Research on off-the-grid energy storage system

Tianlun Ni

Suzhou High School of Jiangsu Province, Jiangsu, China

Abstract. To enable people in remote areas to meet their energy needs during cloudy days or at night, our team decided to tailor an off-the-grid energy storage system for them. To allow this energy storage system to meet the energy usage of a 1,600-square-foot family, we comprehensively consider the factors about continuous power rating, instantaneous power rating, usable capacity, round-trip efficiency, lifetime of battery, and price of batteries to maximize efficiency and lower costs. In the modeling process, the mathematical method we use is the Analytic Hierarchy Process. Verify whether the matrix passes the consistency test by calculating the eigenvalues and eigenvectors of the matrix, and get a score for each battery. Among the five batteries, we found that the battery Deka Solar 8GCC2 6V 198 can best meet our needs. Then, to perfect the model, we added a new consideration, which is personal preference, including all the subjective factors that would affect people’s choice. After incorporating this new consideration into our original model, the rankings of two of the five batteries have changed. However, despite these slight shifts, battery Deka Solar 8GCC2 6V 198 is still our best choice. Nowadays, researchers have discovered some new types of batteries to store energy, such as cement batteries, and this new type of battery has some unique advantages. We compare currently available batteries with the cement batteries, and discover that both batteries have their pros and cons. We believe that with the development of technology, cement batteries will have a very broad prospect and can even surpass currently available batteries.

Keywords: AHP, Impact factor, Off-the-grid energy, Geometric method.

1. Background

Our team is helping the 1600-square-foot homes in remote areas to choose batteries suitable for them to implement an off-the-grid energy storage system. The system could store enough energy and meet the requirements through capturing electricity and store them into other forms of energy. This choice requires consideration of various factors in order to select the best solution, which is able to provide those homes with sufficient energy and can even continue to work on a cloudy day or at night. To solve the power supply problem, our team will choose the most suitable battery from five types. The method we chose is analytic hierarchy process (AHP).

2. Selection of Batteries

Our team is helping to plan the use of solar power to provide electricity to a 1600 square-foot home being built in a remote area. As the house is in a remote area, the cost of connecting to the grid is very expensive, so we decide to go off-the-grid and invest in energy storage. To solve this problem, we did the following work:
(1) To solve the problem 1, we took five batteries and established six factors. Based on AHP, the weight of six factors is compared to establish consistency matrix and carry out consistency test. The final score for each of the five batteries is obtained.
(2) To solve the problem 2, we added a new factor -- personal preference to make our model satisfy personal preference. Then based on the analytic hierarchy process, we got new scores for five batteries.
(3) To solve the problem 3, we compared cement and lithium batteries, analyzed their advantages and disadvantages, and gave a general introduction to their application scenarios.
(4) To solve the problem 4, for convenience, we made a one-page non-technical news article of our research content and results. Finally, we decided that battery Deka Solar 8GCC2 6V 198 was the best choice.

3. Analytic Hierarchy Process

Analytic Hierarchy Process is a simple method for making decisions on vague problems, and it is especially suitable for problems that are difficult to fully quantitatively analyze. It is a simple, flexible, practical and multi-criteria decision-making method proposed by American operations researcher Professor T. L. Saaty in the early 1970s [1].
The use of analytic hierarchy process modeling to solve problems can generally be carried out in the following four steps.

(a) First, establish a hierarchical structure model. When applying AHP to analyze decision-making problems, the problem must be organized and hierarchized first, and a hierarchical structural model must be constructed. These levels can be divided into three categories: the highest level (target level), the middle level (criteria level), and the lowest level (program level). The number of layers in the hierarchical structure is related to the complexity of the problem and the level of details that need to be analyzed. Generally, the number of layers is not limited. There are generally no more than 9 elements dominated by each element in each level.

(b) Second, construct all judgment matrices in each level. Each criterion in the criterion layer does not necessarily have the same weight in the target measurement. In the minds of decision makers, each of them occupies a certain proportion, and the numbers 1-9 and their reciprocals are used as a scale to define the judgment matrix. Different scales represent different importance.

(c) The third is the level list sorting and consistency check. Calculate the consistency index CI, and find the consistency index RI next. Then, calculate the value of the consistency ratio CR by dividing CI by RI. When CR<0.10, it is considered that the consistency of the judgment matrix is acceptable, otherwise, the judgment matrix needs to be appropriately modified.

(d) The final step is to perform hierarchical total ranking and consistency test. Finally, each element, especially the sorting weight of the target in the lowest layer of the scheme, is obtained, and the scheme selection is carried out. In addition, the overall ranking of the hierarchy also needs to be tested for consistency, to calculate the composite weight of the elements of each layer to the total goal of the system, and to sort the selected schemes. To choose the suitable battery among the five batteries, we refer to a few of the most common decision criteria and establish consistency matrix. Finally, the suitable battery is obtained by consistency sorting. The types of batteries are denoted by a, b, c, d and e.

### 4. Symbol Description

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Deka Solar</td>
<td>8GCC2 6V 198</td>
</tr>
<tr>
<td>b</td>
<td>Trojan L-16</td>
<td>-SPRE 6V 415</td>
</tr>
<tr>
<td>c</td>
<td>Discover AES</td>
<td>7.4 kWh</td>
</tr>
<tr>
<td>d</td>
<td>Electriq PowerPod</td>
<td>2</td>
</tr>
<tr>
<td>e</td>
<td>Tesla Powerwall+</td>
<td></td>
</tr>
<tr>
<td>$F_1$</td>
<td>Continuous power rating</td>
<td></td>
</tr>
<tr>
<td>$F_2$</td>
<td>Instantaneous power rating</td>
<td></td>
</tr>
<tr>
<td>$F_3$</td>
<td>Usable capacity</td>
<td></td>
</tr>
<tr>
<td>$F_4$</td>
<td>Round-trip efficiency</td>
<td></td>
</tr>
<tr>
<td>$F_5$</td>
<td>Lifetime of battery</td>
<td></td>
</tr>
<tr>
<td>$F_6$</td>
<td>Price of batteries</td>
<td></td>
</tr>
<tr>
<td>$F_7$</td>
<td>Personal preference</td>
<td></td>
</tr>
</tbody>
</table>

### 5. Solution of problem 1

#### 5.1 Mathematic model

According to the official statistics, the average number of people per family in the United States in the last ten years is between 3.12 and 3.18 [2]. So, there are approximately three people in one typical family. Thus, we can assume that three people in this home will be using energy. Energy is indispensable in people’s daily life at home, and almost everything in the home need energy including items such as heating, hair dryers, washers, air conditioners, ovens, etc. [3]. Different items need different levels of energy, and the Table 1 shows some of their respective energy requirements.

<table>
<thead>
<tr>
<th>Number</th>
<th>Types of items</th>
<th>Energy requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Heating [4]</td>
<td>1.5 kilowatts per hour</td>
</tr>
<tr>
<td>2</td>
<td>Hair dryer</td>
<td>1.2 kilowatts per hour</td>
</tr>
<tr>
<td>3</td>
<td>Clothes washing machine</td>
<td>Between 2.3 kWh per load and 6.3 kWh</td>
</tr>
<tr>
<td>4</td>
<td>Air conditioner</td>
<td>Between 2,000-4,000 watts or .2556 kWh or .5132 kWh</td>
</tr>
<tr>
<td>5</td>
<td>Oven [5]</td>
<td>2.3 kWh</td>
</tr>
</tbody>
</table>

However, people don't use electrical appliances at home all the time of day. The traditional American business hours are 9:00 a.m. to 5:00 p.m., Monday to Friday [6], and a new report released by the U.S. Census Bureau in 2021 shows the average one-way commute in the United States increased to a new high of 27.6 minutes in 2019 [7]. So, people probably stay at home only from 5:30 p.m. to 7:30 a.m. every weekday. And during this period of time, they will use energy in the home. There are several important factors to consider when deciding the "best" battery storage system, and we list six considerations as shown in the Table 2 below, using $F_1$, $F_2$, $F_3$, $F_4$, $F_5$ and $F_6$ to represent them separately.

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Continuous power rating</td>
<td>$F_1$</td>
</tr>
<tr>
<td>2</td>
<td>Instantaneous power rating</td>
<td>$F_2$</td>
</tr>
<tr>
<td>3</td>
<td>Usable capacity</td>
<td>$F_3$</td>
</tr>
<tr>
<td>4</td>
<td>Round-trip efficiency</td>
<td>$F_4$</td>
</tr>
<tr>
<td>5</td>
<td>Lifetime of battery</td>
<td>$F_5$</td>
</tr>
<tr>
<td>6</td>
<td>Price of batteries</td>
<td>$F_6$</td>
</tr>
</tbody>
</table>

By comparing the relative importance of different factors, we determine the weight of each factor. When the two factors are compared, there are different scales to show their relative importance, which is shown in the Table 3 below.
Table 3. Different scales

<table>
<thead>
<tr>
<th>Scale</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The two factors are equally important</td>
</tr>
<tr>
<td>3</td>
<td>One factor is slightly more important than the other</td>
</tr>
<tr>
<td>5</td>
<td>One factor is obviously more important than the other</td>
</tr>
<tr>
<td>7</td>
<td>One factor is strongly more important than the other</td>
</tr>
<tr>
<td>9</td>
<td>One factor is extremely more important than the other</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>The two factors are equally important</td>
</tr>
</tbody>
</table>

We set the consistency matrix $A$ formed among the six indicators, showing the relationship between each factor, which is shown below.

$$
A = \begin{pmatrix}
1 & 5 & 9 & 3 & 7 & 2 \\
1/5 & 1 & 7 & 1/3 & 4 & 1/5 \\
1/9 & 1/7 & 1 & 1/7 & 1/3 & 1/9 \\
1/3 & 3 & 7 & 1 & 3 & 2 \\
1/7 & 1/4 & 3 & 1/3 & 1 & 1/5 \\
1/2 & 5 & 9 & 1/2 & 5 & 1
\end{pmatrix}
$$

Where, $a_{ij}$ represents the weight of index $j$ relative to index $i$.

Our team chooses five types of battery that are available to store energy. They are Deka Solar 8GCC2 6V 198, Trojan L-16 -SPRE 6V 415, Discover AES 7.4 kWh, Electriq PowerPod 2, and Tesla Powerwall+ [8]. We use $a$, $b$, $c$, $d$, and $e$ to represent them separately, Table 4 below shows the battery types and their respective symbols.

Table 4. The battery types and their respective symbols

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deka Solar 8GCC2 6V 198</td>
<td>$a$</td>
</tr>
<tr>
<td>2</td>
<td>Trojan L-16 -SPRE 6V 415</td>
<td>$b$</td>
</tr>
<tr>
<td>3</td>
<td>Discover AES 7.4 kWh</td>
<td>$c$</td>
</tr>
<tr>
<td>4</td>
<td>Electriq PowerPod 2</td>
<td>$d$</td>
</tr>
<tr>
<td>5</td>
<td>Tesla Powerwall+</td>
<td>$e$</td>
</tr>
</tbody>
</table>

Next, we set the consistency matrices below to help us analyze how those factors would affect the various types of battery and further determine the efficiency of energy storage of each battery sort.

The consistency matrix formed by various batteries for factor $F_1$ is shown as $B_1$.

$$
B_1 = \begin{pmatrix}
1 & 3 & 7 & 9 & 9 \\
1/3 & 1 & 3 & 9 & 7 \\
1/7 & 1/3 & 1 & 7 & 5 \\
1/9 & 1/9 & 1/7 & 1 & 3 \\
1/9 & 1/7 & 1/5 & 1/3 & 1
\end{pmatrix}
$$

The consistency matrix formed by various batteries for factor $F_2$ is shown as $B_2$.

$$
B_2 = \begin{pmatrix}
1 & 1/3 & 9 & 5 & 7 \\
3 & 1 & 7 & 4 & 5 \\
1/9 & 1/7 & 1 & 3 & 5 \\
1/5 & 1/4 & 1/3 & 1 & 3 \\
1/7 & 1/5 & 1/5 & 1/3 & 1
\end{pmatrix}
$$

The consistency matrix formed by various batteries for factor $F_3$ is shown as $B_3$.

$$
B_3 = \begin{pmatrix}
1 & 2 & 3 & 5 & 7 \\
1/2 & 1 & 2 & 3 & 5 \\
1/3 & 1/2 & 1 & 3 & 6 \\
1/5 & 1/3 & 1/3 & 1 & 4 \\
1/7 & 1/5 & 1/6 & 1/4 & 1
\end{pmatrix}
$$

The consistency matrix formed by various batteries for factor $F_4$ is shown as $B_4$.

$$
B_4 = \begin{pmatrix}
1 & 1 & 7 & 9 & 3 \\
1 & 1 & 5 & 7 & 2 \\
1/7 & 1/5 & 1 & 3 & 1/3 \\
1/9 & 1/7 & 1/3 & 1 & 1/5 \\
1/3 & 1/2 & 3 & 5 & 1
\end{pmatrix}
$$

The consistency matrix formed by various batteries for factor $F_5$ is shown as $B_5$.

$$
B_5 = \begin{pmatrix}
1 & 1/3 & 5 & 9 & 7 \\
3 & 1 & 7 & 9 & 9 \\
1/5 & 1/7 & 1 & 7 & 5 \\
1/9 & 1/9 & 1/7 & 1 & 1/3 \\
1/7 & 1/9 & 1/5 & 3 & 1
\end{pmatrix}
$$

The consistency matrix formed by various batteries for factor $F_6$ is shown as $B_6$.

$$
B_6 = \begin{pmatrix}
1 & 1/4 & 1/5 & 1/9 & 1/7 \\
4 & 1 & 1/3 & 1/7 & 1/5 \\
5 & 3 & 1 & 1/5 & 1/3 \\
9 & 7 & 5 & 1 & 2 \\
7 & 5 & 3 & 1/2 & 1
\end{pmatrix}
$$

5.2 Model Test

The setting of data standards has a lot of subjectivity, to make the model more perfect, we decide to check the consistency. We judge whether the matrix is a consistent matrix by judging whether the largest eigenvalue $\lambda_{max}$ of the matrix is equal to $n$. The more $\lambda_{max}$ is larger than $n$, the more serious the degree of inconsistent rows of the matrix.

Then, we check the consistency of the judgment matrix as follows. First, we calculate the consistency index $CI$, in which

$$
CI = \frac{\lambda_{max} - n}{n - 1}
$$

Second, we find the corresponding average random consistency index $RI$. For $n = 1, 2, 3, 4, 5, 6, 7, 8, 9$. The value of $RI$ given by Saaty is shown in Table 5.

Table 5. Value of $RI$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$RI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.58</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
</tr>
<tr>
<td>4</td>
<td>1.12</td>
</tr>
<tr>
<td>5</td>
<td>1.24</td>
</tr>
<tr>
<td>6</td>
<td>1.32</td>
</tr>
<tr>
<td>7</td>
<td>1.41</td>
</tr>
<tr>
<td>8</td>
<td>1.45</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

The value of $RI$ is obtained by using a random method to construct 500 sample matrices: randomly from 1 to 9 and its inverse. Then, extract numbers from the numbers to construct a reciprocal matrix, finding the average value of the largest eigenvalue $\max \lambda_{max}$, and define it. Third, we calculate the consistency ratio $CR$, in which

$$
CR = \frac{CI}{RI}
$$
When \( CR <0.10 \), the consistency of the judgment matrix is considered acceptable, otherwise the judgment matrix should be modified appropriately to adjust.

Next, we write a program to check the consistency of the matrix by MATLAB. After checking, we find that \( CR_e=0.0858, CR_2=0.1221, CR_3=0.1912, CR_4=0.0462, CR_5=0.0223, CR_6=0.1222, \) and \( CR_e=0.0555 \). We notice that matrices \( A, B_1, B_4, \) and \( B_6 \) are acceptable because the CR is less than 0.10. However, matrices \( B_2, B_3, \) and \( B_5 \) are not so perfect because the CR of them are greater than 0.10, which are not quite acceptable so that we need to modify the matrices, and the Table 6 below shows our testing results.

### Table 6. Testing results

<table>
<thead>
<tr>
<th>Matrix</th>
<th>( CR )</th>
<th>Acceptable/unacceptable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>0.0858</td>
<td>acceptable</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>0.1221</td>
<td>unacceptable</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>0.1912</td>
<td>unacceptable</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>0.0462</td>
<td>acceptable</td>
</tr>
<tr>
<td>( B_4 )</td>
<td>0.0223</td>
<td>acceptable</td>
</tr>
<tr>
<td>( B_5 )</td>
<td>0.1222</td>
<td>unacceptable</td>
</tr>
<tr>
<td>( B_6 )</td>
<td>0.0555</td>
<td>acceptable</td>
</tr>
</tbody>
</table>

#### 5.3 Model Modification

Since there are three out of seven matrices are imperfect, we have to modify them. We use the geometric average method to calculate the feature vector. First, we multiply the vectors in each row and turn it into a \( 5 \times 1 \) matrix. Next, we take every entry in this new matrix to the fifth power. Add up these five entries to be \( m \), and use the five vectors to divide \( m \) separately, which we mark these five numbers as \( w_1, w_2, w_3, w_4, \) and \( w_5 \). We construct the consistency matrix \( W \) as follows:

\[
W = \begin{pmatrix}
1 & w_1/w_2 & w_1/w_3 & w_1/w_4 & w_1/w_5 \\
w_2/w_1 & 1 & w_2/w_3 & w_2/w_4 & w_2/w_5 \\
w_3/w_1 & w_3/w_2 & 1 & w_3/w_4 & w_3/w_5 \\
w_4/w_1 & w_4/w_2 & w_4/w_3 & 1 & w_4/w_5 \\
w_5/w_1 & w_5/w_2 & w_5/w_3 & w_5/w_4 & 1
\end{pmatrix}
\]

Calculate the disturbance matrix as follows:

\[
D = A - W = \begin{pmatrix}
0 & a_{12} - w_2/w_1 & a_{13} - w_3/w_1 & a_{14} - w_4/w_1 & a_{15} - w_5/w_1 \\
a_{21} - w_2/w_1 & 0 & a_{23} - w_3/w_2 & a_{24} - w_4/w_2 & a_{25} - w_5/w_2 \\
a_{31} - w_3/w_1 & a_{32} - w_2/w_3 & 0 & a_{34} - w_4/w_3 & a_{35} - w_5/w_3 \\
a_{41} - w_4/w_1 & a_{42} - w_2/w_4 & a_{43} - w_3/w_4 & 0 & a_{45} - w_5/w_4 \\
a_{51} - w_5/w_1 & a_{52} - w_2/w_5 & a_{53} - w_3/w_5 & a_{54} - w_4/w_5 & 0
\end{pmatrix}
\]

Where \( d_{ij} = a_{ij} - w_i/w_j \).

We sort the absolute values of non-diagonal elements \( |d_{ij}| \) from largest to smallest, getting the sort result \( \{1, 2, 3, \ldots \} \). We adjust the element value \( a \) corresponding to \( i_s \), where \( s=1, 2, \ldots \).

1. If \( a_{ij} >1 \) and \( d_{ij} >0 \), we decrease the value of \( a_{ij} \) that is \( a_{ij} \). If \( a_{ij} =2 \), we will not adjust.
2. If \( a_{ij} >1 \) and \( d_{ij} <0 \), we increase the value of \( a_{ij} \) that is \( a_{ij} \). If \( a_{ij} =9 \), no adjustment to be made.

Given the general method above, we substitute the matrices \( B_1, B_2, \) and \( B_3 \) into it to adjust. After several attempts to adjust, we finally decided to adjust the element \( B_{13} \) of matrix \( B_1 \) from 7 to 3, and accordingly, the element \( B_{13} \) is changed from 1/7 to 1/3. Then, the consistency ratio of matrix \( B_1 \) could be 0.0971 now, which is acceptable. Similarly, using the method mentioned from above, we finally adjust six elements in the matrix \( B_2, B_{23} \) is changed from 9 to 1, along with the change of \( B_{21}, B_{24} \) from 1 to 9, along with the change of \( B_{21}, B_{24} \) from 1 to 9, along with the change of \( B_{21}, B_{24} \) from 1 to 9. After these adjustments, now the matrix \( B_2 \) has a consistency ratio of 0.0996 which is less than 0.10 and is acceptable.

Finally, we started to improve the matrix \( B_4 \), and through several trials, we made the following changes. \( B_{51} \) is adjusted from 7 to 6 and \( B_{51} \) from 1/7 to 1/6; \( B_{52} \) is adjusted from 7 to 3 and \( B_{52} \) is changed from 1/7 to 1/3; \( B_{54} \) is adjusted from 7 to 4 and \( B_{54} \) is changed from 1/7 to 1/4. Accordingly, the consistency ratio of matrix \( B_5 \) became 0.0997 which is within the standard range because it is less than 0.10, and the modifying vectors are shown in the Table 7 below.

### Table 7. Modifying vectors

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Element</th>
<th>Value before the change</th>
<th>Value after the change</th>
<th>CR before the change</th>
<th>CR after the change</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_1 )</td>
<td>( b_{13} )</td>
<td>7</td>
<td>3</td>
<td>0.1221</td>
<td>0.0971</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( b_{13} )</td>
<td>1/7</td>
<td>1/3</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>( b_{13} )</td>
<td>9</td>
<td>1</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_4 )</td>
<td>( b_{13} )</td>
<td>1/9</td>
<td>1</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_5 )</td>
<td>( b_{13} )</td>
<td>7</td>
<td>6</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_6 )</td>
<td>( b_{13} )</td>
<td>1/7</td>
<td>1/6</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_7 )</td>
<td>( b_{13} )</td>
<td>7</td>
<td>6</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_8 )</td>
<td>( b_{13} )</td>
<td>1/7</td>
<td>1/6</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
<tr>
<td>( B_9 )</td>
<td>( b_{13} )</td>
<td>7</td>
<td>4</td>
<td>0.1912</td>
<td>0.0996</td>
</tr>
</tbody>
</table>

With these minor adjustments, all the seven matrices passed the consistency test. We rewrite the modified matrices \( B_1', B_2', \) and \( B_3' \) as shown below.

\[
B_1' = \begin{pmatrix}
1 & 3 & 3 & 9 & 9 \\
1/3 & 1 & 3 & 9 & 7 \\
1/9 & 1/9 & 1/7 & 1 & 3 \\
1/9 & 1/7 & 1/5 & 1/3 & 1
\end{pmatrix}
\]

\[
B_2' = \begin{pmatrix}
1 & 3 & 1 & 2 & 6 \\
1/3 & 1 & 3 & 5 & 4 \\
1/2 & 1/4 & 1/3 & 1 & 3 \\
1/6 & 1/5 & 1/5 & 1/3 & 1
\end{pmatrix}
\]

\[
B_3' = \begin{pmatrix}
1 & 3 & 1 & 2 & 6 \\
1/3 & 1 & 3 & 5 & 4 \\
1/2 & 1/4 & 1/3 & 1 & 3 \\
1/6 & 1/5 & 1/5 & 1/3 & 1
\end{pmatrix}
\]
5.4 Model Solving

After adjustment, we began to score the five types of selected batteries. The eigenvectors of matrices $A$, $B_1$, $B_2$, $B_3$, $B_4$, $B_5$, $B_6$ and $C$ are shown below.

$$W_A = \begin{pmatrix} 0.3876 \\ 0.0996 \\ 0.0234 \\ 0.2149 \\ 0.0483 \\ 0.2262 \end{pmatrix},$$

$$W_{A_1} = \begin{pmatrix} 0.4663 \\ 0.2905 \\ 0.1653 \\ 0.0459 \\ 0.0319 \end{pmatrix},$$

$$W_{A_2} = \begin{pmatrix} 0.1653 \\ 0.0459 \\ 0.0319 \end{pmatrix},$$

$$W_{A_3} = \begin{pmatrix} 0.4289 \\ 0.2526 \\ 0.1864 \\ 0.0927 \\ 0.0395 \end{pmatrix},$$

$$W_{A_4} = \begin{pmatrix} 0.4005 \\ 0.2526 \\ 0.1864 \\ 0.0927 \\ 0.0395 \end{pmatrix},$$

$$W_{A_5} = \begin{pmatrix} 0.4005 \\ 0.2526 \\ 0.1864 \\ 0.0927 \\ 0.0395 \end{pmatrix},$$

$$W_{A_6} = \begin{pmatrix} 0.0325 \\ 0.0706 \\ 0.1331 \\ 0.4769 \\ 0.2868 \end{pmatrix}.$$ 

We synthesize the weights of each battery corresponding to the weights of various factors, and calculate their respective total ranking weight as shown in the Table 8 below [1].

<table>
<thead>
<tr>
<th>Layer $A$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>...</th>
<th>$A_m$</th>
<th>Total ranking weight of layer $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>$b_{11}$</td>
<td>$b_{12}$</td>
<td>...</td>
<td>$b_{1m}$</td>
<td>$\sum_{j=1}^{m} b_{1j}a_j$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$b_{21}$</td>
<td>$b_{22}$</td>
<td>...</td>
<td>$b_{2m}$</td>
<td>$\sum_{j=1}^{m} b_{2j}a_j$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$B_n$</td>
<td>$b_{n1}$</td>
<td>$b_{n2}$</td>
<td>...</td>
<td>$b_{nm}$</td>
<td>$\sum_{j=1}^{m} b_{nj}a_j$</td>
</tr>
</tbody>
</table>

We substitute the obtained values into the table and get the results as shown in Table 9.

<table>
<thead>
<tr>
<th>criterion</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_4$</th>
<th>$F_5$</th>
<th>$F_6$</th>
<th>Total ranking weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
<td>0.0</td>
<td>0.31</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.0</td>
<td>0.27</td>
</tr>
<tr>
<td>$c$</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.0</td>
<td>0.27</td>
</tr>
<tr>
<td>$d$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.13</td>
</tr>
<tr>
<td>$e$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>0.14</td>
</tr>
</tbody>
</table>

We find that the total scores of the five batteries $a$, $b$, $c$, $d$, and $e$ are 0.3170, 0.2768, 0.1368, 0.1468, and 0.1210. According to the comprehensive consideration of selection factors, the order of batteries from good to bad is $a > b > d > c > e$. That is, in consideration of the six factors related to the efficiency of energy storage system such as continuous power rating, instantaneous power rating, usable capacity, round-trip efficiency, lifetime of battery, and cost, we found that the battery Deka Solar 8GCC2 6V 198 is the best, the battery Trojan L-16 -SPRE 6V 415 is relatively good, the battery Electriq PowerPod 2 is at medium level, the battery Discover AES 7.4 kWh is relatively inefficient, and the battery Tesla Powerwall+ is the worst. The Table 10 below shows their order.
Table 10. The order of batteries

<table>
<thead>
<tr>
<th>Order</th>
<th>Symbol</th>
<th>Name of battery</th>
<th>score</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>Deka Solar 8GCC2 6V</td>
<td>0.3170</td>
<td>best</td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>Trojan L-16 - SPRE 6V 415</td>
<td>0.2768</td>
<td>Relative good</td>
</tr>
<tr>
<td>3</td>
<td>d</td>
<td>Electriq PowerPod 2</td>
<td>0.1468</td>
<td>middle</td>
</tr>
<tr>
<td>4</td>
<td>c</td>
<td>Discover AES 7.4 kWh</td>
<td>0.1368</td>
<td>Relative inefficient</td>
</tr>
<tr>
<td>5</td>
<td>e</td>
<td>Tesla Powerwall+</td>
<td>0.1210</td>
<td>worst</td>
</tr>
</tbody>
</table>

Thus, the battery Deka Solar 8GCC2 6V 198 is our most ideal choice to be used in the energy storage system.

6. Solution of problem 2

6.1 Model Improvement

We think that the choice of battery is only considered by the six factors mentioned above, which seems a bit incomplete and not universal. Therefore, we decided to add some personal needs and preferences to our model so that it could be more generalized. Personal preferences include many factors. For example, some people prefer batteries with good-looking shapes, some prefer batteries with smaller sizes, and others may prefer batteries that are more practical. We divide these subjective factors into a separate category for consideration, that is, personal preference, using $F_7$ to represent it. Then, after incorporating it into our original model, the judgment matrix has been modified to be the following one.

$$A'' = \begin{pmatrix}
1 & 5 & 9 & 3 & 7 & 2 & 4 \\
1/5 & 1 & 7 & 1/3 & 4 & 1/5 & 1/4 \\
1/9 & 1/7 & 1 & 1/7 & 1/3 & 1/9 & 1/7 \\
1/3 & 3 & 7 & 1 & 3 & 2 & 1/3 \\
1/7 & 1/4 & 3 & 1/3 & 1 & 1/5 & 1/5 \\
1/2 & 5 & 9 & 1/2 & 5 & 1 & 3 \\
1/4 & 4 & 7 & 3 & 5 & 1/3 & 1
\end{pmatrix}$$

In addition, we have a comparison of the preference between these seven types of batteries, and how this factor would affect the choice for battery is set below in the matrix $B_7$.

$$B_7 = \begin{pmatrix}
1 & 1/3 & 1/7 & 1/5 & 1/9 \\
3 & 1 & 1/5 & 1/3 & 1/7 \\
7 & 5 & 1 & 3 & 1/3 \\
5 & 3 & 1/3 & 1 & 1/5 \\
9 & 7 & 3 & 5 & 1
\end{pmatrix}$$

6.2 Model Test

After establishing the modified model, we need to check the consistency, and the steps are the same as mentioned in 3.2. The consistency ratio of matrix $A''$ is 0.1142, which is larger than 0.10, so we need to adjust it. The consistency ratio of matrix $B_7$ is 0.0530, which is smaller than 0.10, so there is no adjustment for it. The Table 11 below shows our testing result.

Table 11. Testing Results

<table>
<thead>
<tr>
<th>Matrix</th>
<th>CR</th>
<th>Acceptable/unacceptable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.1142</td>
<td>acceptable</td>
</tr>
<tr>
<td>$B_7$</td>
<td>0.0530</td>
<td>unacceptable</td>
</tr>
</tbody>
</table>

6.3 Model Modification

There is only one matrix for us to adjust, which is $A''$, and the steps of adjusting are the same as mentioned in 3.3. After several trials, we decided to make the following changes. We changed $a''_{23}$ from 7 to 3, along with the adjustment of $a''_{32}$ from 1/7 to 1/3; we also changed $a''_{25}$ from 4 to 3, along with the adjustment of $a''_{52}$ from 1/4 to 1/3. After the adjustment, the consistency ratio of matrix $A''$ dropped to 0.0945, which is quite acceptable. The modifying vectors are shown in the Table 12 below.

Table 12. Modifying vectors

<table>
<thead>
<tr>
<th>Matrix Element</th>
<th>Value before the change</th>
<th>Value after the change</th>
<th>CR before the change</th>
<th>CR after the change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A''$</td>
<td>$a''_{23}$</td>
<td>7</td>
<td>3</td>
<td>0.1142</td>
</tr>
<tr>
<td></td>
<td>$a''_{32}$</td>
<td>1/7</td>
<td>1/3</td>
<td>0.0384</td>
</tr>
<tr>
<td></td>
<td>$a''_{25}$</td>
<td>4</td>
<td>3</td>
<td>0.0568</td>
</tr>
</tbody>
</table>

6.4 Model Solving

After these modifications, we start to again score the five types of selected batteries. We first list the new feature vectors as below so that we can see more clearly.

$$WA'' = \begin{pmatrix}
0.3440 \\
0.0568 \\
0.0208 \\
0.1525 \\
0.0319
\end{pmatrix}$$

$$WB_1 = \begin{pmatrix}
0.4663 \\
0.2905 \\
0.1653 \\
0.0459 \\
0.0319
\end{pmatrix}$$

$$WB_2 = \begin{pmatrix}
0.1779 \\
0.5176 \\
0.1684 \\
0.0971 \\
0.0444
\end{pmatrix}$$

$$WB_3 = \begin{pmatrix}
0.4289 \\
0.2526 \\
0.1864 \\
0.0927 \\
0.0395
\end{pmatrix}$$
Then, the total scores of the five batteries $a$, $b$, $c$, $d$, and $e$ are 0.2653, 0.2284, 0.1601, 0.1540, and 0.1924, respectively. Since $0.2653 > 0.2284 > 0.1924 > 0.1601 > 0.1540$, according to the comprehensive consideration of our selection factors, the order of batteries from good to bad is $a > b > e > c > d$. We found that after incorporating the factor of personal preference, the sorting of batteries has undergone slight changes. The first, second, and fourth positions are still $a$, $b$, and $c$, respectively. But the rankings of $d$ and $e$ are swapped. In other words, battery Deka Solar 8GCC2 6V 198 is still the best; battery Trojan L-16 -SPRE 6V 415 is still the relatively good one; battery Discover AES 7.4 kWh is still relatively inefficient. But now battery Tesla Powerwall+ is of the middle level while battery Electriq PowerPod 2 becomes the worst choice. Consequently, we would still consider battery Deka Solar 8GCC2 6V 198 as the best option to suit the energy requirements of a 1,600-square-foot family, which is our ultimate choice. The Table 13 below shows the order of the batteries.

**Table 13. The order of batteries**

<table>
<thead>
<tr>
<th>Order</th>
<th>Symbol</th>
<th>Name of battery</th>
<th>score</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$a$</td>
<td>Deka Solar 8GCC2 6V 198</td>
<td>0.2653</td>
<td>best</td>
</tr>
<tr>
<td>2</td>
<td>$b$</td>
<td>Trojan L-16 -SPRE 6V 415</td>
<td>0.2284</td>
<td>Relative good</td>
</tr>
<tr>
<td>3</td>
<td>$e$</td>
<td>Tesla Powerwall+</td>
<td>0.1924</td>
<td>middle</td>
</tr>
<tr>
<td>4</td>
<td>$c$</td>
<td>Discover AES 7.4 kWh</td>
<td>0.1601</td>
<td>Relative inefficient</td>
</tr>
<tr>
<td>5</td>
<td>$d$</td>
<td>Electriq PowerPod 2</td>
<td>0.1540</td>
<td>worst</td>
</tr>
</tbody>
</table>

**7. Solution of problem 3**

**7.1 The Advantages and Disadvantages of Cement Batteries**

Cement batteries have a lot of unique advantages compared to traditional batteries such as lithium-ion batteries.

1. The storage capacity of concrete solutions could be expanded by incorporating them into an entire concrete building. This means that the entire concrete buildings could function as giant cement-based batteries in the future, and the units could supply enough energy to households even on a cloudy day or during winter [9].

2. Cement batteries can also solve the cost problems because they can play different roles at the same time, serving as both energy storage and building materials, making them a rather cost-effective solution for storing energy compared to traditional ones. However, each coin has two sides. Although cement batteries overwhelmed the traditional ones in some extents, they still have some shortages.

1. The density of cement is relatively low, being only at 0.8 Watt hours per liter, which is hundreds of times less energy-dense than a lithium-ion battery, and is completely useless for transportation of electricity [10].

2. The battery’s life might not match the long periods over which concrete structures are expected to last so that how to keep the battery work while the building is still in use is a potential concern to be addressed.

3. Concrete currently has an immense environmental cost, the production of it generally accounts for 4-8 percent of global carbon emissions which would harm the environment in a large extent.

Figures 1-3 below shows some of the comparisons between cement battery and lithium-ion battery based on the collected data.
consideration, and all of them are closely related to the batteries, these factors should also be taken into performance of cement batteries and currently available selection process, if we want to additionally compare the cost, and personal preference in the previous battery usable capacity, round-trip efficiency, lifetime of battery, because we have considered the seven factors of continuous power rating, instantaneous power rating, usable capacity, round-trip efficiency, lifetime of battery, cost, and personal preference in the previous battery section process, if we want to additionally compare the performance of cement batteries and currently available batteries, these factors should also be taken into consideration, and all of them are closely related to the efficiency of solar-power storage.

8. Non-technical news article

Overall, we chose five batteries, Deka Solar 8GCC2 6V 198, Trojan L-16 -SPRE 6V 415, Discover AES 7.4 kWh, Electriq PowerPod 2, and Tesla Powerwall+, as options. In order to find the best battery that can meet the electricity demand of a 1600-square-foot home, we set six parameters for reference and modeling. They are continuous power rating, instantaneous power rating, usable capacity, round-trip efficiency, lifetime of battery, and cost. After comprehensive comparison, we found that the battery Deka Solar 8GCC2 6V 198 is the most ideal choice. It can not only meet the electricity needs of households in remote areas, but also has the most comprehensive advantages. Then, in order to make our judgments and choices more general and universal, we added a more subjective factor of personal preference into consideration. Ultimately, after careful analysis and researches, we found that the battery Deka Solar 8GCC2 6V 198 is still the best and most suitable one, so we finally determined our choice. After that, we made a comprehensive comparison between the traditional lithium-ion battery and the new cement battery. We discovered that although the cement battery still has some shortcomings and imperfections compared to the lithium-ion battery, some of its advantages are very significant and important such as a very long lifetime. Therefore, we believe that in the near future, with the further development of science and technology and the further improvement of research abilities, cement batteries will have a very broad application prospect, and can help us optimize the energy storage system, as well as extending the time of usage. In terms of both space and time, it will brought a qualitative leap for us, and our team strongly recommend people to try this kind of cement battery which will definitely satisfy their energy requirements very well.

9. Conclusion

This article uses the analytic hierarchy process to solve the problem of battery selection, and ultimately we successfully find out a method that can provide suitable power supply for 1600-square-foot households in remote areas. The specific conclusions are as follows.

(1) Among the five types of batteries, Deka Solar 8GCC2 6V 198, Trojan L-16 -SPRE 6V 415, Discover AES 7.4 kWh, Electriq PowerPod 2, and Tesla Powerwall+, battery Deka Solar 8GCC2 6V 198 is the best option when considering the six objective factors of continuous power rating, instantaneous power rating, usable capacity, round-trip efficiency, lifetime of battery, and cost.

(2) After adding the subjective factor of personal preference, battery Deka Solar 8GCC2 6V 198 is still the most suitable choice among all the five kinds of batteries.

(3) Comparing cement batteries with traditional lithium-ion batteries, we find that both of them have their pros and cons. Cement batteries have very long lifetime and high electrical energy utilizing rate but very low energy density. While traditional lithium-ion batteries have relatively short life span and low electrical energy utilizing rate, their advantages are their high energy density.

(4) Since the cement battery has many prominent and crucial advantages over the traditional ones, it will definitely replace the old ones and has a broad prospect in the future as the technology develops.
References


