The analysis of asbestos-cement shell pile performance under off-center loading and forced torsion

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Abstract. The paper discusses the new design of a round pile with a relatively smoother surface, developed in an attempt to eliminate excessive negative friction forces. A theoretical study has been conducted into the performance of asbestos-cement shell pile exposed to off-center loading, which occurs in response to uneven deformations in the surrounding soil and seismic loads. The cross section of the upper eccentrically compressed part of a flexible asbestos-cement-pipe-concrete pile element is considered. When considering the stress-strain state of flexible eccentrically compressed asbestos-cement-pipe-concrete piles, the premise of compliance with the hypothesis of flat sections is accepted. With the predicted intensive development of loading friction forces, a method is proposed for preliminary turning these round piles around the axis to remove the main part of the adhesion between the side surface of the pile with the base soil. Theoretically, the stress-strain state of the indicated pile, which is formed during its torsion around the axis, was estimated. The pile is considered as a composite element consisting of a concrete cylinder and an asbestos-cement pipe. As a first approximation, we assume that there are no body forces, and the concrete core and the asbestos-cement shell are rigidly linked. According to the Saint-Venant principle, it can be considered that for a twisted rod at a sufficiently large distance from its ends, the stresses depend on the torque and do not depend on the method of distribution of the forces that give this moment over the end sections. Based on the basic equations of the theory of elasticity, it follows that in a pile of circular cross section, bounded by a concentrically round shell, the cross sections remain flat during torsion.

1 Introduction

The increasing scope of capital construction requires increased industrialization and reduced labor intensity, especially when it comes to engineering and installing the underground part of buildings and structures.

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One of the solutions is offered by the widespread use of pile foundations. The comparative analysis of different types of foundations installed in challenging soil conditions has shown that the majority of projects use pile foundations as the most cost-effective and technically feasible solution.


When pile foundations are installed in macroporous subsiding soils, there occur negative friction forces which often lead to unacceptable subsidence in piles. The engineering methods for solving this geotechnical challenge are presented in monographs by N. M. Glotov, A. A. Luga, K. S. Silin & K. S. Zavriev (1975), V. I. Krutov, A. S. Kovalev & V. A. Kovalev [4], among others.

One solution for eliminating the negative friction forces is offered by a round pile design [5] which is manufactured on-site in a leave-in-place form, the latter represented by a standard asbestos-cement pipe. The purpose of the asbestos-cement pipe is to form the outer protective shell and smooth the side surface of the pile.

2 Method for studying the operation of a pile with an asbestos-cement shell under eccentric loading

Exposed to unevenly spread deformation in the base and seismic loads, the round piles with asbestos-cement shell experience off-center loads of various strength and duration. Therefore, it is of interest to analyze the load-bearing capacity of a reinforced pile, when stresses in the most compressed section of asbestos-cement pipe and in the reinforced concrete reach their maximum, causing the reinforced concrete core and the asbestos-cement shell to develop microcracks (independent process) and change to plastic state.

Let us consider the cross-section of the upper eccentrically compressed part of a ductile asbestos-cement-pipe pile element. Let us assume that since force \( H \) which applies itself to the element, is far from its limit value, the stress curves in the asbestos-cement pipe and the reinforced concrete can be assumed to be triangular (Figure 1).

The following conditions apply to the cross-section under analysis:

\[
\sum N = 0; \tag{1}
\]

\[
\sum M = 0. \tag{2}
\]

From condition (1)
\[ N = 2r\delta\sigma_A \left( \int_{\alpha'}^{\pi} \frac{\cos\alpha' - \cos\beta'}{1 + \cos\alpha'} \, d\beta' - \int_{0}^{\alpha'} \frac{\cos\beta' - \cos\alpha'}{1 + \cos\alpha'} \, d\beta' \right) + \\
+ 2r\sigma_B r^2 \left( \int_{\alpha'}^{\pi} \frac{\cos\alpha' - \cos\beta'}{1 + \cos\alpha'} \, d\beta' - \int_{0}^{\alpha'} \frac{\cos\beta' - \cos\alpha'}{1 + \cos\alpha'} \, d\beta' \right) \]

where \( \sigma_A \) and \( \sigma_B \) = maximum stresses in the compressed zone of the asbestos-cement pipe and the reinforced concrete.

### 3 Results of the study of the operation of the considered pile under off-center loading

Let us solve equation (3):

\[ N = \frac{\pi r\cos\alpha'}{1 + \cos\alpha'} (2\delta\sigma_A + r\sigma_B). \]  

From condition (2)

\[ M = 2r^2 \delta\sigma_A \left( \int_{0}^{\alpha'} \frac{\cos\beta' - \cos\alpha'}{1 + \cos\alpha'} \, d\beta' - \right. \\
- \left. \int_{\alpha'}^{\pi} \frac{\cos\alpha' - \cos\beta'}{1 + \cos\alpha'} \, d\beta' \right) + \\
+ 2\sigma_B r^3 \left( \int_{0}^{\alpha'} \frac{\cos\beta' - \cos\alpha'}{1 + \cos\alpha'} \, d\beta' - \right. \\
- \left. \int_{\alpha'}^{\pi} \frac{\cos\alpha' - \cos\beta'}{1 + \cos\alpha'} \, \sin^2 \beta' \, d\beta' \right) \]  

Let us solve equation (5) and perform relevant transformations:

\[ M = \frac{\pi r^2}{1 + \cos\alpha'} (\delta\sigma_A + 0.5r\sigma_B). \]  

The following dependence is true for eccentrically compressed elements:

\[ I = N(e_0 + f), \]  

where \( e_0 \) and \( f \) are the eccentricity of the load and the distance from the center of the pile to the center of the load, respectively.
where $e_0$ is initial eccentricity of the external load applied to the pile; $f$ is deflection in the pile.

As follows from (4) and (5):

$$e_0 + f = \frac{M}{N} = \frac{r}{2\cos \alpha}.$$  \hspace{1cm} (8)

Here, $\alpha$ is half of the central angle limiting the compressed zone of the pile (Figure 2).

In our analysis of the stress-strain state of ductile eccentrically compressed asbestos-cement-pipe piles, we assume that the conditions of plain-sections hypothesis are met. We further assume that the ductility of pile follows a sinusoid. In this case, the deflection in the middle of the pile equals

$$f = \frac{\ell^2}{\pi^2 \rho},$$  \hspace{1cm} (9)

where $\ell$ = length of pile; $\rho$ = curvature radius of the pile.

It is known that

$$\frac{1}{\rho} = \frac{M}{B},$$  \hspace{1cm} (10)
where \( B = \) cross-sectional stiffness of the pile.

After plugging \( M \) (7) into equation (10) and \( 1/\rho \) (10) into (9), we obtain the expression for determining the deflection:

\[
f = \frac{Ni^2e_0}{Bn^2 - Ni^2},
\]

(11)

\[
B = E_AI_d^A,
\]

(12)

where \( E_A \) = elastic modulus of the asbestos cement; \( I_d^A \) = moment of the pile cross-section inertia applied to asbestos-cement pipe.

Once the deflection values are determined, it is possible to calculate the stresses in the asbestos-cement pipe and in the reinforced concrete of the compressed section; \( \cos \alpha' \) can be calculated with formula (8).

If the stress diagrams in the reinforced concrete and the asbestos-cement pipe are assumed to be triangular for the limit state of the ductile pile, and the stresses in the most compressed section of the asbestos-cement pipe and the reinforced concrete are assumed to equal \( \alpha_{BH}R_A \) and \( \beta_{BH}R_{1D} \), then the formula for calculating the bearing capacity of the eccentrically loaded (compressed) asbestos-cement-pipe piles will be as follows:

\[
N = \frac{\pi r^2}{2(e_0 + f + r)} (2\alpha_{BH} \delta R_A + \beta_{BH} R_{1D}),
\]

(13)

where \( \alpha_{BH} \) and \( \beta_{BH} \) = coefficients for combined performance of the reinforced concrete and the asbestos-cement pipe in the asbestos-cement-pipe pile.

The above formulas are suitable if the stress diagram in the cross-section of the pile is two-digit, i.e., according to expression (8), at \( \cos \alpha' = 1 \):

\[
2e_0 \geq r.
\]

(14)

When eccentricities are low, it is assumed that the compressive stress diagram in the reinforced concrete and the asbestos-cement pipe is unambiguous and has the form of a trapezoid. At the same time, the following dependence is true

\[
N = N_A + N_B,
\]

(15)

where \( H_A \) is force communicated to the asbestos-cement pipe; \( H_B \) is force communicated to the reinforced concrete.

\( H_A \) and \( H_B \) can be calculated as follows:

\[
\sigma_A = N_A \left( \frac{1}{2\pi r \delta} + \frac{e_0 + f}{w_A} \right);
\]

(16)

\[
\sigma_B = N_B \left( \frac{1}{\pi r^2} + \frac{e_0 + f}{w_B} \right),
\]

(17)

where \( \sigma_A \) and \( \sigma_B \) are maximum stress values in the asbestos-cement pipe and the reinforced concrete; \( w_A, w_B \) are moment resistance in the asbestos-cement and the reinforced concrete.

Let us plug \( N_A \) (16) and \( N_B \) (17) into (15) and perform relevant transformations:

\[
N = \pi r \left[ \frac{2\delta \sigma_A}{1 + 2\pi r^2 (e_0 + f)} + \frac{2r \sigma_B}{1 + 2\pi r^2 (e_0 + f)} \right].
\]

(18)

The deflections can thus be calculated as follows:
where \( w_A \) = moment resistance for the asbestos-cement; \( \partial \) is inner diameter of the asbestos-cement-pipe pile.

Let us plug (19) into (9):

\[
f = \frac{2N_e \ell^2}{\pi^2 w_A^2 \ell A d - 2N_0 \ell^2}.
\] (20)

In the limit state at \( \sigma_A = \alpha_{BH} R_A \) and \( \sigma_B = \alpha_{BH} R_{ID} \),

\[
N = \pi r \left[ \frac{2\delta a_{BH} R_A w_A}{w_A + 2\pi \delta (e_0 + f)} + \frac{r b_{BH} R_{ID} w_B}{w_B + \pi r^2 (e_0 + f)} \right].
\] (21)

Thus, the boundary condition between the case of large and small eccentricities can be represented as:

\[
e_0 \geq 0,25D.
\] (22)

where \( D \) is loaded diameter of the asbestos-cement-pipe pile.

## 4 Method for examining a pile with an asbestos-cement sheath under forced torsion

With an assumption that the loading friction forces will be growing in intensity, a method is proposed for turning these round piles around their axes to remove the main bulk of friction between the lateral surface of the pile and the foundation soil.

In some cases, as a way to remove the negative friction forces, some of the piles in an installed foundation are pulled out to be driven back to the design depth. Since in our case the pile has a circular cross-section, for the negative friction forces to be guaranteeingly removed, it is sufficient to turn the pile, using a special rotating device, at a certain shear angle between the asbestos-cement surface and foundation soil; the measure of the angle should be determined in laboratory conditions using a dedicated shear device. The stress-strain state, which occurs in the pile in question while rotating around its axis, has been considered by us only theoretically. The pile is considered as a composite element consisting of a concrete cylinder and an asbestos-cement pipe.

As a first approximation, we assume that there are no volumetric forces and that the concrete core and the asbestos-cement shell are rigidly coupled. According to Saint-Venant principle, it can be assumed that the stresses occurring in a twisted rod at a sufficiently large distance from its ends, depend on torque \( M \) and do not depend on the way these forces, that produce this moment, are distributed over the end sections. Any law that governs the distribution of these forces across the end sections of the rod causes only local stresses – at the points that are farther than the linear dimensions of the cross section of the rod.

When torsion occurs, all sections rotate around the axis at a certain angle \( \varepsilon \). It is assumed that until the moment when the lateral surface of the pile breaks away from the surrounding soil, the pile toe remains stationary. In this case, angle \( \varepsilon \) is proportional to distance \( z \) between the section under analysis and the toe:

\[
\varepsilon = \gamma \cdot z,
\] (23)

where \( \gamma = \) relative angle of torsion.
5 Results of the study of the considered pile under forced torsion

Let us assume that the sections do not remain flat, but bend in one and the same manner. Then, displacement components are expressed as:

\[ u = -\gamma z y; \ \theta = \gamma z y; \ \omega = \gamma \phi(x, y), \]  

where \( \phi(x, y) \) = some torsion function of \( x \) and \( y \) to be determined.

The stresses in each region will be as follows:

\[ \tau_{Hzx} = \gamma G_k \left( \frac{\partial \phi}{\partial x} - y \right); \tau_{Hzz} = \gamma G_k \left( \frac{\partial \phi}{\partial y} + x \right). \]  

where \( G_k \) is shear modulus within a given region; \( k \) is ordinal index of the region under analysis.

Function \( \phi \) must be a harmonic function of variables \( x \) and \( y \). Since \( \phi \) does not depend on \( z \), it is sufficient to consider one normal cross-section of the element.

Let us assume the following boundary:

1) the lateral surface of the pile is free from external stresses:

\[ \tau_{Hzx} \cos(n, x) + \tau_{Hzz} \cos(n, y) = 0; \]  

2) the forces acting on the surfaces between the concrete and the asbestos cement have equal magnitude and are oppositely directed:

\[ \left[ \tau_{Hzx} \cos(n, x) + \tau_{Hzz} \cos(n, y) \right]_N = \left[ \tau_{Hzx} \cos(n, x) + \tau_{Hzz} \cos(n, y) \right]_K, \]  

where \( n \) is normal to the corresponding contour; \( N, K \) are indices that indicate belonging to different regions;

3) the displacement components \( u, \omega, \omega \) remain continuous when passing through the interface (strong coupling conditions). This condition reduces to the requirement that torsion function \( \phi \) remains continuous when moving from one region to another.

The moment of the external stresses applied to the upper base can be determined from the following equation:

\[ M = \iint_S (x \tau_{Hzz} - y \tau_{Hzx}) \, dxdy = \gamma G \iint_S \left( x^2 + y^2 + x \frac{\partial \phi}{\partial y} - y \frac{\partial \phi}{\partial x} \right) \, dxdy, \]  

\[ M = \gamma D, \]  

\[ D = \sum_{k=1}^2 \iint_S G_k \left( x^2 + y^2 + x \frac{\partial \phi}{\partial y} - x \frac{\partial \phi}{\partial x} \right) \, dxdy. \]  

The boundary condition (26) can be written as:

\[ \left( \frac{\partial \phi}{\partial x} - y \right) \cos(n, x) + \left( \frac{\partial \phi}{\partial y} + x \right) \cos(n, y) = 0, \]  

(31)
from which

$$\frac{\partial \phi}{\partial x} \cos(n,x) + \frac{\partial \phi}{\partial y} \cos(n,y) = \frac{\partial \phi}{\partial n} = cos(n,x) - x \cos(n,y). \tag{32}$$

In this case, when the origin of coordinates is set at the center of the circles represented by the inner surface (radius $r_1$) and the outer surface (radius $r_2$) of the asbestos-cement pipe, condition (26) will take the form:

$$y \cos(n,x) - x \cos(n,y) = 0, \tag{33}$$

Hence, $\partial \phi / \partial n = 0$ along on the entire boundary, and $\phi = \text{const}=0$.

The displacements and the stresses will be expressed by formulas:

$$u = -\gamma z y; \quad v = \gamma z x; \quad \omega = 0;$$
$$\tau_{kxz} = -\gamma G_K y; \quad \tau_{kxy} = -\gamma G_K x \tag{34}$$

The torsional stiffness of the composite pile is determined from the following equation:

$$D = \sum_{k=1}^{2} \iint_S G_K (x^2 + y^2) \, dx \, dy = G_S J_{SC} + (G_c - G_S) J_c, \tag{35}$$

where $J_{SC}$ = polar moment of inertia in the composite pile relative to the center; $J_c$ is polar moment of inertia in the concrete core of the pile relative to the center; $G_c$ is shear modulus of concrete; $G_S$ is shear modulus of asbestos cement.

The polar moments of inertia in the pile and its concrete core relative to the center, are determined from formulas:

$$J_{SC} = \frac{\pi r_1^4}{4}; \tag{36}$$
$$J_{SC} = \frac{\pi r_2^4}{4}. \tag{37}$$

### 6 Discussion of the obtained results

It follows from the basic equations of the theory of elasticity that in a round pile with a concentrically circular shell, the cross-sections remain flat during torsion.

The above conclusion is quite satisfactory to us. It substantiates our method for eliminating the negative friction forces, which, compared to the above mentioned conventional method, appears less energy-intensive as these are only the friction forces occurring between the lateral surface of the pile and the surrounding soil that are being overcome.

### 7 Conclusions

1. When pile foundations are installed in macroporous soils, there occur negative friction forces which often lead to unacceptable subsidence in piles.

2. A new design of a round pile has been developed, that uses the asbestos-cement shell with a relatively smoother surface.
3. A theoretical study has been conducted into the performance asbestos-cement shell pile exposed to off-center loading, which occurs in response to uneven deformations in the surrounding soil and seismic loads.

4. A method for eliminating negative friction forces is developed and analyzed, that uses forced torsion for twisting the pile around its axis and is characterized by lower energy consumption.

References


2. R. Rajapakse, Pile design and construction rules of thumb (2016)

