

# ANALYSIS OF THE CHARACTERISTICS OF A BIHARMONIC OSCILLATOR WITH TWO SYNCHRONOUS MODES

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**Abstract.** The paper investigates the influence of the coupling coefficient of the double-loop filter forming a biharmonic oscillator on the quality factor and resonant impedance of the input impedance at the resonant frequencies of the double-loop filter. The resonant frequencies of the filter are exactly multiples of 2. In this paper show numerical results that simplify the choice of filter parameters to provide conditions for self-excitation of two oscillations at exactly multiples of frequencies. Two synchronous modes allow to create a generator with compensation for fluctuations in the reactive component of the output impedance of the active element, which increase the phase noise of the oscillator.

## 1 Introduction

Standard methods for suppressing the phase noise (PhN) of sources of quasi-sinusoidal oscillations: improving the stability of the reactive parameters of the filter of oscillator (FO), including the active element (AE), and performing the oscillatory circuit (OC) of the FO on a higher quality resonant circuit. The method of using mutual synchronization of oscillations in a biharmonic oscillator (BHO) with multiple frequencies is less known [1]. The latter is a consequence of the poor elaboration of the design methodology for the OC BHO.

In [2] considers the case of synchronization of several oscillators in order to reduce PhN. In [3, 4] the synchronization scheme of two nanoscale spintronic oscillators was investigated, for which the possibility of PhN reduction in the phase synchronization mode was also demonstrated. Therefore, the mechanism of phase and amplitude noise transformation during synchronization of auto oscillations is of considerable interest from the fundamental point of view for oscillators of different physical nature.

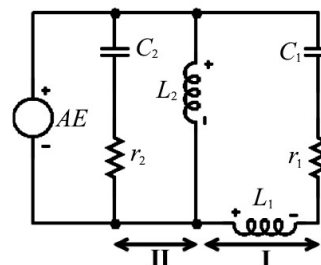
G.M.Utkin [5, 6] showed that when the frequency of the harmonic loop is tuned, the effect of pulling the frequency of the 1<sup>st</sup> harmonic occurs, which is similar to the usual synchronization process in a double-loop single-frequency oscillator. D.P. Tsarapkin [7] showed that the presence of a falling section on the frequency trapping curve can be used for nonlinear compensation of frequency deviations caused by changes in the output reactance of the AE.

The block diagram of a BHO on an active two-port network is shown in Figure 1, where the 1<sup>st</sup> circuit is determined by the elements  $L_1+L_2$ ,  $C_1$  and  $r_1$ , and the 2<sup>nd</sup>

circuit –  $L_2$ ,  $C_2$  and  $r_2$ . The purpose of this work is to study in detail the influence of the parameters of a double-loop filter with reactive coupling, which forms a typical oscillatory circuit of a BHO, on the resonant characteristics of the input impedance of the filter, that is, on the parameters of the equivalent circuit at the coupling frequency  $f_{c1} = 1/\sqrt{2\pi(L_1+L_2)C_1}$  and Q-factor  $Q_1 = 2\pi f_{c1}(L_1+L_2)/r_1$ , which determines the "fundamental" generated frequency  $f_1$ , and the circuit at the frequency coupling  $f_{c2} = 1/\sqrt{2\pi L_2 C_2}$  and Q-factor  $Q_2 = 2\pi f_{c2} L_2 / r_2$ , excited by a current with the frequency of the selected  $n^{\text{th}}$  harmonic  $f_2$ .

## 2 Results

In the synchronous mode, the BHO  $f_2 = n \cdot f_1$ . For this article, let's set  $n = 2$ .



**Fig. 1.** The block diagram of a BHO (I – main circuit, II – harmonic circuit).

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The input impedance normalized to the characteristic impedance of the 2nd circuit of such an oscillatory system is described by equation 1 [8]:

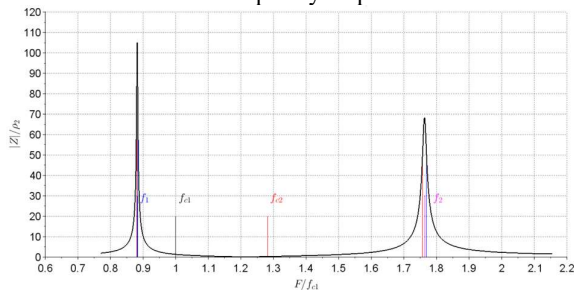
$$Z_{in}(f) = j \frac{f}{f_{c2}} \cdot \frac{\alpha_1(f) \cdot \alpha_2(f) - \alpha_1(f) \cdot j \frac{f}{f_{c1}} k - \alpha_1(f) \cdot j \frac{f}{f_{c2}} - \frac{f^2}{f_{c1} \cdot f_{c2}} k}{\alpha_1(f) \cdot \alpha_2(f) + \frac{f^2}{f_{c1} \cdot f_{c2}}}, \quad (1)$$

where  $k = L_2 / (L_1 + L_2)$  is the coupling coefficient of the circuits,  $\alpha_i(f) = \frac{1}{Q_i} + j \left( \frac{f}{f_{ci}} - \frac{f_{ci}}{f} \right)$ ,  $j = \sqrt{-1}$ ,  $Q_i$  are the intrinsic Q-factors of the resonant circuits,  $i = 1, 2$  – number of OC.

The features of the filter with multiple frequencies are illustrated in Figure 2, where the frequency response is presented for the frequency multiplicity  $n = 2$  with the coupling coefficient  $k = 0,32$ , and the values of the quality factors of the partial loops are equal to 200.

The position of the partial frequency of the "harmonic circuit"  $f_{c2} = 1,282$  is determined by fulfilling the requirement  $f_2 = 2f_1$  at the selected  $k$ . The normalized coupling frequencies are  $f_1 = 0,882$  and  $f_2 = 1,763$ , respectively. The Q-factors of resonances at coupling frequencies are determined by the level  $1/\sqrt{2}$  from the maximum value of the resistance. Qualitatively,  $Q_1 \ll Q_{c1}$ ,  $Q_2 \ll Q_{c2}/2$ . Significant degradation of  $Q_1$  in comparison with  $Q_{c1}$  is a consequence of the strong reactive coupling of the circuits under the conditions of  $Q_{c2} \ll Q_{c1}$ . This circumstance determines the direction of further calculations.

Partial frequencies are identified in Figure 2 by vertical lines with a height of 20 conventional units. This value is also used to scale the frequency response.



**Fig. 2.** The input impedance of the OS of the double-circuit filter normalized to the characteristic impedance of the 2<sup>nd</sup> circuit with  $k=0.32$ ,  $Q_1=Q_2=200$ ,  $f_{c1}=13$  MHz ( $\rho_2 = 2\pi f_{c2} L_2$ ).

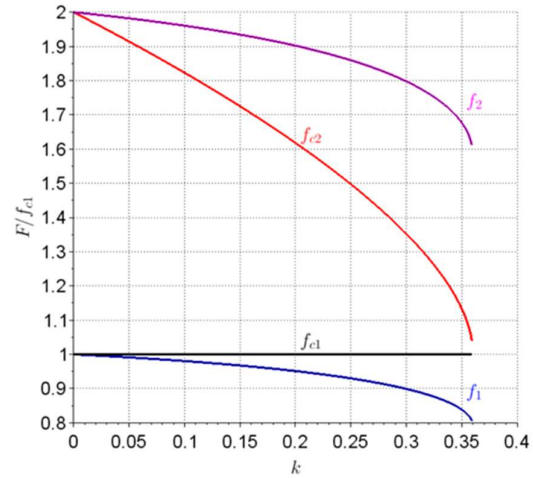
The resonant frequencies of the circuits can be found from (1) by equating the denominator to zero, they depend little on the Q-factors of the circuits, therefore:

$$f_{1/2} \approx \sqrt{\frac{f_{c1} \cdot f_{c2}}{2 \cdot (1-k)}} \cdot \sqrt{\frac{f_{c2}^2 + f_{c1}^2}{f_{c1}^2 + f_{c2}^2} - 2 \cdot (1-2k) \pm \left( \frac{f_{c2}}{f_{c1}} + \frac{f_{c1}}{f_{c2}} \right)}. \quad (2)$$

To fulfill the exact multiplicity  $f_2/f_1 = 2$  you need:

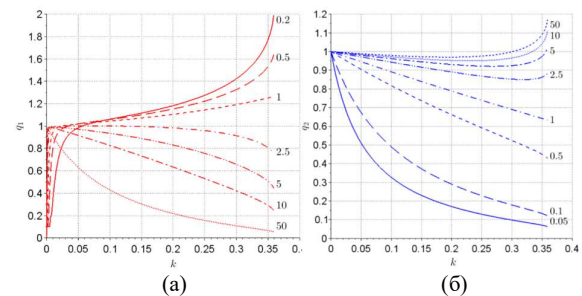
$$f_{c2} \approx f_{c1} \cdot \frac{5\sqrt{1-k} + \sqrt{9-25k}}{4}. \quad (3)$$

Figure 3 shows the dependences of the intrinsic and partial frequencies normalized to  $f_{c1}$  on the coupling coefficient. It can be seen that  $\lim_{k \rightarrow 0.36} f_{c2} \rightarrow f_{c1}$ , therefore the exact frequency multiplicity equal to 2 can be performed only for  $0 < k < 0.36$ .



**Fig. 3.** Dependences of natural and partial frequencies normalized to  $f_{c1}$  on the coupling coefficient.

Figure 4 (a) shows the dependences of the normalized Q-factor of the main loop  $q_1 = Q_1/Q_{c1}$ , and Figure 4 (b) shows the dependence of the normalized Q-factor  $q_2 = Q_2/Q_{c2}$  on the coupling coefficient of the circuits  $k$ . The parameter is the ratio of the intrinsic Q-factors of the circuits  $\chi = Q_{c1}/Q_{c2}$ .

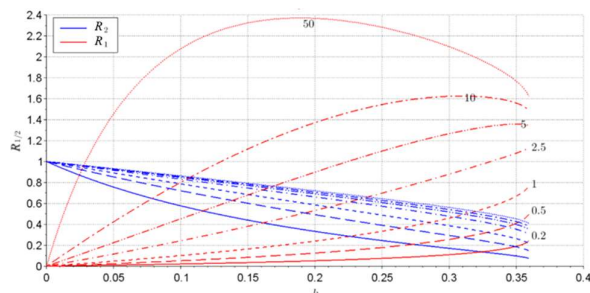


**Fig. 4.** Dependences of the normalized loaded Q-factor ( $Q_1/Q_{c1}$ ) of the 1<sup>st</sup> circuit (a) and ( $Q_2/Q_{c2}$ ) of the 2<sup>nd</sup> circuit (b) on the coupling coefficient of the circuits at different ratios of the intrinsic Q-factors of the resonant circuits ( $Q_{c1}/Q_{c2}$ ).

It follows from the graphs in Fig. 4 that for small  $\chi$  ( $Q_{c1} < Q_{c2}$ ) the loaded Q-factor  $q_1$  can be more than 1. A change in the coupling coefficient of the circuits strongly affects  $Q_1$  and  $Q_2$ . At  $\chi$  about 1, a gentle maximum of the Q-factor of the main circuit is observed. At the same time, with an increase in  $\chi$ , the Q-factor of the circuit at the lower couple frequency decreases rapidly with an increase in the coupling coefficient.

The graphs of the input resistances of the filter  $R_1$ ,  $R_2$  normalized to the characteristic impedance of the 2<sup>nd</sup> circuit at the resonant frequencies  $f_1$  and  $f_2$ , respectively, presented in Figure 5, allow you to consciously choose the factors of regeneration of the BHO by two modes. Increasing  $Q_1$  in order to increase the Q-factor of the oscillatory system of the BHO in the fundamental

harmonic (formally, an increase in  $\chi$ ) modifies the  $R_1(k)$  dependence, but has little effect on the  $R_2(k)$  function. As a rule, we have  $R_1 > R_2$ .



**Fig. 5.** Dependence of resonant resistances at coupling frequencies on the coupling coefficient of the circuits

Similar calculations performed at a ratio of frequencies of the used harmonics of 1: 3 gave similar results. As a useful difference from the case  $n = 2$ , we can note here a noticeable weakening of the negative influence of the coupling of the circuits on the Q-factor of the resonance corresponding to the lower coupling frequency.

### 3 Conclusion

This paper presents the results of calculating the characteristics of a double-loop OC: resonance resistances, coupling frequencies and loaded Q-factors for the two modes, if the condition of exact multiplicity of coupling frequencies is fulfilled. The influence of the coupling coefficient on the listed parameters of the oscillating system is analyzed.

The analytical forms of dependences of the 2<sup>nd</sup> circuit eigenfrequency on the coupling coefficient and the main circuit eigenfrequency, as well as the coupling frequencies, were found to fulfill the condition of exact multiplicity of coupling frequencies.

Numerical results were obtained for the influence of the coupling coefficient and the ratio of intrinsic quality of the circuits on their loaded Q-factors and resonant impedances.

In the future, it is planned to analyze the influence of the coupling coefficient of the circuits on the phase noise of the biharmonic oscillator.

This work will make it possible to make measurements more accurately, thereby reducing the likelihood of false-positive and false-negative alarms of measuring instruments that can be used in certain anthropogenic spheres of activity.

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