left[e + \left(\psi/a\right)^n\right]\right\}^{m+1}}\frac{mn\left(\frac{\psi}{a}\right)^{n-1}}{a\left[e + \left(\psi/a\right)^n\right]} \quad (6)$$

Where:

$$C'(\psi) = \frac{-1}{(Cr+\psi)\ln\left(1+1000000/C_r\right)} \quad (7)$$

3 Measured data

3.1 Laboratory test

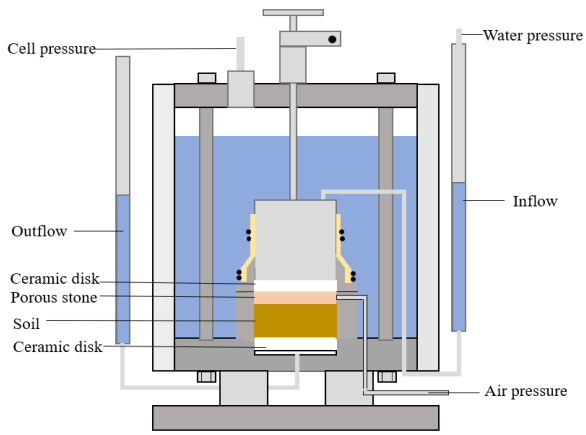


Fig. 1. Sketch of test device.

A device used for measuring k_{uns} is shown in Fig. 1. The two ceramic disks used allow water flow through them, but air cannot flow through (for air pressure less than the air entry value of the disks). The ceramic disks used had an air entry value of 300 kPa. The function of the porous stone is to ensure a uniform air pressure applied on top of a soil sample. During a test, the value of matric suction was controlled by adjusting air pressure and water pressure. The cell pressure was kept always 50 kPa higher than the applied air pressure.

Four sandy soils were prepared from decomposed granite (called Masado in Japan) by sieving it with sieves of opening size of 850 μm, 425 μm, 250 μm and 106 μm, and the soils are named as M850, M425, M250 and M106 respectively. Values of specific gravity (G_s), initial void ratio (e) and saturated permeability (k_s) of the soils are listed in Table 1 and the grain size distributions are presented in Fig. 2.

The main test procedures are as follows:

(1) Soil sample, ceramic disks and porous stone were saturated in a vacuum chamber.

(2) Soil sample was placed in the equipment and compressed under 50 kPa pressure, and then the sample was trimmed to the desired height (Height:

15 mm, radius: 30 mm). Then, the equipment was set up and all the tubes were connected.

(3) Firstly, the saturated water permeability test was conducted. The amounts of water inflow and the outflow were recorded. After the inflow and outflow reached balance. The amount of water inflow was recorded for three times for permeability calculation.

(4) The air pressure and water pressure were adjusted to measure unsaturated permeability for different suction value. The average suction was used and calculated as follows:

$$\psi = \frac{(u_a - u_w) + u_a}{2} \tag{8}$$

Where u_a is the air pressure, u_w is the water pressure.

(5) Finally, the unsaturated permeability was calculated as follows:

$$k_{uns} = \frac{\frac{a_1 L}{a_2(t_2 - t_1)} ln \frac{h_1}{h_2}}{(1 - \frac{L_E}{k_E} \frac{a_1}{a_2(t_2 - t_1)} ln \frac{h_1}{h_2})} \times \frac{1}{100} \tag{9}$$

Where a_1 and a_2 is area of inflow and outflow burette, L is the height of soil sample. L_E is the height of ceramic disk and porous stone. k_E is the equivalent permeability coefficient of ceramic disk and porous stone, which can be determined as follows:

$$k_E = \frac{L_c + L_p}{L_p} \frac{k_p}{1 + \frac{k_p}{k_c} \frac{L_c}{L_p}} \tag{10}$$

Where L_c and L_p is the height of ceramic disk and porous stone. k_c and k_p is the permeability of ceramic disk and porous stone.

Table 1. Values of specific gravity (G_s), initial void ratio (e) and saturated permeability (k_s)

Soil	G_s	e	k_s
M850	2.62	0.905	8.50E-07
M425	2.66	0.887	4.84E-07
M250	2.65	1.043	7.85E-07
M106	2.60	1.046	4.16E-07

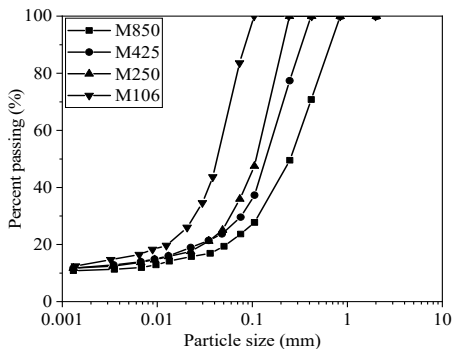


Fig. 2. Grain size distribution of tested soils.

3.2 Collected data from literature

Twelve sets of published data, i.e. SWCC and unsaturated permeability data, were collected from the literature as listed in Table 2.

Table 2. Data collected from literature.

Soil number	Soil name	k_s (m/s)	Source
S1	Volcanic Sand	8.10×10^{-5}	[17]
S2	Hygiene Sandstone	1.25×10^{-5}	[4]
S3	Mine tailings	9.19×10^{-7}	[18]
S4	Weld Silty Clay	5.67×10^{-6}	[17]
S5	Guelph loam	3.92×10^{-6}	[19]
S6	Superstition	1.83×10^{-5}	[20]
S7	Touchest silt	3.51×10^{-5}	[4]
S8	Sandy loam (2242)	4.76×10^{-5}	[21]
S9	Grenoble sand	4.28×10^{-5}	[22]
S10	Glass beads	1.15×10^{-4}	[4]
S11	Mixed sand	1.12×10^{-4}	[23]
S12	Poudre river sand	2.47×10^{-4}	[4]

4 Comparing measured and estimated results

Before making the prediction of unsaturated permeability, all the fitting parameters for SWCC were determined by employing equation (2) and (5) to fit the SWCC data. The best-fitting parameters were determined by the least square method. All the parameters for SWCC are presented in Table 3. It should be noted that residual degree of saturation is determined with a graph method [16]. After parameters for SWCC were determined, all parameters were introduced into equation (3), (4) and (5) to predict relative permeability.

Table 3. SWCC fitting parameters.

Number	Type	Fitting parameters					
		a (kPa)	n	m	α (1/kPa)	b	S_r
M1	M	5.105	4.055	0.574	0.179	2.114	0.187
M2	M	10.47	3.836	0.553	0.064	2.284	0.216
M3	M	10.104	6.967	0.438	0.074	2.579	0.216
M4	M	22.501	5.202	0.483	0.032	2.736	0.245
S1	C	1.826	10.428	0.605	0.4	4.000	0.132
S2	C	10.723	7.566	0.400	0.07	4.668	0.349
S3	C	13.699	1.984	0.996	0.069	1.906	0.089
S4	C	6.372	11.152	0.415	0.124	5.000	0.284
S5	C	6.257	2.204	0.499	0.119	2.087	0.373

S6	C	2.736	11.461	0.444	0.300	4.137	0.251
S7	C	8.191	8.690	0.482	0.095	3.747	0.227
S8	C	3.336	6.823	0.558	0.231	3.638	0.201
S9	C	2.247	2.305	1.15	0.4138	2.1972	0.051
S10	C	2.973	12	0.864	0.3091	15.625	0.035
S11	C	2.925	6.751	1.234	0.3058	5.073	0.026
S12	C	1.46	8.226	0.946	0.5756	5.214	0.035

M: Measured data, C: Collected data.

4.1 Test data from this study

The comparisons are presented in Figs. 3 (a), (b), (c) and (d) for four soils respectively. As shown in Fig. 3(a), for M850, there are discrepancies between predictions and the measured data for both models (MVG and FXM). While for M425, M250 and M106, two models resulted in acceptable predictions. However, we judge that MVG model performance better than FXM model. At low suction range, both FXM and MVG models show a similar predicted result. But a relatively large difference is observed between two models at higher suction range, and MVG predictions closer to the test data.

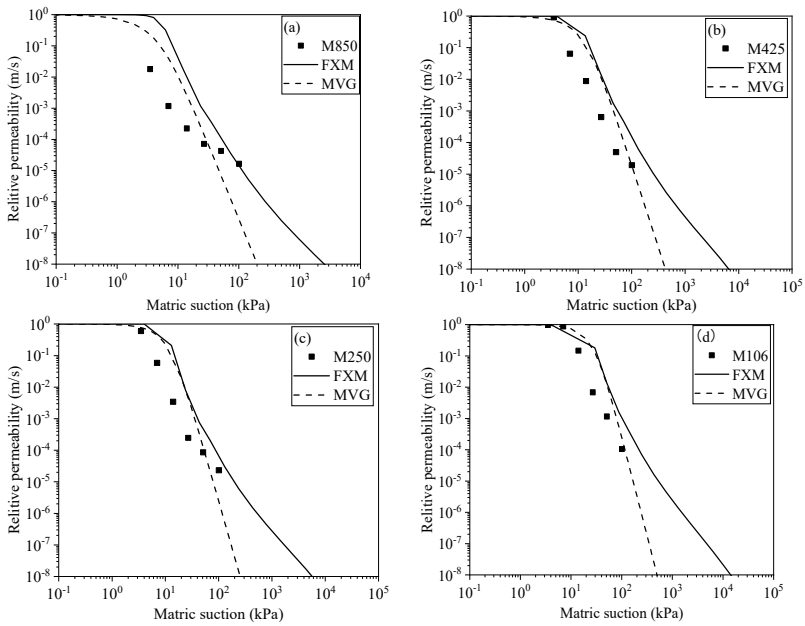
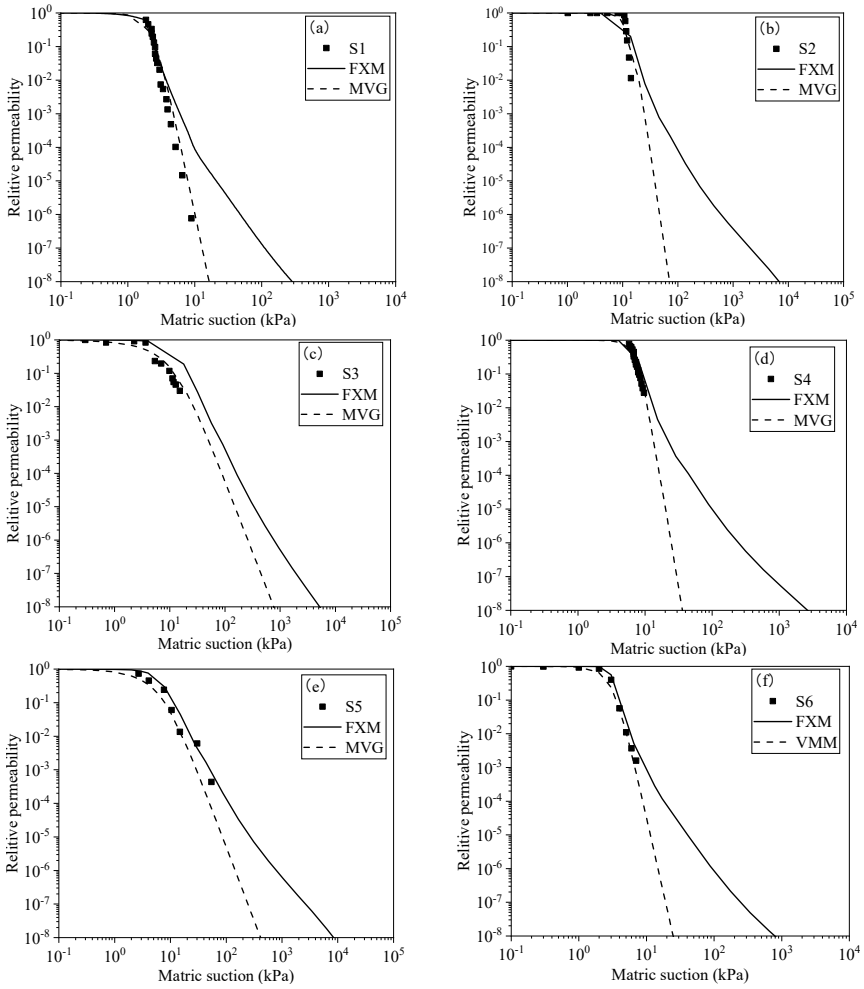


Fig. 3. Comparison of predicted k_r and test data: (a) M850, (b)M425, (c)M250, (d)M106.

4.2 Data from literature

Fig. 4 presents the comparisons of measured and predicted unsaturated permeability and matrix suction curves of twelve data sets from the literature. Overall, MVG model performed better than FXM model. One main reason for difference performance of the FXM and MVG models is that two models employ different

SWCC equations which have an important influence on unsaturated permeability estimation [24]. However, FXM model has an advantage that it can predict unsaturated permeability for a full suction range, while MVG model can only predict unsaturated permeability when the degree of saturation is larger than residual degree of saturation.



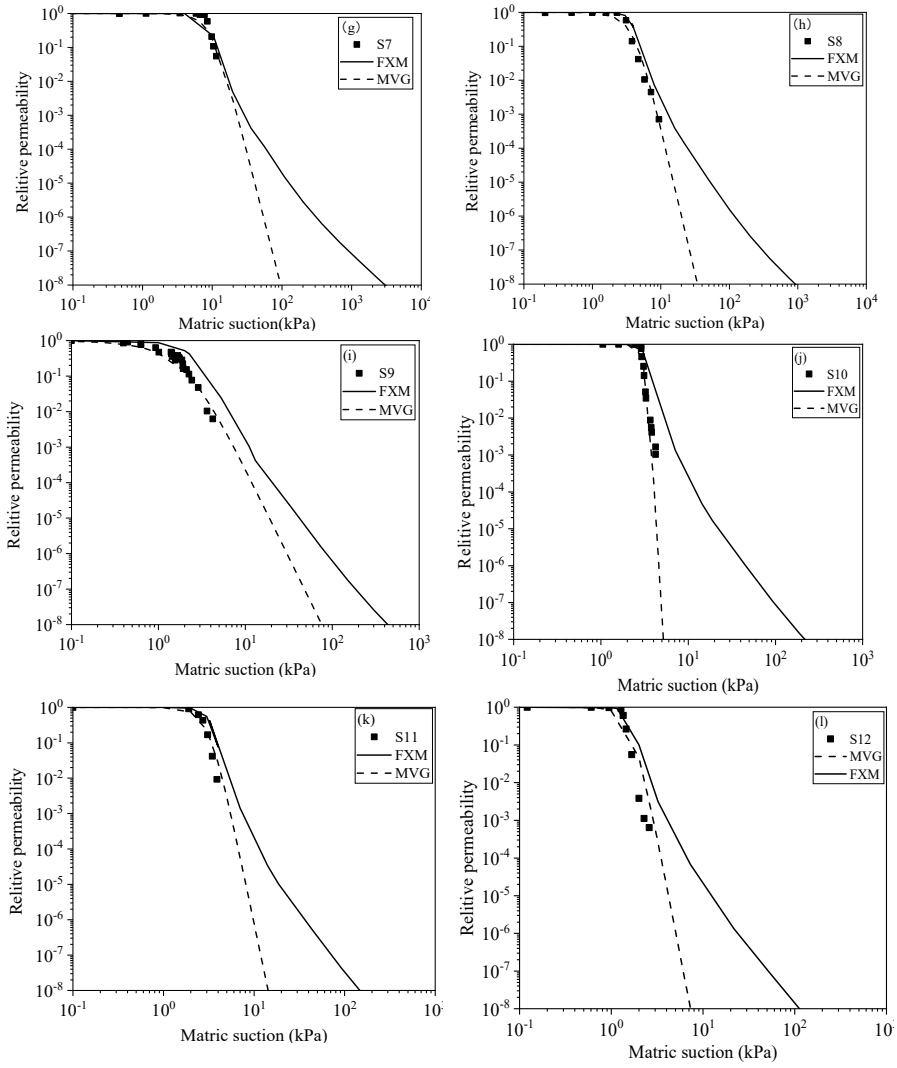


Fig. 4. Comparison of predicted k_r and collected data: (a) S1, (b) S2, (c) S3, (d) S4, (e) S5, (f) S6, (g) S7, (h) S8, (i)S9, (j)S10, (k)S11, (l)S12.

5 Conclusion

The performance of the two mostly used unsaturated permeability estimation models were investigated by comparing the predicted and measured values. The two models MVG model which combined Mualem's [12] unsaturated permeability equation with Van Genuchten's [15] soil water characteristic curve, and FXM model which was developed from Fredlund and Xing's [16] SWCC equation. Four measured data sets from this study and twelve data sets from literature were used in the comparisons. Based on the analysis results, the following conclusions can be drawn.

(1) Although both MVG and FXM models resulted in acceptable predictions, generally MVG model performed better than that of FXM model.
(2) FXM model has an advantage that it can predict unsaturated permeability for a full suction range, while MVG model can only predict unsaturated permeability when the degree of saturation is larger than residual degree of saturation.

References

1. Q. Zhai and H. Rahardjo, Eng. Geol. **199**, 148 (2015)
2. E. C. Leong and H. Rahardjo, J. Geotech. Geoenvironmental Eng. **123**, 1118 (1997)
3. G. P. Wind, Netherlands J. Agric. Sci. **3**, (1955)
4. R. Brooks and A. Corey, Hydrol. Pap. Color. State Univ. **3**, 26 (1964)
5. S. T. Yuster, in *World Pet. Congr. Proc.* (1951), pp. 437–445
6. S. F. Averjanov, English Collect. **7**, 19 (1950)
7. S. Irmay, Eos, Trans. Am. Geophys. Union **35**, 463 (1954)
8. Y. Mualem, Water Resour. Res. **14**, 325 (1978)
9. Y. Zou, Acta Geotech. **7**, 129 (2012)
10. N. T. Burdine, J. Pet. Technol. **5**, 71 (1953)
11. N. Childs, E.C., Collis-George, Proc. R. Soc. London. Ser. A. Math. Phys. Sci. **201**, 392 (1950)
12. Y. Mualem, Water Resour. Res. **12**, 513 (1976)
13. Y. Mualem, in *Methods Soil Anal. Part 1 Phys. Mineral. Methods* (1986), pp. 799–823
14. D. G. Fredlund, Anqing Xing, and Shangyan Huang, Can. Geotech. J. **31**, 533546 (1994)
15. M. T. Van Genuchten, Soil Sci. Soc. Am. J. **44**, 892 (1980)
16. D. G. Fredlund and Xing, Can. Geotech. J. **31**, 521 (1994)
17. J. D. Valiantzas, J. Irrig. Drain. Eng. **137**, 223 (2011)
18. D. W. Rassam and D. J. Williams, Geotech. Test. J. **22**, 144 (1999)
19. D. E. Elrick and D. H. Bowman, Soil Sci. Soc. Am. J. **28**, 249 (1964)
20. L. A. Richards, Proc. Int. Symp. Desert Res. Jerusalem pp. 523 (1952)
21. U.S.D.A, (1999)
22. J. Touma and M. Vauclin, Transp. Porous Media **1**, (1986)
23. O. Dury, U. Fischer, and R. Schulin, J. Contam. Hydrol. **33**, (1998)
24. A. Rahimi, H. Rahardjo, and E. C. Leong, Soils Found. **55**, 1400 (2015)