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Acoustic Splitter Waves Based on Ramified System Made of Waveguides

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Abstract. In this paper, we studied the propagation of acoustic waves in an acoustic ramified system. Our proposed system contains an input waveguide of length d_0 and three output lines (three channels), each output line contains a semi-infinite waveguide. The theoretical analysis is based on the Transfer Matrix Method (TMM), which allows us to calculate the three transmission rates T_1, T_2, T_3 and the reflection rate R. We demonstrate that our proposed three-output channels system can be used to design a multifunctional device that functions as an amplitude splitter: an incident sound wave is splited to three output channels. This system is capable of achieving various wave-guiding characteristics with perfect channels transmissions.

1 Introduction

Acoustic is the science of sound and factors related to hearing. It can be divided into various subfields, including physical acoustics, applied acoustics, architectural acoustics, music acoustics, physiological acoustics and psychological acoustics. Physical acoustic studies audible airborne sound waves, infrasound and ultrasound. It examines the propagation and absorption of all frequencies in the atmosphere and in gases, liquids and solids. In gases, only longitudinal waves are important, while in liquids and solids, the transverse waves and surface waves occur.

Several acoustic devices exist, such as acoustic couplers [1], demultiplexers [2], filters [3], and waveguides, which are often used to develop devices with many industrial applications. For example, the real time signal processing in radars or in TV sets use Rayleigh acoustic waves propagating at the surface of a piezoelectric solid [4].

A manipulation of audio frequency acoustic waves [5] is demonstrated experimentally and numerically by ring resonator structures. Two ring resonator systems are studied: a single ring structure between two parallel waveguides that acts as an add-drop filter and a simple single ring structure that acts as a comb/notch filter. Experiments are conducted in linear waveguides using the impulse response

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method. Numerical simulations were performed using COMSOL software [5]. Acoustic filters have been extensively studied over the past century [6]. These systems can act as low-pass, high-pass or band-pass filters [7].
Many works focus on the design of multichannel devices (demultiplexer) able of guiding waves from a specific input line to different output lines, the application of wave splitters has always been a great challenge for many physical applications [8]. Furthermore, Acoustic demultiplexers have been realized either through add-drop filters [9] or multiport channels. Recently, a three-port acoustic system has been used to study the control of absorption below the incident wavelength [10] using resonators on each channel. A. Mouadili et al have studied an acoustic Y demultiplexer to obtain Fano and AIT type resonances, this demultiplexer is based on two different configurations (cross and U-shaped structures) [11]. Moreover, I. El Kadmiri et al. studied a Y-shaped phononic demultiplexer based on Fano and acoustically induced transparency resonances in a one-dimensional asymmetric/symmetric loop structure. The demultiplexer is a Y-shaped waveguide with one input line and two output lines. The output line contains a segment and two balanced/unbalanced loops. This system can used as high performance acoustic demultiplexer and noise filtering. The position and the quality factor of resonances depend on to the geometrical parameters [12].

The objective of our recent work is to study an acoustic ramified system (1D) with several output channels in order to split the amplitude of the incident wave on the three output channels. The proposed ramified acoustic system consists of one-dimensional waveguides with one input line and three output transmission lines; each output line contains a semi-infinite acoustic waveguide. We use the transfer matrix method to calculate the transmission and reflection rates. In all our calculations, we assume that the cross sections of the guides are small compared to its linear dimension (length).

2 Theoretical formalism: Transfer Matrix Method

The transfer matrix method (TMM) consists in writing relations between output and input acoustic variables of simple geometries, the transfer matrix method has many advantages. It is a very useful algorithm, very suitable for the calculation of reflectivity and transmission in multilayer structures. This method can be applied to lossy and absorbing materials, and it can also handle any number of layers in a multilayer structure. Moreover, these layers can be ordered in any way and it is not necessary to be periodic [13]. Hence, A. Ezzarfi et al. present the theoretical study to determine the transmission coefficient for multi-quantum wells, composed of two periodic semiconductor materials, and containing a material defect layer. The transfer matrix (for each unit cell and for the system with N cells) is utilized then to compute the electronic band structure. They showed that the thickness and concentration of the defect layer have an impact on the electronic states localized in the band gap [14]. They also studied the propagation of electronic wave in one-dimensional multi-quantum well composed of alternating layers of barriers and wells using the transfer matrix (TM) method, they showed that as the barrier thickness increases, the energies of the corresponding Eigen modes shift to lower energies [15]. Moreover, the transfer matrix method (TMM) is classically used to predict acoustic properties, the transmission was determined using the Transfer Matrix Method (TMM) algorithm, the pressure P can be described in one-dimensional direction (following x for example) as:

$$\Delta P - \frac{1}{V2}\frac{\delta P}{\delta x} = 0 \qquad (1)$$

3 Theoretical analysis: Transmission and reflection rates of a ramified acoustic system

In this section, we consider the structure shown in Figure 1. This structure is composed of an input line of length d_0 and three semi-infinite guides in the output lines. The calculation of the transmission and reflection rates are made using the transfer matrix method.

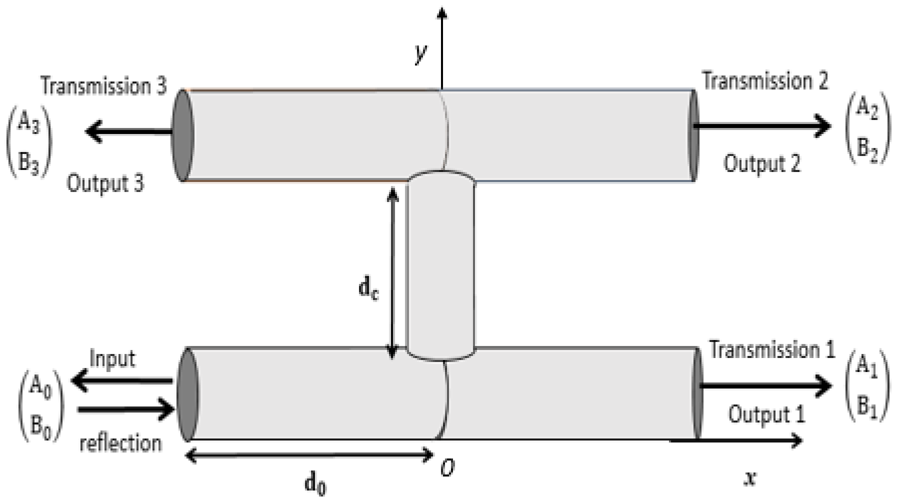

Fig. 1. Acoustic ramified system without resonators.

The transfer matrix of our proposed system allows to relate the amplitudes of the incoming wave and the amplitudes of the outgoing wave is:

$$\begin{pmatrix}A_0\\B_0\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26}\end{pmatrix}\begin{pmatrix}A_1\\B_1\\A_2\\B_2\\A_3\\B_3\end{pmatrix} \quad (2)$$

The a_{ij} are the elements of the matrix obtained by applying Hook's laws as well as the continuity conditions on the expressions of pressure and velocity for an acoustic wave.

$$\begin{cases} P_0(x) = A_0 e^{ik(x+d_0)} + B_0 e^{-ik(x+d_0)} & -d_0 \leq x \leq 0 \\ P_c(y) = A_c e^{iky} + B_c\, e^{-iky} & 0 \leq y \leq d_c \\ P_1(x) = A_1 e^{ikx} + B_1 e^{-ikx} & x \geq 0 \\ P_2(x) = A_2 e^{ikx} + B_2 e^{-ikx} & x \geq 0 \\ P_3(x) = A_3 e^{ikx} + B_3 e^{-ikx} & x \leq 0 \end{cases}$$

$$\begin{cases} V_0(x) = -\frac{1}{z_0}\left(A_0 e^{ik(x+d_0)} - B_0 e^{-ik(x+d_0)}\right) & -d_0 \leq x \leq 0 \\ V_C(y) = -\frac{1}{z_c}\left(A_c e^{iky} - B_c\, e^{-iky}\right) & 0 \leq y \leq d_c \\ V_1(x) = -\frac{1}{z_1}\left(A_1 e^{ik(x+d_0)} - B_1 e^{-ik(x+d_0)}\right) & x \geq 0 \\ V_2(x) = -\frac{1}{z_2}\left(A_2 e^{ikx} - B_2 e^{-ikx}\right) & x \geq 0 \\ V_3(x) = -\frac{1}{z_3}\left(A_3 e^{ikx} - B_3 e^{-ikx}\right) & x \leq 0 \end{cases}$$

A_i (i=0, 1, 2, 3) are the amplitudes of the progressive acoustic waves in the 4 channels.
B_i (i=0, 1, 2, 3) are the amplitudes of the regressive acoustic waves in the 4 channels with $z_i = \frac{\rho c}{s}$, ρ and c are the density and speed of sound in air. It can be shown analytically that the transmission 2 through the output channel 2 and the transmission 3 through the channel 3 are equal.
The transmission and reflection coefficients can be calculated by the following relations:

$$t_1 = \frac{A_1}{A_0}\Big|_{\substack{B_1=0\\B_2=0\\B_3=0}},\ t_2 = \frac{A_2}{A_0}\Big|_{\substack{B_1=0\\B_2=0\\B_3=0}},\ t_3 = \frac{A_3}{A_0}\Big|_{\substack{B_1=0\\B_2=0\\B_3=0}},\ r = \frac{B_0}{A_0}\Big|_{\substack{B_1=0\\B_2=0\\B_3=0}} \quad (3)$$

With $t_2 = t_3$.

After a calculation, we can show that:

$$t_1 = \frac{a_{23} + a_{25} + (a_{23} + a_{25})e^{2ikd_0}}{(a_{13} + a_{15})(e^{ikd_0} - a_{21}) + a_{11}(a_{23} + a_{25})} \quad (4)$$

$$t_2 = \frac{\left(a_{21} - e^{ikd_0}\right) + a_{11}e^{ikd_0}}{(a_{23} + a_{25})(a_{21} - e^{ikd_0}) - a_{11}(a_{23} + a_{25})} \quad (5)$$

$$r = [\, t_1 - e^{ikd_0}]\, e^{ikd_0} \quad (6)$$

The transmission rates in the output lines are given by:

$$T_1 = |t_1|^2 \quad T_2 = |t_2|^2 \quad T_3 = |t_3|^2 \quad (7)$$

The reflection rate in the input line is given by:

$$R = |r|^2 \quad (8)$$

The reflection and transmission rates satisfy the energy conservation:

$$R + T_1 + T_2 + T_3 = 1 \quad (9)$$

4 Results and discussions

4.1 Transmission spectrum of a ramified system

In this part, we show in Figure 2 the transmission rate T_1 through the first output line (red color), the transmissions T_2 and T_3 through the second and third output lines (blue color) and the reflection rate R in the input line (black color) as a function of the reduced frequency Ω for d_c= 1D and d_0 = 1D. The reduced frequency is given by Ω=2fD/c, with c represents the velocity of sound of the acoustic wave (c = 348 m/s) and f is the real frequency (s^{-1}). We notice that when the transmission rate T_1 in the first output line reaches 0.65, the transmission in the second and third output lines T_2 and T_3 and the reflection R in the input line take minimum values (T_2 =T_3=0.15 and R =0.05). There are the coincidence of transmission and reflection rates when the reduced frequencies Ω =0, 1, 2, 3, 4, 5.

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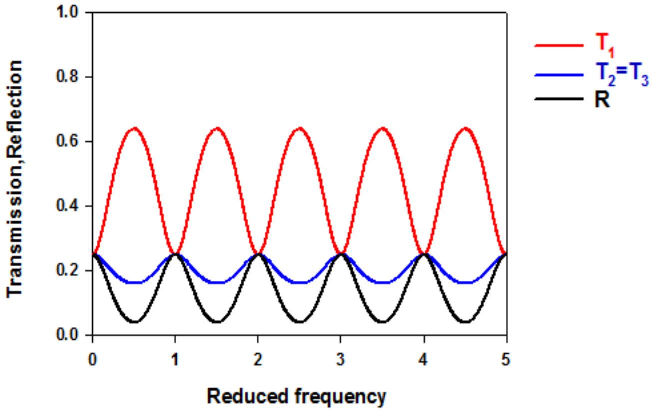

Fig. 2. Variation of the transmission and reflection rates of the ramified system as a function of reduced frequency.

4.2 Effect of the guide length d_c

In the case where d_c=1D and d_0=1D, we note the appearance of transmission peaks in the first channel for Ω=0.5, 1.5, 2.5, 3.5, 4.5 and the transmission peaks in the second channel for Ω =0, 1, 2,3,4,5. In the case where d_c=2D and d_0=1D, we notice that the number of modes increases by a multiple of 2, and we observe the appearance of transmission peaks in the first channel for Ω=0.25, 0.75, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75, 4.25, 4.75 and the transmission peaks in the second channel for Ω = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5,5. These results show that the modes can be tuned by appropriately choosing different lengths d_c constituting the system. We find that these modes are very sensitive to the variation of the length d_c.

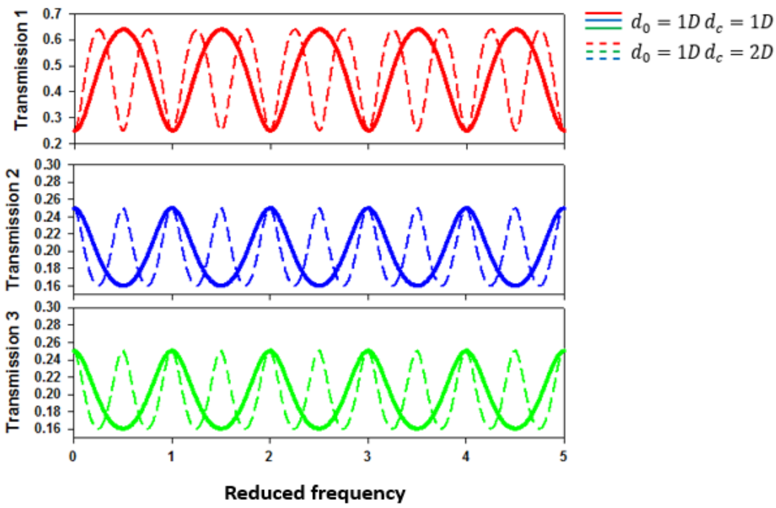

Fig. 3. Variation of transmission rates in the first output channel (red color), in the second output channel (blue color) and in the third output channel (green color) as a function of the reduced frequency.

4.3 Discrete modes of a ramified system

In this section, we are interested in the confined modes of a ramified system. We study the variation of the reduced frequency as a function of the guide length d_c, we keep d_0=1D .The red branches represent the maximum of T_1, the blue branches represent the maximum of T_2 and T_3. For each value of the guide length d_c, we obtain one or more discrete modes that correspond to a well-defined frequency (figure 4(a)) is established for d_c= 1D. The red branches appear from Ω = 0.5 but the blue branches appear from Ω = 0. If we increase the length of the guide d_c, we notice that the discrete modes move to lower frequencies as well as the number of discrete modes increases.

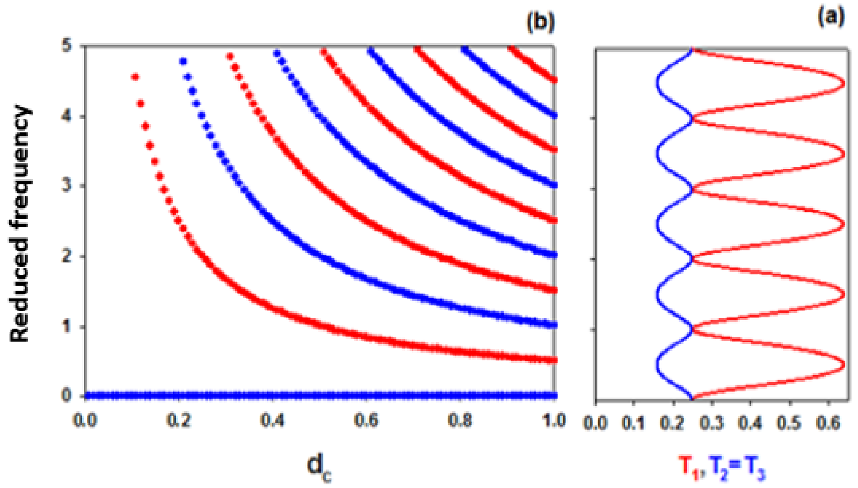

Fig. 4. (a) Variation of the reduced frequency as a function of the transmission rates for d_0 = 1D and d_c = 1D. (b) Variation of the reduced frequency as a function of the guide length dc.

5 Conclusion

In this paper, we have studied a ramified acoustic system consisting of an input line and three output channels; each transmission channel is composed of a semi-infinite waveguide. This three-channel acoustic ramified system can be used to design a multi-functional device that functions as a wave splitter for different frequencies returning from the input line and as an amplitude splitter: an incident sound wave is split to three output channels. This system is capable to various waveguide characteristics with a perfect transmission.

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