

# Elastic impedance inversion method based on Quantum Annealing MH algorithm

Chen Zhao, Fengming Jin, Guomeng Han, Shuwen Guo, Pengtao Wang, Delu Wang, Wei Guan

Dagang Oilfield Company, PetroChina, Tianjin, China.

**Abstract.** Seismic inversion is one of the important methods to realize reservoir prediction in oil and gas field exploration. The seismic stochastic inversion based on Metropolis-Hastings algorithm makes full use of well logging data and improves the vertical resolution of inversion results. However, conventional Metropolis-Hastings algorithm has slow convergence speed and poor stability. In this paper, an elastic impedance inversion method based on optimized Metropolis-Hastings algorithm is established by combining quantum annealing algorithm with Metropolis-Hastings algorithm. The feasibility of the method is verified by model calculation and actual data test.

## 1. Introduction

Exploration of oil and gas is extremely important to secure energy supplies. As the exploration of oil and gas become more and more extensive, exploration target has been transferred from structural reservoirs to complicated reservoirs. Seismic inversion is an important method to provide efficient and reliable support for the identification of complicated oil and gas reservoirs.

How to describe the complicated reservoir based on seismic inversion has become current research hotspots. For conventional linear inversion method, it is often difficult to meet the identification requirements of the complicated reservoir because of seismic is band-limited. Then, we study the stochastic inversion which uses seismic data as a soft constraint to improve the vertical resolution of the inversion results. The Metropolis-Hastings (MH) algorithm is widely used in stochastic inversion. However, the MH algorithm has some limits. Due to the limited computing time Settings or inappropriate search strategy, MH algorithm is easy to fall into local extremum for a complex parameter space.

In 2006, Wei Chao realized the inversion of log data based on Quantum annealing. In 2015, Badr Alulawi and Mrinal K. Sen applied QA algorithm in a seismic pre-stack inversion to Cana field. The idea of Quantum annealing applied to optimization is come from simulated annealing. Get inspiration from it, we also can improve the MH method based on the theory of Quantum annealing.

Connolly (1999) first proposed the concept of elastic impedance. Whitcombe (2002) applied a new normalized form of elastic impedance to improve the stability of parameters extraction. Elastic impedance inversion based on angle partial stack gathers has higher efficiency, higher stability, higher noise immunity and lower dependence on the quality of seismic data than AVO inversion.

In this paper, we propose a new fast stochastic inversion method based on Quantum MH method to deal with the stochastic elastic inversion problems.

## 2. Quantum MH method

Quantum Annealing (QA) is a global optimization method that is inspired from the mechanics of the quantum fluctuations known as tunneling effect. Compared to Simulated Annealing, Quantum Annealing algorithm converges faster than SA and escapes from being trapped in a local minimum. The construction of Hamiltonian of the system is crucial.

To apply Quantum Annealing to seismic inversion, the kinetic energy of the system  $H_0$  is analogue to the error function  $\Delta E$ , thus the Hamiltonian  $H$  can be rewritten as:

$$H = \Delta E + C\tau \quad (1)$$

Where  $C$  is a constant,  $\tau$  is the transverse field acting on the system.

Then the probability of the quantum jump can be written as:

$$\rho(H) = (e^{-H/PT})^P \quad (2)$$

Where  $P$  is the number of model parameter.

Get inspiration from the Quantum Annealing algorithm, the acceptance probability  $\omega$  of the Quantum Metropolis-Hastings method can be expressed as:

$$\omega = \begin{cases} 1, & \text{if } \Delta E \leq 0 \\ \exp[-(\Delta E + \varphi)], & \text{if } \Delta E > 0 \end{cases} \quad (3)$$

Where  $\varphi$  is a variable which is gradually decreased.

We realize the elastic inversion based on the Quantum Metropolis-Hastings method to test the feasibility of the inversion method. The flow chart of the Quantum Metropolis-Hastings method is shown in Figure

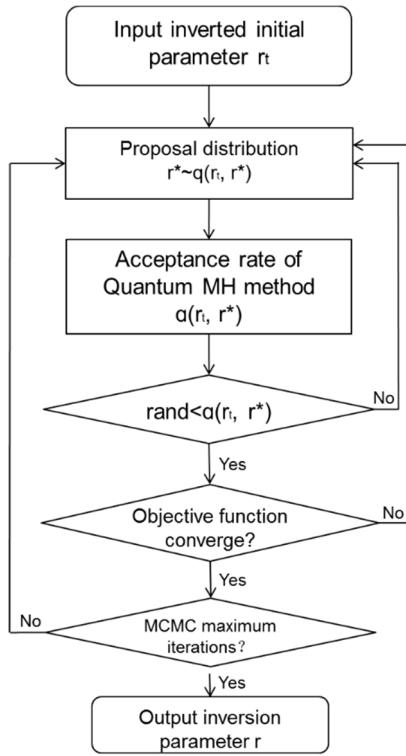


Figure 1 The Process of Quantum MH method

### 3. Elastic impedance equation

Connolly (1999) first proposed the concept of elastic impedance based on Zoeppritz equation. The elastic impedance can be expressed as:

$$EI(\theta) = \alpha^{(1+\tan^2\theta)} \beta^{-8K\sin^2\theta} \rho^{(1-4K\sin^2\theta)} \quad (4)$$

The changes of the incident angle will affect scale characteristics in the elastic impedance equation. In order to eliminate the influence, Whitcombe (2002) applied a new normalized form of elastic impedance to improve the stability of parameters extraction. Then the elastic impedance can be written as:

$$EI(\theta) = \alpha_0 \rho_0 \left[ \left( \frac{\alpha}{\alpha_0} \right)^{(1+\tan^2\theta)} \left( \frac{\beta}{\beta_0} \right)^{(-8K\sin^2\theta)} \left( \frac{\rho}{\rho_0} \right)^{(1-4K\sin^2\theta)} \right] \quad (5)$$

Where  $\alpha$  is P-wave velocity,  $\beta$  is S-wave velocity,  $\rho$  is density.  $\theta$  is incident angle, and  $K = \beta^2 / \alpha^2$ .  $\alpha_0, \beta_0, \rho_0$  are the average value of P-wave velocity, S-wave velocity, density respectively.

### 4. The posterior probability density of inverted parameter

Based on Bayes' theorem, the posterior probability density of inverted parameter is expressed as:

$$p(r|d) \propto p(r) \cdot p(d|r) \quad (6)$$

Where  $d$  is seismic observation data,  $r$  is reflection coefficient,  $p(r|d)$  is posterior probability.  $p(d|r)$  is likelihood function.

Assuming seismic background noise obeys independent Gaussian distribution with zero-mean and  $\sigma_n^2$ -variance, it can be expressed as:

$$p(d|r) = \frac{1}{2\pi\sigma_n^2} \exp \left[ -\sum \frac{(d-g(r))^2}{2\sigma_n^2} \right] \quad (7)$$

Where  $g$  is the forward operator.  $n$  is the sample number of seismic data.

Assuming that the priori information of reflection coefficient obeys Cauchy distribution to stand out weak reflection of the underground media, the priori information is expressed as:

$$p(r) = \frac{1}{(\pi\sigma_m)^M} \prod_{i=1}^M \left[ \frac{1}{1+r_i^2/\sigma_m^2} \right] \quad (8)$$

So objective function can be expressed as:

$$J = \frac{1}{2\sigma_n^2} \sum (d - g(r))^2 + \frac{1}{\sigma_m^2} \sum_{i=1}^M \ln(1 + r_i^2/\sigma_m^2) + \varepsilon \sum (I - I_0)^2 \quad (9)$$

Where  $\varepsilon \sum (I - I_0)^2$  represents the smooth constraint of elastic impedance,  $I_0$  is smooth elastic impedance constraint information constructed according to the log data,  $\varepsilon$  is elastic impedance smoothing factor

To obtain the posterior probability distribution, we apply Quantum MH method to obtain the inversion results.

### 5. Case study with 1D data

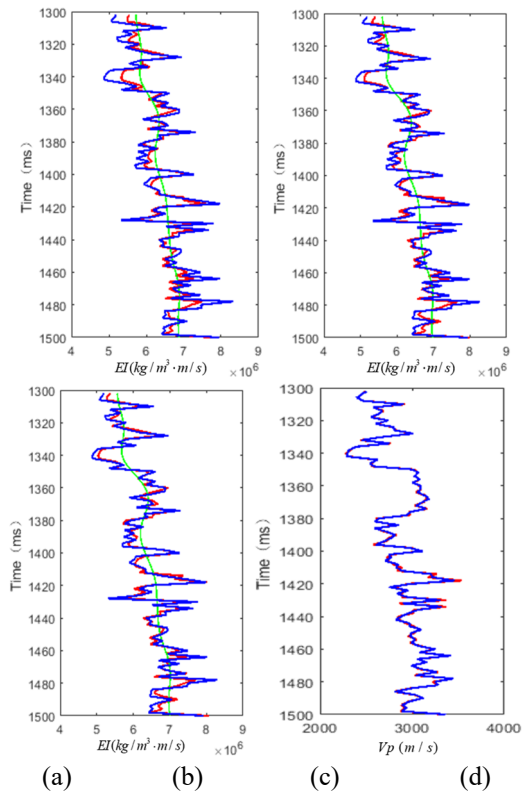
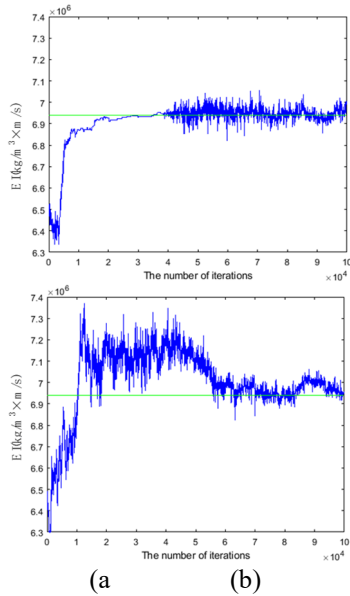


Figure 2 Elastic impedance inversion results of small angle (a), the middle angle (b), large angle (c) and the extracted P-velocity result (d) by using quantum MH method when SNR=3.

To test the feasibility of the Quantum Metropolis-Hastings method, we carry out the feasibility test of one well. The elastic inversion test is to prove the feasibility and the superiority of the Quantum MH method.

From Figure 2 and Figure 3, We can see that the inversion results match the model data well and convergence and stability are greatly improved for quantum MH algorithm compared with MH algorithm. The factor  $\varphi$  in quantum

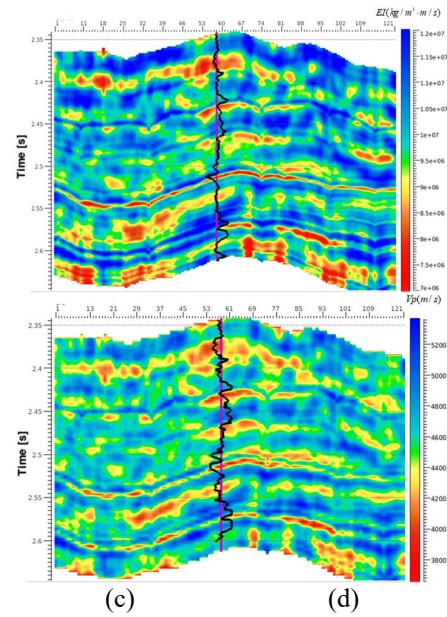
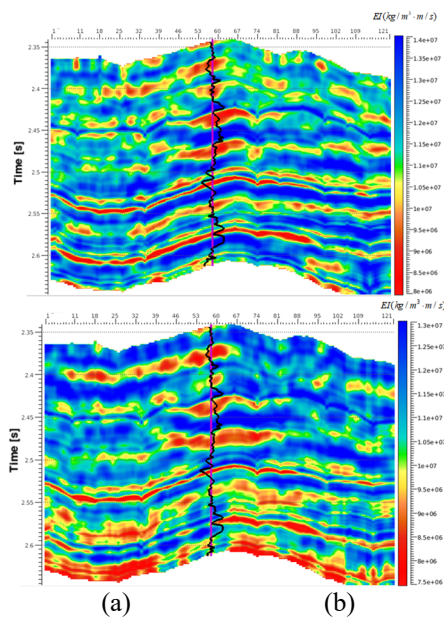
MH algorithm can reduce the unnecessary transfer of the current state, so that the Markov chain can converge to the posterior probability density more rapidly and steadily.



**Figure 3** The change of elastic impedance value (the middle angle) along with the change of the number of iterations for an sampling point in quantum MH algorithm (a) and MH algorithm (b).

## 6. Case study with 2D real data

To test the feasibility of the elastic impedance inversion based on Quantum MH method, we apply this method to 2D real seismic data. The inversion results are shown in Figure 4. The red region represent oil and gas reservoir.



**Figure 4** The elastic inversion result of small angle (a), medium angle (b), large angle (c) and the extracted P-velocity profile (d).

We can see from Figure 4 that the results of the elastic inversion match the logging data well. The elastic inversion method based on quantum MH method is feasible in practical applications.

## 7. Conclusions

Quantum Metropolis-Hastings method is a feasible inversion method that is inspired from Quantum MH method and the objective function can be quickly convergent. The 1D data test and 2D real data test show that quantum MH algorithm can effectively obtain the elastic inversion results.

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