

# Modelling ventilation and convective heat transfer in deeply buried underground tunnels based on boundary layer theory

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**Abstract.** Underground traffic tunnels serve as the entry and exit buildings for underground spaces. Most of the ventilation and air-conditioning systems in underground spaces rely on traffic tunnels for cooling or preheating to save energy. The temperature distribution of traffic tunnels has always attracted the attention of researchers. In this study, a heat transfer model of a traffic tunnel was established based on energy conservation and boundary layer theory, and the field test was carried out. The calculation model provided in this study was in good agreement with the field measurement results. It is found that the dimensionless air temperature of the traffic tunnel decays exponentially along the tunnel at the same time. The air temperature in the traffic tunnel fluctuates with time, and the simple harmonic fluctuation decreases along the tunnel. In summer, the air temperature decreases and increases exponentially along the tunnel during the day and night, respectively. In this study, a heat transfer model for calculating the temperature distribution of underground traffic tunnels was theoretically established, which provided an important theoretical support for underground traffic tunnels as natural air conditioners. The establishment of heat transfer model of underground traffic tunnel has significant economic and environmental benefits.

## 1 Introduction

As one of the clean energy sources, hydropower resources play an important and positive role in energy conservation, emission reduction and environmental protection in the world [1,2]. Most countries in the world are committed to the development of hydropower industry. China is rich in hydropower resources, with 542 million gigawatt hours of hydropower resources available for development, ranking first in the world [3]. The ventilation system is essential as an important system for conditioning the indoor heat and humidity environment to ensure the safety of workers and the safe operation of electromechanical equipment [4-8]. Traffic tunnel services are huge ventilation ducts that bring fresh outside air into hydropower station. The traffic tunnel can cool and heat the air from the outside in summer and winter respectively, reducing the energy consumption of air conditioning [9,10].

Most of the traffic tunnels are buried deep underground, and the extension distance can reach 1000m. M. Krarti et al. [11] simplified the tunnel heat transfer model into one-dimensional heat transfer and established an analytical model to determine the energy performance of the underground tunnel. Li et al. modelled the effect of roughness on heat transfer in an underground traffic tunnel and pointed out that the air temperature decays exponentially along the underground traffic tunnel [12], and the exponential decay law of air temperature is more popular in tunnel air temperature prediction [13]. In addition, Liu [14], Gao [15] and Mao [16] et al. established a simple numerical model of heat and mass transfer in the underground tunnel which can accurately describe the field test results. However, the accuracy of the above models all depend on the accuracy of the convective heat

transfer coefficient. Based on the convective heat transfer of the pipe, the researchers [13,14,16] modelled the heat transfer of the underground tunnel. However, the heat exchange process in the underground tunnel is more complicated than the convection heat transfer in the pipe. In order to accurately calculate the cooling or heating performance of underground traffic tunnels, a universally applicable convective heat transfer coefficient model is needed.

The goal of this paper is to analyse and establish a convective heat transfer model suitable for underground traffic tunnels. The airflow and wall temperatures along the tunnel were obtained via a field test in a traffic tunnel. A new temperature distribution model is theoretically established, verified, and applied to calculate the air temperature in the traffic tunnel.

## 2 Theoretical Methodology

According to the narrow and long characteristics of underground traffic tunnels, the heat transfer model can be simplified as a steady heat convection in an infinitely long cylinder. Thermal balance in a control volume along the tunnel (as shown in Fig.1) requires that the amount of convective heat transfer between the airflow and the rock wall is equal to the amount of internal energy change in the control volume, and the thermal balance equation is expressed as follows:

$$\rho c_p u_m F \frac{\partial [t_f(x)]}{\partial x} dx = \alpha U [t_w - t_f(x)] dx + Q dx \quad (1)$$

Where,  $t_f(x)$  is the air temperature at  $x$ , °C;  $x$  is the distance from tunnel entrance, m;  $t_w$  is tunnel wall temperature, °C;  $\alpha$  is convective heat transfer coefficients of airflow and tunnel walls,  $W \cdot m^{-2} \cdot K^{-1}$ ;  $U$

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is the perimeter of tunnel section, m;  $F$  is the area of the tunnel section, m<sup>2</sup>;  $\rho$  is the air density, kg·m<sup>-3</sup>;  $c_p$  is the air specific heat capacity, J·kg<sup>-1</sup>·K<sup>-1</sup>;  $u_m$  is the ventilation velocity, m·s<sup>-1</sup>;  $Q$  is heat source in the tunnel, W·m<sup>-3</sup>.

According to equation (1), When  $x=0$ ,  $t_f(0)=t_0$ ,  $Q=0$  as shown in Fig.1, then:

$$t_f(x) = t_w - (t_w - t_0) e^{-\frac{\alpha U x}{\rho c_p u_m F}} \quad (2)$$

Where,  $t_0$  is the air temperature at  $x=0$ , °C.

From equation (2), it can be seen a relationship of exponential decay on the air temperature along with the tunnel [13], as shown in Fig.1.

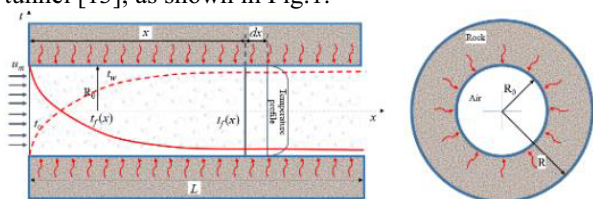


Fig.1 Schematic of the air temperature curve in the underground traffic tunnel of the hydropower station

The accurate calculation of the air temperature distribution curve in the tunnel depends on the convective heat transfer coefficient  $\alpha$ . Based on the boundary layer theory, this paper established turbulent boundary layer model for the air flow field and obtains the analytical solution to  $\alpha$ . The development of the boundary layer in the tunnel is shown in Fig.2.

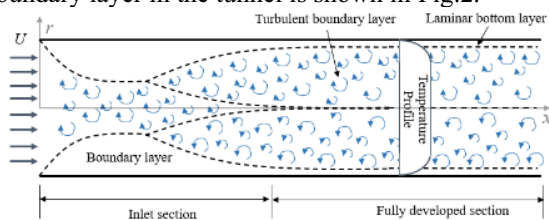


Fig.2 The development of the boundary layer in the circular tunnel

The turbulent boundary layer energy equation [17] can be described as follows:

$$\bar{u} \frac{\partial \bar{t}}{\partial x} + \bar{v}_r \frac{\partial \bar{t}}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\lambda}{\rho} \frac{\partial \bar{t}}{\partial r} - \bar{t}' \bar{v}' \right) \right] = 0 \quad (3)$$

This study assumes that the air flow state in the traffic tunnel is in the fully developed section. Then there is  $\bar{v}_r=0$  and  $\bar{u}$  depends only on  $r$ . When heat transfer turbulent diffusivity  $\varepsilon_H = -\bar{t}' \bar{v}' / (\partial \bar{t} / \partial r)$  is introduced based on the mixing-length and eddy diffusivity methods, Equation (3) be simplified as follows:

$$\bar{u} \frac{\partial \bar{t}}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\lambda}{\rho} + \varepsilon_H \right) \frac{\partial \bar{t}}{\partial r} \right] \quad (4)$$

The dimensionless temperature  $((t_0 - t) / (t_0 - t_m))$  profile is independent of  $x$  in the fully developed section. Then its mathematical expression is as follows:

$$\frac{\partial t}{\partial x} = \frac{dt_0}{dx} = \frac{dt_m}{dx} \quad (5)$$

Furthermore, since the turbulent velocity profile is flat, an approximation can be made as follows  $\bar{u}=V$ , where  $V$  is the air average velocity in the tunnel, and  $V$  is a constant. Then, through the variable transformation

$y = r_0 - r$ , the turbulent boundary layer model for the air flow field in the tunnel can be established:

$$V \frac{dt_m}{dx} = \frac{1}{r_0 - y} \frac{\partial}{\partial y} \left[ (r_0 - y) \left( \frac{\lambda}{\rho} + \varepsilon_H \right) \frac{\partial \bar{t}}{\partial y} \right] \quad (6)$$

The boundary conditions are as follows:

$$y=0 \quad \bar{t} = t_0 \quad (7)$$

$$y=r_0 \quad \frac{\partial \bar{t}}{\partial y} = 0 \quad (8)$$

By the Reichardt equation:

$$\frac{\varepsilon_H}{\nu} = \frac{\kappa y^+}{6} \left( 1 + \frac{r}{r_0} \right) \left[ 1 + 2 \left( \frac{r}{r_0} \right)^2 \right] \quad (9)$$

Taking the von Karman constant  $\kappa=0.40$  and the turbulent Prandtl number  $Pr_t = 0.90$ , the integral result in equation (6) is:

$$t^+ = 13.2 Pr + 2.25 \ln \left[ y^+ \frac{1.5 \left( 1 + \frac{r}{r_0} \right)}{1 + 2 \left( \frac{r}{r_0} \right)^2} \right] - 5.8 \quad (10)$$

Assuming that  $t^+$  at the centreline of the tunnel is  $t_c^+$ , then the expression of  $t_c^+$  is:

$$t_c^+ = 13.2 Pr + 2.25 \ln(1.5 r_0^+) - 5.8 \quad (11)$$

According to the energy conservation law, there is the following relationship:

$$t_c^+ = \frac{(\bar{t}_c - t_0) \sqrt{\tau_0 / \rho}}{q / \rho c} \quad (12)$$

Based on the physical meaning of fluid mechanics, there are the following relationships:

$$q = \alpha (t_0 - t_m) \quad (13)$$

$$\frac{r_0 \sqrt{\tau_0 / \rho}}{\nu} = \frac{Re}{2} \sqrt{c_f / 2} \quad (14)$$

$$Nu = \frac{2 t_0 \alpha}{\lambda} \quad (15)$$

According to equations from (11) to (15), the theoretical analytical solution of  $Nu$  is:

$$Nu = \frac{(\bar{t}_c - t_0) Re Pr \sqrt{c_f / 2}}{(t_m - t_0) [13.2 Pr + 2.25 \ln(0.75 Re \sqrt{c_f / 2}) - 5.8]} \quad (16)$$

At moderate Reynolds numbers, temperature can be approximated by a 1/7 power equation when  $Pr \approx 1.00$ . The current analysed fluid is air ( $Pr=0.71$ ), so the temperature curve can be expressed as:

$$\frac{\bar{t} - t_0}{t_c - t_0} = \left( 1 - \frac{r}{r_0} \right)^{1/7} \quad (17)$$

From  $t_m = 2 / V r_0^2 \int_0^{r_0} u r t dr$ , integrating Equation (17) can get:

$$\frac{t_m - t_0}{t_c - t_0} = 0.833 \quad (18)$$

$Nu$  can be expressed as:

$$Nu = \frac{Re Pr \sqrt{c_f / 2}}{0.833 [13.2 Pr + 2.25 \ln(0.75 Re \sqrt{c_f / 2}) - 5.8]} \quad (19)$$

The friction coefficient  $c_f$  can be calculated as follows:

$$\text{Smooth surface} \quad \frac{c_f}{2} = 0.023 Re^{-0.2} \quad (20)$$

$$\text{Rough surface} \quad \frac{c_f}{2} = \left[ 2.46 \ln \left( \frac{D}{k_s} \right) + 3.22 \right]^{-2} \quad (21)$$

### 3 Experimental Methodology

A field test was conducted at the SSL Hydropower Station, which is located in Beijing, China. An isometric view of the SSL hydropower station is shown in Fig.3. There is a traffic tunnel to introduce fresh air into the underground space. Traffic tunnel has an important energy-saving effect on the ventilation and air-conditioning systems of hydropower stations, so this study focuses on the traffic tunnel, which is marked in red in Fig.3. The traffic tunnel is an underground arched building structure with an average burial depth of about 70m. The actual map and dimensions of the traffic tunnel is shown in Fig.4, respectively.

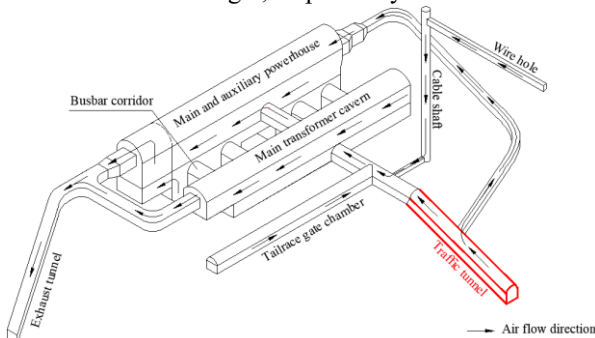


Fig.3 The isometric diagram of the SSL hydropower station.

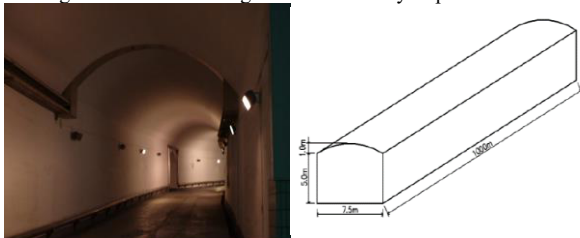


Fig.4 The actual map and dimensions of the traffic tunnel

In order to clarify the heat transfer potential of the traffic tunnel, overall goal of the measurements needs to be developed. The measurement profiles were designated at entrance, 5m, 30m, 60m, 90m, 170m, 340m, 510m, 680m, 850m along with the traffic tunnel and the traffic tunnel exit, as shown in Fig.5 (a). Measurement points in each profile were evenly arranged, as shown in Fig.5 (b). In addition, 510m is taken as the typical profile for measuring the air velocity.

Table 1 Apparatus Accuracy and Test Purpose

Apparatus	Accuracy	Testing purpose
American TSI8386A hand-held hot-wire thermocetor	Temperature (k) ± 0.1	Air temperature and velocity
	Velocity (m/s) ± 0.01	
American raytek P800615 infrared thermocetor	Temperature (k) ± 0.01	Wall temperature
RL002 automatic temperature recorder	Temperature (k) ± 0.3	Indoor and outdoor temperature

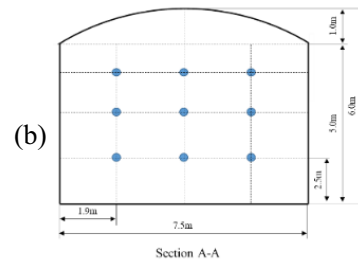
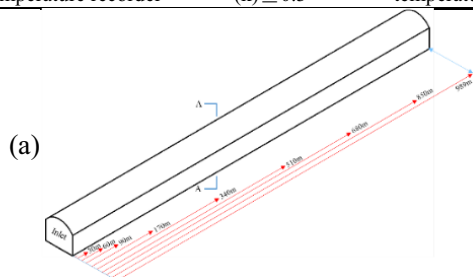


Fig.5 Distribution of measuring points in the traffic tunnel. (a) Longitudinal arrangement of the measurement profiles; (b) Section A-A: Measuring points in each testing profile (•: Measuring point).

### 4 Results and discussion

#### 4.1 Experimental results and analysis

Field testing of the SSL hydropower station lasted five days. The test results show that the air temperature at the entrance of the traffic tunnel approximates a simple harmonic fluctuation with time, as shown in Fig.6. The peak air temperature at the inlet occurred at about 14:00 pm and 4:00 am, respectively. In addition, at the same time, especially when the peaks and troughs occur, the air temperature is distributed exponentially along the traffic tunnel. Taking 14:00 on August 31st as an example, the air temperature dropped from 29.4°C to 23.8°C, and the air temperature decay curve has been marked in red in Fig.6.

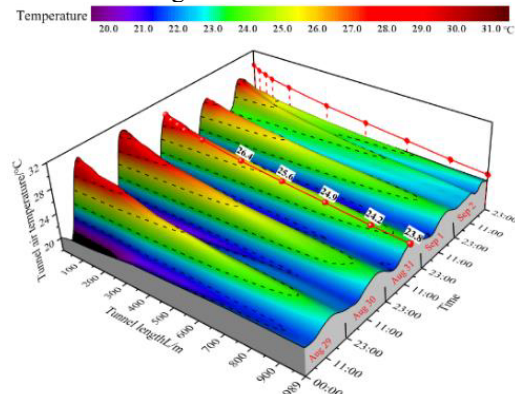


Fig.6 The air temperature test results in the traffic tunnel

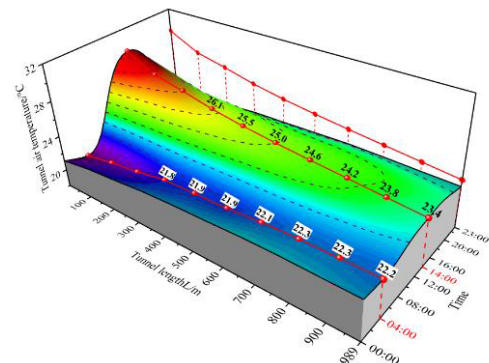


Fig.7 Average air temperature in the traffic tunnel

#### 4.2 Dimensionless analysis of air temperature along with traffic tunnel

According to the section size of the underground traffic tunnel, the perimeter  $L$  and area  $F$  of the section are

calculated to be 25.39m and 42.71m<sup>2</sup>, respectively, and the hydraulic diameter is 3.36m. The air flow velocity is about 0.8m/s by measuring the air velocity at 510m. Then the ventilation volume of the underground traffic tunnel is 34.17m<sup>3</sup>/s. The Reynolds number Re is 1.82×10<sup>5</sup> calculated by the Reynolds number definition formula. The wall of the traffic tunnel is cement plastered, and the rock wall is smooth. However, there are lighting, wire pipes, etc. along the traffic tunnel. In this study, the roughness is 0m and 0.01m for calculation, and the Nu is 299.2 and 339.6 in the smooth and rough cases, respectively. Therefore, the convective heat transfer coefficients are 2.31 W·m<sup>-2</sup>·K<sup>-1</sup> and 4.05 W·m<sup>-2</sup>·K<sup>-1</sup> under the smooth and rough conditions, respectively. Applying the above parameters to (2), the air temperature distribution model under smooth and rough conditions were obtained:

$$\text{Smooth condition } t_f(x) = t_w - (t_w - t_0)e^{-3.979 \times 10^{-3} x} \quad (22)$$

$$\text{Rough condition } t_f(x) = t_w - (t_w - t_0)e^{-6.976 \times 10^{-3} x} \quad (23)$$

The air and wall temperatures at the inlet and outlet collected by TSI are 29.4°C and 23.8°C. Using formula (2), the actual convective heat transfer coefficient can be obtained. Then, the air temperature distribution model from the test results can be expressed as:

$$\text{Test condition } t_f(x) = t_w - (t_w - t_0)e^{-5.719 \times 10^{-3} x} \quad (24)$$

The air temperature in the traffic tunnel (data in Fig.7) are dimensionless and plotted in Fig.8. The test data at different times can be compared under the same conditions after dimensionless. It can be seen from Fig.8 that the dimensionless air temperature decays exponentially along the traffic tunnel. The heat transfer model under smooth conditions can describe the exponential decay law along the traffic tunnel, but it is different from the experimental data. The heat transfer model under rough conditions is in good agreement with the test data, and the difference between the heat transfer model under rough conditions and the heat transfer model under test conditions is not significant. This proves that the convective heat transfer coefficient calculated by the theory can well predict the heat transfer between the air and the rock wall in the tunnel.

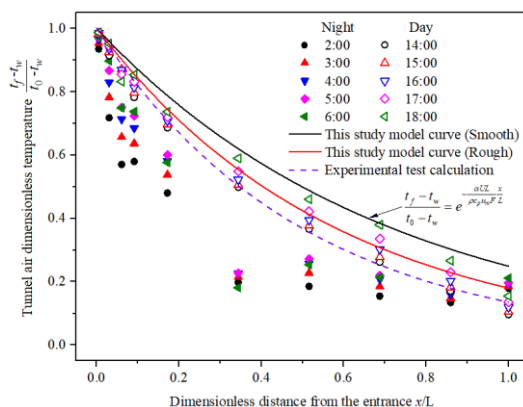


Fig.8 Experimental and theoretical results comparison of the dimensionless air temperature

## 5 Conclusions

In this study, a theoretical model of heat transfer in an underground traffic tunnel was established. The air temperature distribution in the traffic tunnel was tested

on site. The main conclusions are summarized as follows:

(1) The distribution law of air temperature in the deep underground traffic tunnel was tested on site. The air temperature of the deep underground traffic tunnel fluctuates periodically with time, and the air temperature fluctuation weakens along the traffic tunnel. At the same time in summer, the air temperature decreases along the tunnel during the day and the air temperature increases along the tunnel at night.

(2) Based on the steady-state heat transfer theory, the convective heat transfer model of the deep underground traffic tunnel was established, and the convective heat transfer coefficient suitable for the deep underground traffic tunnel was obtained. The air dimensionless temperature decays exponentially along the underground tunnel. Through the boundary layer theory, the convective heat transfer coefficient is solved theoretically in this research, and the influence of roughness on the heat transfer of the underground traffic tunnel is analysed, which is consistent with the actual heat transfer condition.

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