Optimization of container traffic distribution on the railway network

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Abstract. The paper considers the problem of optimal redistribution of transit container flows on the railway network. The problem is proposed to be solved as the problem of a multi-commodity flow of minimum cost using the relaxation method. The relaxation method allows choosing the most efficient variant of the next solution at each iteration step. To search for the optimal distribution of container flows on the network, graph theory was used, i.e., the network is presented as a multigraph with a set of vertices and arcs, for which the costs of transporting a unit of production, the maximum and minimum values of the flows between the vertices are given, the conditions for maintaining the flow at the vertices of the graph and restrictions on the amount of flow along the arcs are given. It has been established that in order to implement the task, it is necessary to take into account the unit costs for the movement of container flows between the nodes of the railway network, taking into account the capacity of the branches between the junction stations. The developed mathematical model for the redistribution of container traffic allows you to identify areas with limited throughput, develop options for optimization measures, and, as a result, achieve an economic effect by reducing the cost of transporting containerized cargo.

1 Introduction

In order to increase the transport system sustainability in the global economy, it is important to study the issues of accessibility, standardization of multimodal transportation and global transportation management. This will optimize the logistics processes for the delivery of goods and increase the level of transportation process controllability. As a result, this will make it possible to develop optimal options for promoting traffic flows [1].

In the current conditions of the global transport system operation, the main ways of transporting goods are rail and water container transportation. This type of transportation is recognized as the most efficient, economical and environmentally friendly [2].

The globalization processes involve minimizing the size of stocks in a dynamically changing demand. This condition increases the requirements for reducing the delivery time of goods when transporting between Asia and Europe. In the Eurasian space, the most

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promising segment of transit traffic is transportation along the PRC-EU-PRC routes [3]. In the EU, the Green Deal program is implementing a project to promote rail transport as the most environmentally friendly mode of transport. In recent years, there has been an increase in the number of shippers choosing options for a more sustainable future, i.e., makes a choice in favor of the railways. Therefore, most of the transit cargo traffic is container transportation by rail. Today, the land Eurasian transit corridors are already capable of providing acceptable terms and costs for the delivery of transit goods. The infrastructure basis of these corridors is the latitudinal railway lines of Kazakhstan, Russia and Belarus. A significant contribution to improving the efficiency of transit traffic across the Eurasian territory is made by the United Transport and Logistics Company – Eurasian Railway Alliance (UTLC ERA). The Alliance provides a service for organizing transit transportation of containers as part of regular container trains through the territories of Kazakhstan, Russia and Belarus. To date, the UTLC ERA container train covers a distance of 5430 km between the stations Dostyk (Kazakhstan) and Brest (Belarus) in 5.4 days at a speed of over 1000 km per day. The ideas of sustainable development are also actively supported by the railway administrations of Kazakhstan, Russia and Belarus.

According to the UTLC ERA [4], the volume of transit in the direction of the CNP-EU-PRC through Kazakhstan, Russia and Belarus has shown a stable growth in recent years (Fig. 1).

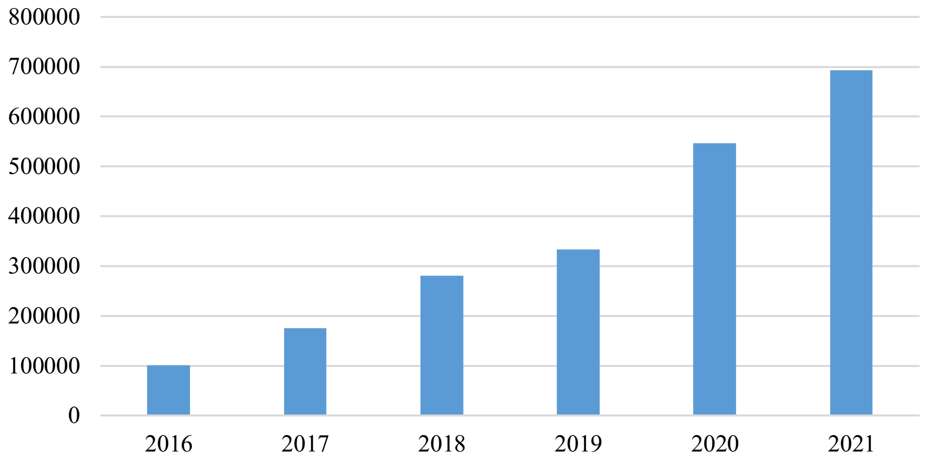

Fig. 1. Transportation volume of UTLC ERA JSC, thousand TEU.

The dynamics of traffic volumes reflects the growing role of container transportation by rail in trans-Eurasian trade. Moreover, the period of the global pandemic did not become a deterrent to the development of container transit traffic.

When organizing container transportation, time is an important factor determining the reliability of the railway container chain. For strict compliance with the standards for the timing of container trains on the way and at intermediate points, it is necessary to increase the level of management of the railway container transportation chain and optimize the routes of container trains [5, 6].

In the Republic of Kazakhstan, the growth in the volume of transit container traffic was positively affected by the implementation of a number of large-scale infrastructure projects. Among them, it is worth noting the construction of new railway lines and the modernization of sections with a shortage of capacity [7]. To date, processes aimed at increasing the speed of transit container trains have been improved, and flexible transit tariffs for container traffic have been introduced. In order to attract transit cargo, it is necessary to minimize the delivery time in transit directions and the cost of container transportation. This can be realized taking into account the intensive development of container terminals and promising logistics routes, as well as the automation of traffic management [8]. Thus, increasing the volume of containerized cargo transportation is a strategically important direction in the development of the transport system as a whole.

The analysis of research works allowed us to conclude that the issues of optimizing container transportation are still relevant in the world community [9, 10]. The papers substantiate that in conditions of limited investment resources, it is most expedient to carry out optimization measures. For example, forecasting the size of container trains and rationalizing the distribution of container traffic along the railway network are effective measures [11, 12]. The implementation of optimization measures will increase the volume of container traffic along international transport corridors.

2 Materials and Methods

The problem of distributing container train flows can be represented as a problem of a multi-commodity flow of minimum cost, which boils down to determining the optimal option for transporting goods in a network setting [13]. The multiproduct transport problem is of great practical importance and was used by a number of authors to optimize railway network routes [14–18].

This process can be modeled by constructing a multigraph $G(V, A)$, which consists of a pair of sets: V – vertices and A – directed arcs [19].

Each vertex $i \in V$ is characterized by the intensity value s_i. For example, if the intensity of a vertex is more than 0 ($s_i > 0$), then such a vertex is the source from where it's necessary to send the amount of cargo s_i. If the intensity of a vertex is less than 0 ($s_i < 0$), then such a vertex is a drain vertex to which it is required to deliver the amount of cargo $|s_i|$. If the intensity of a vertex is equal to 0 ($s_i = 0$), then such a vertex will be classified as neutral, in which the cargo flow will only be redistributed.

A direction is assigned to each arc $(i, j) \in A$, and the following parameters are set:

- c_{ij} - shipping costs from point i to point j;
- a_{ij} and b_{ij} - the smallest and largest sizes of cargo flows x_{ij} along the arc (i, j).

In the mathematical model of the task in question, the condition is set that the total size of sent goods is equal to the total size of arrivals, i.e., the balance of cargo flows is ensured on the network:

$$\sum_{i \in V} s_i = 0. \tag{1}$$

Solving the problem of a multi-commodity flow of minimum cost, it is necessary to minimize the objective function:

$$z(x)=\sum_{(i,j)\in A}\sum_{k=1}^{N}c_{ij}^{(k)}x_{ij}^{(k)}\to\min \tag{2}$$

Given the following constraints:

$$\sum_{j|(i,j)\in A}x_{ij}^{(k)}-\sum_{j|(j,i)\in A}x_{ji}^{(k)}=s_i^{(k)},\quad \forall\, i\in V,\quad (k=\overline{1,N}), \tag{3}$$

$$a_{ij}^{(k)}\le x_{ij}^{(k)}\le b_{ij}^{(k)},\quad \forall\,(i,j)\in A,\quad (k=\overline{1,N}), \tag{4}$$

$$a_{ij}\le\sum_{k=1}^{N}x_{ij}^{(k)}\le b_{ij},\quad \forall\,(i,j)\in A. \tag{5}$$

In the problem of a multi-commodity flow of minimum cost, for the vertices, the intensity values for each k -th type of cargo $(k=\overline{1,N})$ are additionally introduced, and for the arcs, the values of the costs of transporting the k -th type of cargo $c_{ij}^{(k)}$ and the values of the minimum and maximum sizes of cargo flows of the k -th type, $a_{ij}^{(k)}$, $b_{ij}^{(k)}$.

From expression (2), the total costs for the transportation of goods are determined using the multigraph $G(V,A)$, and their minimum value is selected. At each vertex of the multigraph $i\in V$, the flow must be stored taking into account its intensity. In expression (3), two sums are presented: the first is the outgoing flow of all types of goods, the second is the incoming flow of all types of goods. If the vertex i is the source, then the difference between the outgoing and incoming flows will be s_i. If the vertex i is a drain, then the difference between the incoming and outgoing flows will be $|s_i|$. If the vertex i is neutral, then the outgoing and incoming flows will be equal.

It follows from expression (4) that the following conditions must be met in each arc $(i,j)\in A$: the value of the flow of the k -th type of cargo $x_{ji}^{(k)}$ must be no less than $a_{ij}^{(k)}$ and no more than $b_{ij}^{(k)}$. In the problem under consideration, a value of the lower limit of the flow size is always non-negative $x_{ji}^{(k)}\ge 0$, thus $a_{ij}=0$ is set for each arc $(i,j)\in A$.

It follows from expression (5) that the total size of all goods transported along the arc (i,j) must be within certain limits from a_{ij} to b_{ij}, ($\forall\,(i,j)\in A$).

The problem is solved when the balance condition is met for all types of cargo $\sum_{i\in V}s_i^{(k)}=0$, $(k=\overline{1,N})$.

So, the task is to determine the minimum cost of transportation of cargo flows $x = \{x_{ij}^{(k)}\}$, $(i, j) \in A$, $(k = \overline{1, N})$, which pass along the arcs of the multigraph $G(V, A)$ and satisfy the constraints (3), (4) and (5). In the problem posed, the requirements for keeping the sizes of flows at the vertices and limiting the sizes of flows along the arcs of the multigraph must be strictly feasible.

When modeling traffic flows, research should be carried out in two areas – network loading and flow dynamics. In this case, it is necessary to distribute flows over the network strictly in accordance with its capacity [19, 20]. Modeling traffic flows using graph theory makes it possible to simultaneously solve a number of problems – justifying routes with redistribution of the traffic flow on the network, determining the feasibility of carrying out optimization measures, and obtaining additional profit [21, 22]. The stated optimization problem can be solved using linear programming methods, for example, the relaxation method [23]. The effectiveness of the relaxation method in solving network flow optimization problems lies in the high speed of selecting the best option at each iteration step.

3 Results

Transit container flows passing through the territory of Kazakhstan are visually represented on the multigraph $G(V, A)$, which has 19 vertices and 25 arcs (Fig. 2). The calculations were carried out for four routes of container trains on the railway network of the Republic of Kazakhstan:

- EU (European Union);
- CA (Central Asia);
- AP (Aktau Port);
- TRK (Turkmenistan).

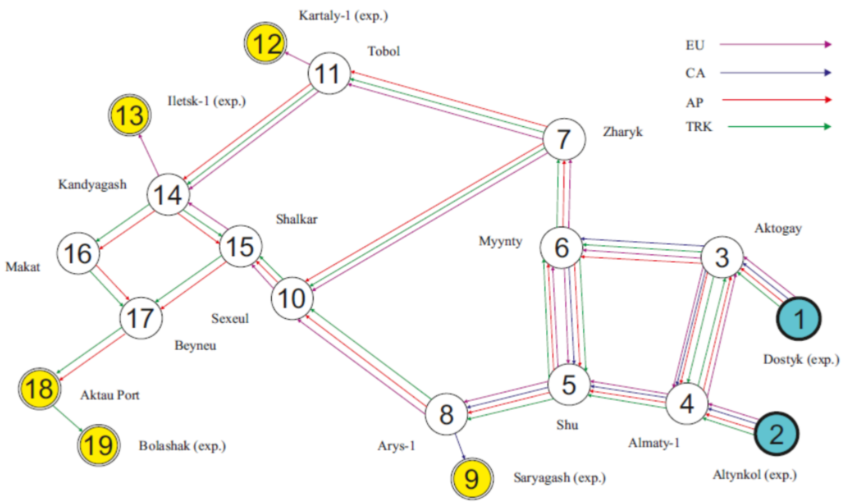

Fig. 2. Scheme of transit cargo flows on the railway network.

In the problem posed, container flows are considered as multi-product flows, therefore, on the multigraph, some vertices are connected by several equally directed arcs. There are also some arcs on the multigraph, along which container flows follow in both directions - these are arcs (3, 4) and (4, 3); (5, 6) and (6, 5); (14, 15) and (15, 14).

On the railway network, container flows originate at two points - these are vertices 1 (Dostyk) and 2 (Altynkol), which are sources on the multigraph. Container flows follow the network sections in four directions. Therefore, four container flows are sent from vertices 1 and 2. In the problem being solved about a multi-product flow of minimum cost, it should be assumed that container flows of different directions are heterogeneous and should not mix with each other along the way.

On the railway network, container flows are canceled at five points - these are vertices 12, 13, 9, 18 and 19. Vertices 12 and 13 are drains for the route of container trains following towards the EU, vertex 9 - towards CA, vertex 18 - towards AP, and vertex 19 - towards TRK.

The remaining vertices of the multigraph (vertices 3, 4, 5, 6, 7, 8, 10, 11, 14, 15, 16, and 17) are neutral, because the sizes of incoming and outgoing container flows do not change in them.

The initial data for solving the problem posed are the characteristics of vertices and arcs. Each vertex is characterized by what point it is (source, drain or neutral), direction of container flows and intensity s_i (container trains/day). The characteristics of the arcs include the length of the section l_{ij} (km), the cost of transporting 1 container train c_{ij} (conventional units), and the capacity of the section b_{ij} (trains/day).

The calculations were carried out for the incoming container traffic in the amount of 12 container trains, which is formed at vertices 1 and 2. Of the total number of trains, 7 container trains follow the EU route, 3 trains follow the CA route, 1 train follows the AP route, and 1 train follows the TRK route.

Of the 7 container trains following the EU route, 2 trains are redeemed at vertex 12, and 5 trains at vertex 13.

It is necessary to find the optimal variant of redistribution of transit container flows at the railway network site. To do this, it is necessary to modify the original multigraph. That is, an additional vertex 0 should be introduced, which is the source and has the intensity of $s_0 = 12$ container trains/day. After the introduction of an additional vertex 0, vertices 1 and 2 will be neutral. After introducing an additional vertex, it is necessary to introduce additional arcs (0, 1) and (0, 2) with the capacity of $b_{01} = b_{02} = 12$ trains, and the costs of transportation along these arcs are taken equal to $c_{01} = c_{02} = 0$. Thus, transportation along arcs (0, 1) and (0, 2) will not be included in the objective function (2). This multigraph transformation allows us to automatically obtain the optimal sizes of container flows through vertices 1 and 2.

The results of solving the problem of a multi-product flow of minimum cost showed that the minimum value of unit costs for the movement of container flows along the sections of the railway range was obtained with the following distribution of 12 container trains – 7 trains will go through vertex 1, 5 trains will go through vertex 2. The calculation results can also be used to determine the values of multi-product flows through all arcs $(i, j) \in A$. Table 1 shows the multi-product flow values for $s_0 = 12$ container trains.

Table 1. Multi-product flow values for $s_0 = 12$ container trains.

Arc no.	Arc designation, (i, j)	Multi-product flow EU+CA+AP+TRK through the arc (i, j)	Total flow through the arc (i, j), x_{ij} (c.t./day)	Capacity, b_{ij} (c.t./day.)
1	(1, 3)	7+0+0+0	7	9
2	(2, 4)	0+3+1+1	5	5
3	(3, 4)	1+0+0+0	1	10
4	(4, 3)	-	0	10
5	(3, 6)	6+0+0+0	6	6
6	(4, 5)	1+3+1+1	6	71
7	(5, 6)	-	0	71
8	(6, 5)	-	0	71
9	(5, 8)	1+3+1+1	6	71
10	(6, 7)	6+0+0+0	6	71
11	(7, 10)	-	0	3
12	(8, 9)	0+3+0+0	3	71
13	(7, 11)	6+0+0+0	6	71
14	(8, 10)	1+0+1+1	3	7
15	(11, 12)	2+0+0+0	2	71
16	(10, 15)	1+0+1+1	3	71
17	(11, 14)	4+0+0+0	4	4
18	(14, 13)	5+0+0+0	5	10
19	(14, 15)	-	0	71
20	(15, 14)	1+0+0+0	1	71
21	(14, 16)	-	0	4
22	(15, 17)	0+0+1+1	2	3
23	(16, 17)	-	0	10
24	(17, 18)	0+0+1+1	2	5
25	(18, 19)	0+0+0+1	1	3

Knowing the values of multi-product flows, one can also determine the number and routes of container trains. So, in the considered problem for the flow of 12 container trains, all 7 container trains that are formed at vertex 1 (Dostyk) follow towards the EU. At vertex 2 (Altynkol), 5 trains are formed, of which 3 trains follow towards CA, 1 train – towards AP, and 1 train – towards TRK.

The arcs emerging from vertices 1 (Dostyk) and 2 (Altynkol) have the maximum capacity of $s_0 = 14$ c.t./day, i.e., are equal to $b_{1,3} = 9$ and $b_{2,4} = 5$, respectively. We assume that the intensity of drains has the following values: $s_{12} = -2$ (Kartaly 1), $s_{13} = -6$ (Iletsk 1), $s_9 = -3$ (Saryagash), $s_{18} = -2$ (Aktau Port), $s_{19} = -1$ (Bolashak).

Table 2 shows the multi-product flow values for $s_0 = 14$ container trains.

Table 2. Multi-product flow values for $s_0 = 14$ container trains.

Arc no.	Arc designation, (i, j)	Multi-product flow EU+CA+AP+TRK through the arc (i, j)	Total flow through the arc (i, j), x_{ij} (c.t./day)	Capacity, b_{ij} (c.t./day.)
1	(1, 3)	8+0+0+1	9	9

2	(2, 4)	0+3+2+0	5	5
3	(3, 4)	2+0+0+1	3	10
4	(4, 3)	-	0	10
5	(3, 6)	6+0+0+0	6	6
6	(4, 5)	2+3+2+1	8	71
7	(5, 6)	-	0	71
8	(6, 5)	-	0	71
9	(5, 8)	2+3+2+1	8	71
10	(6, 7)	6+0+0+0	6	71
11	(7, 10)	-	0	3
12	(8, 9)	0+3+0+0	3	71
13	(7, 11)	6+0+0+0	6	71
14	(8, 10)	2+0+2+1	5	7
15	(11, 12)	2+0+0+0	2	71
16	(10, 15)	2+0+2+1	5	71
17	(11, 14)	4+0+0+0	4	4
18	(14, 13)	6+0+0+0	6	10
19	(14, 15)	-	0	71
20	(15, 14)	2+0+0+0	2	71
21	(14, 16)	-	0	4
22	(15, 17)	0+0+2+1	3	3
23	(16, 17)	-	0	10
24	(17, 18)	0+0+2+1	3	5
25	(18, 19)	0+0+0+1	1	3

For the case under consideration, the following redistribution of the input flow $s_0 = 14$ c.t./day was obtained between vertices 1 and 2: $s_0 = s_1 + s_2 = 9 + 5 = 14$, i.e., 9 container trains pass through Dostyk and 5 container trains through Altynkol. At the same time, the number and directions of formed container trains in the terminals can be as follows: 9 container trains leave Dostyk, including 8 c.t. – towards the EU and 1 c.t. - to TRK, and 5 container trains leave Altynkol, 3 of them – towards CA and 2 – to AP.

Analyzing the results obtained (table 2), it can be seen that at the value of the flow $s_0 = 14$ c.t./day, there are arcs in which the flow x_{ij} is equal to the arc capacity b_{ij}. Such arcs are called saturated. In saturated arcs, there is no reserve capacity, so it is impossible to increase the size of train traffic on them. Identification of saturated arcs on the railway network is of great practical importance. These are ‘bottlenecks’ in the existing railway network that hinder the implementation of plans to increase container traffic through the territory of the Republic of Kazakhstan.

For example, if you do not change the value of the flow, but increase the capacity of saturated arcs by 2 times, then it is possible to significantly reduce the total transport costs.

Calculations have shown that with the existing capacity of saturated arcs, the minimum cost of flow transportation costs in the amount of 14 c.t./day is 257,930,928 tenge (644,827 USD). With an increase in the capacity of saturated arcs, the minimum transport costs for the transportation of the flow in the amount of 14 c.t./day amount to 243,396,270 tenge (608490 USD). Thus, the increase in the capacity of saturated arcs allows achieving savings of about 14.5 million tenge (about 36,000 USD) per day when transporting the flow of $s_0 = 14$ c.t. per day.

It should be noted that an increase in the capacity of bottlenecks will increase the flow values s_0, while the size of the resulting savings will increase accordingly.

4 Discussion

The proposed method for solving the optimization problem makes it possible to distribute transit container flows on the railway network with minimal costs for organizing their routes.

To solve the problem, a model of cargo flows on the network was developed - a multigraph. The multigraph shows the points of origin and extinction of cargo flows, the directions of cargo flows. On the multigraph, cargo flows following different routes are represented by arcs of different colors. The considered transport problem was presented as a problem of multicommodity flows of minimum cost. The multi-product transport problem is reduced to determining the maximum size of heterogeneous cargo flows at minimum transportation costs.

One of the most effective iterative methods, the relaxation method, was used to solve the multi-product transport problem. This method allows us to quickly determine the optimal variant of traffic flow distribution on the network under the conditions of conservation of flows when passing through the vertices and constraints on the capacity of arcs on the multigraph.

When solving the problem of the minimum cost flow, the minimum cost of transporting $s_0 = 12$ container trains/day was obtained with the following variant of container flow distribution: at vertex 1 (Dostyk), 7 trains destined for the EU are formed; at vertex 2 (Altynkol), 5 trains are completed, 3 of them – towards CA, 1 – to AP, and 1 – to TRK. With an input flow in the amount of $s_0 = 14$ container trains/day, the minimum transportation costs were obtained with the following option for distributing container flows: 9 trains are formed at vertex 1 (Dostyk), including 8 towards the EU and 1 to TRK; at vertex 2 (Altynkol), 5 trains are completed, 3 of them – towards CA and 2 – to AP.

The analysis of the obtained results for the input flow of $s_0 = 14$ container trains/day revealed the presence of saturated arcs, i.e., bottlenecks in the existing railway network where it is not possible to further increase the existing flow. It is possible to achieve daily savings by eliminating bottlenecks due to increasing the capacity of saturated arcs.

5 Conclusions

Thus, the developed mathematical model for the redistribution of container flows makes it possible to make optimal decisions when choosing a rational investment option for the development of transport infrastructure and increasing the capacity of network sections. The use of the proposed model makes it possible to identify areas with limited capacity in order to most effectively redistribute flows. These solutions will allow us to get the maximum possible economic effect by reducing transportation costs.

Due to the universality of the model, it can be easily used and adapted not only for the transport network of Kazakhstan, but also beyond its borders.

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