Channel Estimation Evaluation For a Massive MIMO System Considering Spatially Correlated Channels in an Urban Network

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Abstract. Channel estimation (CE) is an important process that is done during the pilot transmission phase in each base station. This work addresses this process for the massive multiple-input multiple-output systems by studying the scenario where the channels are spatially correlated. Throughout this work, the spatial correlation between channels is modeled using the exponential correlation model. The minimum mean square error (MMSE) estimator's performance for uncorrelated and correlated channels is compared and examined using the normalized mean square error (NMSE) metric, where the correlated scenario is presented through two array designs, namely proposed uniform planar array (UPA) and uniform linear array (ULA). In comparison to the uncorrelated situation, the correlated channels scenario is a more practical scenario that represents the real-world environment and provides superior channel estimate quality since the spatial correlation is advantageous for CE. Hence, we proposed a proposed UPA arrangement for correlated channels based on the Kronecker prod- uct of the ULA arrangement that outperforms the ULA arrangement and offers superior performance comparedto ULA. Numerical results are offered in order to support our analytical study.

1 Introduction

Current wireless network systems allow various advancements by implementing Massive multiple-input multiple-output (M-MIMO) technology [1] (energy efficiency, spectrum efficiency, and dependability) [2– 4], where the M-MIMO technology combined with a traditional wireless network is commonly regarded as future wireless network architecture. Each M-MIMO system is built on the usage of hundreds of antennas at the base station to serve a relatively small number of users over the same time-frequency transmission [5]. Several studies in the literature seek to show the multiple benefits of M-MIMO systems, and they are based on the assumption that the base stations (BSs) have accurate channel knowledge [5–7], which must be calculated in practice using finite-length pilot sequences (PSs)[8–10]. Unfortunately, the limited length of the coherence block also constrains the length of the PS, which in turn restricts system performance, as the pilot interference from surrounding cells makes it difficult to acquire sufficiently accurate channel estimates, resulting in a particular type of interference, namely, pilot contamination [1]. The main purpose of this work is to investigate the Channel estimation (CE) process in the uplink (UL) phase since there are also related concerns in the down-link phase that can severely restrict the performance of such M-MIMO network systems. We refer for more information on the issues incurred in the down-link to be found in [11–15].

2 Related works

This section is intended to provide some related works regarding CE in M-MIMO systems for spatially correlated channels (SCC). Several models in the literature seek to investigate spatial channel correlation, such as the exponential correlation model and the local scattering model, where when local scattering model is adopted, three distinct deviation distributions are available (i.e., Gaussian, uniformly, and Laplace distributed deviations). The uniform distribution of deviation is referred to as the one ring model, as the scatterers are seen as surrounding the user in a ring. In [6], the authors dealt withthe CE process for uncorrelated channels under selective fading model, whereby multipath channels model that is employed to link every two antennas in both ends.The authors have proposed an estimator to tackle the impractical property provided by the MMSE estimator, which is characterized by foreknowledge of large-scale fading (LSF) coefficients of interfering users. They conclude that the proposed estimator surpasses the unrealistic task caused by the MMSE estimator. In [7], the authors deal with the CE process for uncorrelated channels, where the

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authors have proposed an estimator to resolve the unrealistic property generated through the MMSE estimator, which is represented by the prior knowledge of LSF coefficients regarding the interfering users. They concluded that the proposed estimator is overcome the unrealistic task provoked by the MMSE estimator. In [16], the authors address the spatial correlation by considering the LSF variation over the grid, concluding that the capacity increases unboundedly as the number ofantennas (NoA) in the system increases regardless of whether the pilot contamination existed.

3 System Model

This section discusses the system model utilized in this study, where the channels are considered to be correlated since they describe the most realistic situation. The g_{jlk} signify the channel vector from the k_{th} user in the j_{th} cell to the N antennas at the l_{th} BS, which is defined as $\mathbf{g}_{jlk} = [g_{jlk1}, g_{jlk2} \dots, g_{jlkN}]^T \sim \mathbb{CN}(0_N, \mathbf{R}_{jlk})$, here $\mathbf{R}_{jlk} \in \mathbf{C}^{\mathbf{N}\times\mathbf{N}}$ depict a semi-defined positive channel covariance matrix. It is worth noting that $\mathbf{R}_{jlk}$ isn't a unity matrix, since it defines macroscopic phenomena such as path loss in multiple directions and spatial correlation of the channel. In the other hand, users in all cells transmit their UL PS at the same time, whereas PSs utilized in one cell are replicated in all others (i.e., giving rise in the PC issue). In other words, the reuse of frequency is one. Moreover, the PS dispatched by the k_{th} user is denoted by $\psi_k \in \mathbb{C}^{\tau}$, where τ indicates the PS length. To define a global matrix regarding each cell, we denote $\Psi \in \mathbb{C}^{\tau\times K}$ as the global matrix, which also exhibits the following feature $\Psi^H\Psi = \mathbb{I}_N$ where $\Psi = [\psi_1, \psi_2, \dots, \psi_K]$. Each PS ψ_k exhibits the following feature $\psi_k^H\psi_k = 1, \forall k$. For all users in each cell (i.e., K users), the global matrix $\tau \times K$ of the PS becomes $\Psi^H\Psi = \mathbb{I}_N$, $\forall k$, where $\Psi = [\psi_1, \psi_2, \dots, \psi_K]$. The UL signal Y_j received at the j_{th} BS can be written as

$$Y_j = \sqrt{\eta} \sum_{l=1}^{L} G_{jl} \ \Psi^H + B_j \qquad (1)$$

Here, η indicates the UL transmit power whereas, $B_j^{N\times\tau}$ represents the noise matrix, and every element of $B_j^{N\times\tau}$ follow a $\mathcal{CN}(0,1)$.

4 Channel Estimation

An acceptable and adequate statistic is used to compute the estimation of channel g_{jjk}, which may be presented as follows [17]

$$\chi_{jk} = \hat{g}_{jjk} = \frac{1}{\sqrt{\eta}} Y_j \phi_k = \sum_{l=1}^{L} g_{jlk} + w_{jk} \quad (2)$$

Here, w_{jk} is a vector originating from the product of the noise matrix with the PS of the k_{th} user divided by the square root of pilot power, η, as shown in the equation bellow

$$\boldsymbol{w_{jk}} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\eta}}} \boldsymbol{W_j \phi_k} \sim \boldsymbol{\mathcal{CN}}(\mathbf{0}_{\boldsymbol{N}}, \frac{\mathbf{1}}{\boldsymbol{\eta}} \mathbb{I}_{\boldsymbol{N}}) \qquad \mathbf{(3)}$$

4.1 MMSE-based Channel Estimation

The MMSE CE is a statistics estimator that belongs to the category of Bayesian estimators as stated in [17-19]. For computing, the estimate of the channel $\mathbf{g}_{jjk}$, the expression of the estimated vector is written as

$$\hat{\mathbf{g}}_{jjk} = \mathbf{R}_{jjk} \mathbf{\Psi}_{jk}^{-1} \chi_{jk} \qquad (4)$$

The independence between the vector estimate $\hat{\mathbf{g}}_{jjk}$ and the estimation error $\tilde{\mathbf{g}}_{jjk} = \mathbf{g}_{jjk} - \hat{\mathbf{g}}_{jjk}$ is a specific property of the MMSE channel estimator under the Gaussian model. In addition, $\hat{\mathbf{g}}_{jjk}$ and $\tilde{\mathbf{g}}_{jjk}$ are randomly distributed vectors as $\hat{\mathbf{g}}_{jjk} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{R}_{jjk} \mathbf{\Psi}_{jk}^{-1} \mathbf{R}_{jjk})$ and $\tilde{\mathbf{g}}_{jjk} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{R}_{jjk}(\mathbf{I}_N - \mathbf{\Psi}_{jk}^{-1} \mathbf{R}_{jjk}))$. So, the estimated vector $\hat{\mathbf{g}}_{jjk}$ is not correlated with the de-spread vector (i.e., received vector) χ_{jk} (i.e., $\mathrm{Cov}(\hat{\mathbf{g}}_{jjk}, \chi_{jk}) = 0$). Consequently, both the estimated vector and the estimation error vector have no influence on χ_{jk}. Using the MMSE estimator, the NMSE expression is written as follows

$$\Pi_{jk} = \frac{1}{N} \mathbb{E}\{\parallel \hat{g}_{jjk} - g_{jjk} \parallel^2\} = \frac{1}{N} Tr[\mathbf{R}_{jjk} - \mathbf{R}_{jjk} \mathbf{\Psi}_{jk}^{-1} \mathbf{R}_{jjk}] \qquad (5)$$

5 Spatial Correlation Model

The exponential correlation model (ECM) is investigated in our study for a ULA, adopting E.Bjornson's work [16]. As a result of modeling the channels correlation, the ECM can be represented as

$$[\mathbf{R}]_{m,n} = \beta r^{|n-m|} e^{i\varphi(n-m)} 10^{(f_n+f_m)//20} \qquad (6)$$

Here, β, φ, and r represent, respectively, the LSF coefficient, the Angle of Arrival (AoA) (it is considered to be in the range of $-\pi$ to π), and the correlation coefficient. In addition, $(f_n, f_m) \in f_1, \dots, f_M \sim \mathcal{CN}(0, \sigma^2)$ allows for independent LSF variations throughout the array. The ECM shown in equation (6) is adequate only for the ULA arrangements, it is not applicable for the proposed UPA arrangement. Luckily, the authors in [20] have shown that the covariance matrix of proposed UPA arrangement can be proxied using the Kronecker product of two ULA arrangements, i.e., the first one being in the horizontal dimension while the second is in the vertical dimension (VD), which allows us to write the covariance matrix of the channel for the proposed proposed UPA arrangement in the following form

$$\mathbf{R} \approx \mathbf{R}_h \otimes \mathbf{R}_v \qquad (7)$$

Here, we assume that we use a co-polarised antenna array. In addition, $\mathbf{R}_h \in \mathbb{C}^{N_h\times N_h}$ and $\mathbf{R}_v \in \mathbb{C}^{N_v\times N_v}$ represent the horizontal and vertical channel covariance matrix successively, where the NoA in horizontal and VD must obey the following constraint $N = N_h N_v$. In addition, the vertical AoA is indicated by $\phi \in [-\pi/2, +\pi/2)$, whereas the horizontal AoA is denoted by $\varphi \in [-\pi, +\pi)$.

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6 Simulation Results

In this segment, analytical results are provided to sup- port our theoretical result. Besides, since we deal with SCC, an ECM is provided to illustrate the correlation among channels [16, 21]. Duringthis section, we have taken into account that the NoA at the BS is setting to $N = 100$ (except for the result shown in Figs. (1) and (2)), whereas the standard deviation is adjusted to $\sigma = 4$ (except for the result shownin Fig. (3)) and the correlation coefficient is fixed to $r = 0.5$ (except for the result shown in Fig. (4)). In Fig. 1, we

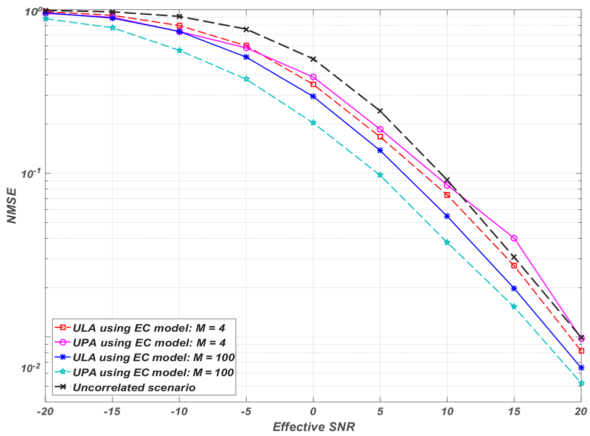

Figure 1. NMSE versus the effective SNR, η

present the NMSE versus the effective Signal-to-noise ratio (SNR), η, for different NoA at the BS (i.e., $M = 4$ and $M = 100$ only for correlated scenario). The MMSE estimator's performances are investigated for the correlated and uncorrelated situations. According to Figure (1), increasing the NoA at the BS in the ULA or proposed UPA arrangements improves the MMSE estimator's performance. In other words, when the BS is fitted with ULA or proposed UPA and the NoA at the BS increases, the performance of MMSE is improved. In addition, the proposed UPA arrangement is provided an improved performance compared to the studied situations, even if for a small NoA at the BS as the proposed UPA arrangements have a more confined configuration compared to ULA.

In Fig. 2, we show the NMSE versus the NoA at the BS, N. Accordingly, the MMSE estimator under the uncorrelated situation displays a consistent NMSE value that remains unchanged over the whole range of BS antennas examined.

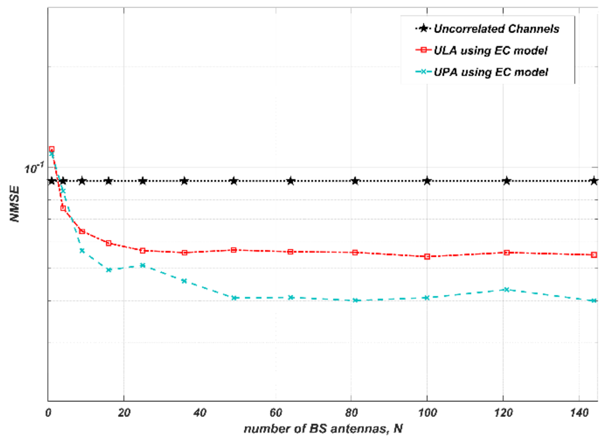

Figure 2. NMSE with respect to the number of BS antennas, N.

Furthermore, the correlatedsituation is given by two cases: In the first case, the BS is fitted with a ULA, whereas, in the second case, theBS is fitted with a proposed UPA. The MMSE estimator with proposed UPA arrangement provides better performance compared to the ULA arrangement, which can be explained by the fact that the proposed UPA arrangements have a more confined configuration compared to ULA. Onthe other hand, the implementation of the proposed UPA arrangement can provide superior performance in termsof CE compared to ULA since it may be installed on the front of enormous buildings in order to send and receive signals to all directions.

The Fig. 3 presents the NMSE performance against

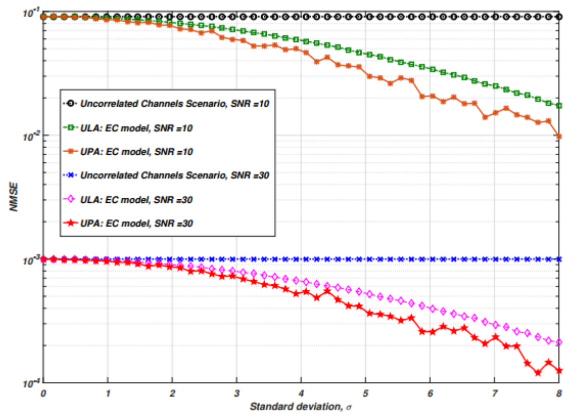

Figure 3. NMSE with respect to the standard deviation, σ.

the standard deviation, σ for two SNR value (i.e., the *SNR* is adjusted to 10 and 30 respectively). According to the figure, the MMSE estimator under the uncorrelated situation displays a consistent NMSE value that remains unchanged throughout the whole range of standard deviationvalues. Furthermore, the correlated situation is givenby two cases: In the first case, the BS is fitted with a ULA, whereas, in the second case, the BS is fitted with a proposed UPA. The MMSE estimator with proposed UPA arrangement provides better performance compared to the ULA arrangement for all standard deviation values and in both studied SNR situations. It's worth noting that a higher shadow fluctuation leads to a higher degree of spatial correlation, which makes the CE process more promotes, thereby improving the CE method.

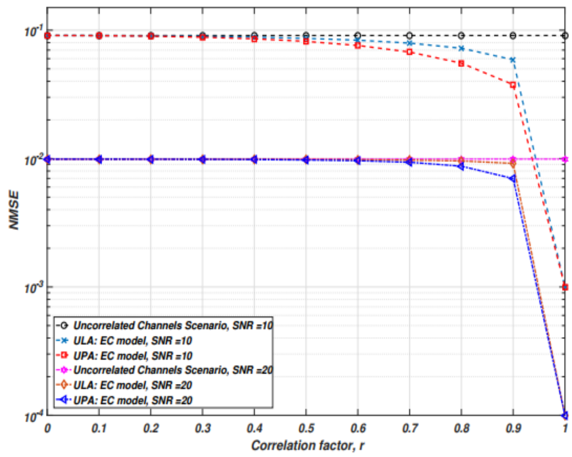

Figure 4. NMSE versus the correlation factor, *r*

The Fig. 4 presents the NMSE performance against the correlation factor, *r* for two SNR value (i.e., the *SNR*is adjusted to 10 and 20 respectively). According to the figure, the MMSE estimator under the uncorrelated situation displays a consistent NMSE value that remains unchanged throughout the whole range of correlation factor values. Furthermore, the correlated situation is givenby two cases: In the first case, the BS is fitted with a ULA, whereas, in the second case, the BS is fitted witha proposed UPA. The MMSE estimator with proposed UPA arrangement provides better performance compared to the ULA arrangement for all correlation factor values and in both studied SNR situations. The spatiality is not sufficiently strong to give any advantage to the CE processuntil it reaches a degree of correlation of about 0.6. Once the correlation factor approaches the completely correlated case (i.e., $r = 1$), a CE error two times smaller than the uncorrelated case can be achieved using ULA and proposed UPA, as can be seen in the two SNR cases studied.

7 Conslusion

This paper has addressed the CE process for the M-MIMO system by considering the scenario where the channels are spatially correlated. Specifically, this is done by modelingthe spatial correlation using the ECM. Furthermore, throughout this work, we have analyzed and evaluated the MMSE estimator's performance for uncorrelated presented through two array designs, namely proposed UPA and ULA. Conclude that the spatial correlation is advantageous for CE since the proposed UPA configuration offers superior performance compared to ULA in all studies circumstances. In addition, we concluded that the correlated situation provides a higher CE quality compared to the uncorrelated situation even if the BS is fitted with ULA only.

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